A Theory of Liquidity and Regulation of Financial Intermediation

Emmanuel Farhi, Mikhail Golosov, and Aleh Tsyvinski*

November 28, 2007

Abstract

This paper studies a Diamond-Dybvig model of financial intermediation in which agents receive unobservable liquidity shocks. If agents can engage in unobservable trades competitive markets provide no insurance across agents. We characterize the constrained efficient allocation in presence of trade and show that it can be implemented by imposing a liquidity floor on interemediaries. The optimal interest rate is lower than that achieved by markets as it reduces incentives to falsely claim liquidity shocks.

Keywords: Optimal Regulations, Financial Intermediation, Optimal Contracts, Market Failures, Mechanism Design.

1 Introduction

The role of financial intermediaries in providing liquidity is one of the central features of a modern financial system. Accordingly, the regulation of financial intermediaries is an important function of central banks and is a topic of frequent debates in the policy-making community. In this paper we answer several important questions. Can markets provide the correct amount of liquidity? What is a precise nature of market failure if such exists? Can a regulator design a simple policy to improve on the allocations provided by competitive markets alone?

We study a mechanism design model of financial intermediaries as providers of liquidity similar to Diamond and Dybvig (1983), Jacklin (1987), and Allen and Gale (2004). In this model, some agents receive liquidity shocks that make them value only early consumption. There are two informational frictions in the modes. The first friction is that the type of an agent is unobservable.

*Farhi: Harvard University and NBER; Golosov: MIT and NBER; Tsyvinski: Harvard and NBER. Golosov and Tsyvinski acknowledge support by the National Science Foundation. Tsyvinski thanks Ente *Luigi Einaudi* for hospitality. We thank Daron Acemoglu, Stefania Albanesi, Franklin Allen, Marios Angeletos, Ricardo Caballero, V.V. Chari, Ed Green, Christian Hellwig, Skander Van den Heuvel, Oleg Itskhoki, Bengt Holmstrom, Guido Lorenzoni, Chris Phelan, Bernard Salanié, Jean Tirole, Ivan Werning, and Ruilin Zhou for comments. We also thank the audiences at the Bank of Canada, Minneapolis Federal Reserve, Harvard, MIT, UT Austin, Wharton, and Society of Economic Dynamics for useful comments.

The second friction is that consumers can trade assets unobservably on a private market by engaging in hidden side trades.[1] Since the contribution of Allen (1985) and Jacklin (1987), the possibility of agents engaging in hidden side trades has been recognized as an important constraint on the provision of liquidity by financial intermediaries any eliminates redistribution of resources across agents in a competitive equilibrium.[2] This is in contrast to an environment without retrading in which it is optimal to allocate a higher present value to the agents affected by liquidity shocks. The reason for absence of redistribution in competitive markets is that financial intermediaries in competitive markets cannot affect the interest rate at which agents can trade on the private markets. Arbitrage leads to the interest on the private markets to be equal to the marginal rate of transformation.

This paper characterizes the constrained efficient allocations in presence of retrading. In contrast to the competitive markets, the social planner can indirectly manipulate the interest rate on the private markets by affecting the aggregate amount of resources available in various periods. Lowering interest rate on the private market relaxes incentive constraints of the optimal program. The intuition for this effect is as follows. The planner wants to allocate a higher present value of resources to the agent affected by liquidity shocks. Given such redistribution, an agent not affected by a liquidity shock has an incentive to pretend to be an early consumer and then save on the private markets. Lower interest rate reduces return on such deviations and allows the planner to redistribute resources across agents. We analytically characterize the optimal interest rate and show that the constrained efficient program with retrading coincides with the constrained efficient problem without retrading.

We then propose a simple regulation imposed on the financial intermediaries in a competitive market that implements the constrained efficient allocation. A regulator can impose a liquidity floor that stipulates the minimal amount of the investment in the short asset. This liquidity regulation by increasing the amount of first period assets drives down the interest rate on the private markets and mimics the behavior of the social planner.

In addition to a description of a theoretical mechanism that addresses critique of the financial intermediation literature that retrading puts significant limitation on redistribution, this paper also highlights some effects that may be interesting to the policymakers or regulators. The main economic intuition of the paper is that either a liquidity regulation that indirectly causes a lower interest rate or direct lowering of the interest rate leads to improvement in the provision of liquidity. There is a market failure in presence of unobservable liquidity needs and ability of agents to enter into transactions with other parties. Financial intermediaries without regulations hold too little liquid assets, and the market interest rate is too high to implement optimal insurance against liquidity shocks. Lowering interest rate allows the financial system to better screen those with

[1] A different interpretation of unobservability of consumption is non-exclusivity of contracts. It is difficult for an individual financial intermediary to preclude an agent to enter in additional risk sharing contracts with other intermediaries.

[2] The importance of access to credit markets as a constraint on the optimal program was also emphasized Chiappori, Macho, Rey, and Salanie (1994).

genuine liquidity needs from those who falsely pretend to be affected by liquidity shocks.

The final part of the paper extends the constrained efficient solution and implementation to a case of more general preferences. We show that the structure of Diamond-Dybvig preferences is somewhat special, and that constrained efficient allocations with and without retrading do not necessarily coincide. We then show that depending on the nature of the preferences and, therefore, on the direction of deviations, the optimal interest rate may be higher or lower than that on the competitive markets and that the optimal implementation may stipulate either a minimal or maximal amount of investment in the short asset.

2 Relationship to the literature

This paper builds on a large literature of risk sharing by in the presence of liquidity shocks (Diamond and Dybvig 1983; Jacklin 1987; Bhattacharya and Gale 1987; Hellwig 1994; Diamond 1997; Von Thadden 1999; Caballero and Krishnamurthy 2003; Allen and Gale 2003, 2004[3]). More generally, our paper fits in the literature of optimal allocations with unobservable taste shocks following Atkeson and Lucas (1992).

Our paper uses the mechanism design framework of an important paper by Allen and Gale (2004) to analyze the model of intermediation in the presence of private markets. Our results in the model with private markets differ significantly from their work. The result of Allen and Gale (2004) that an equilibrium is inefficient relies on exogenously imposed incompleteness of markets for trades among intermediaries when there are aggregate shocks. In the absence of incomplete markets for aggregate shocks or in the absence of aggregate shocks, Allen and Gale (2004) conclude that there is no role for regulation of liquidity or any other regulatory intervention. We show that a liquidity requirement can improve upon the competitive equilibrium by eliminating above described externality even when there are complete markets for aggregate shocks or when there are no aggregate shocks. The mechanism of how liquidity requirements affects interest rates on private markets and the characterization of the optimal liquidity adequacy requirement is new to the literature on the provision of liquidity by financial intermediaries. Moreover, we provide a theoretical characterization of the optimal liquidity adequacy requirement for a general specification of shocks.

Our paper shares a common goal with the work of Allen and Gale (2004) in studying whether laissez-faire markets provide too little or too much liquidity and whether a specific policy intervention that occurs at an aggregate level can be Pareto improving or even optimal. Both of the papers direct regulations at intermediaries rather than individual consumers. A government regulates intermediaries while intermediaries on their own solve incentive problems via direct interactions with consumers.

Holmstrom and Tirole (1998) provide a theory of liquidity in a model in which intermediaries have borrowing frictions. Similar to our paper they do not assume incomplete markets. In their

[3] For a survey of the literature see Freixas and Rochet (1997) and Gorton and Winton (2002).

model, a government has an advantage over private markets as it can enforce repayments of borrowed funds while the private lenders cannot. They show that availability of government provided liquidity leads to a Pareto improvement when there is aggregate uncertainty. The role of the government in our model is to correct an inefficiency arising because of an externality associated with private information and possibility of hidden trades. In our paper, in contrast with Holmstrom and Tirole (1998) and Allen and Gale (2004), a liquidity requirement improves upon a market allocation even when there is no aggregate uncertainty.

Our paper also differs conceptually from the seminal paper of Jacklin (1987). That paper compares a competitive equilibrium *with* private markets $\left(CE^3\right)$ to the social optimum *without* private market $\left(SP^2\right)$ which is, essentially, equivalent to the statement that prohibition of private markets leads to a Pareto improvement. In our paper, we find the optimal liquidity requirement and show that it implements the solution of the social planner's problem who is faced with both unobservable types and private markets $\left(SP^3\right)$ which, for this specification of preferences, coincides with SP^1 and SP^2. In contrast with Jacklin, there is no need to prohibit private markets to achieve superior or even unconstrained allocations. A regulator can impose a liquidity adequacy requirement that achieves such optimal allocations.

Lorenzoni (2006) considers a Diamond-Dybvig model of banking with financial markets. The focus of Lorenzoni (2006) is on the models of money and on implementation of the optimum and advantages of various policy interventions. In his model, a specification of technology for intertemporal transfer of resources allows him to consider tradeoffs of various policy interventions. Another paper that is related to our results in the Diamond-Dybvig setup is Caballero and Krishnamurthy (2003). They develop a model of an emerging market crisis in which there is a market for external borrowing and a domestic private market. The domestic market in their model is similar to the private market in our formulation. They show that the equilibrium coincides with the optimal allocation in the presence of private markets. They further show that a range of financial instruments including liquidity requirements and taxes on external borrowing can implement the optimal allocation *without* private markets that coincides with *the full information optimum.* In our general model, a competitive equilibrium with the optimal liquidity adequacy requirement is different from the competitive equilibrium without private markets and, therefore, is different from unconstrained "first-best" allocation. However, we show that in the case of the Diamond and Dybvig (1983) environment, the optimal liquidity regulation implements the unconstrained optimum.

While the focus of this paper is on the models of financial intermediation, we also contribute to the literature on optimal taxation in the presence of hidden trades[4]. In particular, Golosov and Tsyvinski (2006) study an optimal dynamic Mirrlees taxation with endogenous private markets. There are two main differences between our paper and their work. The first difference is conceptual. In Golosov and Tsyvinski (2006) as in most of the models of dynamic Mirrlees taxation (see,

[4] See, for example, Arnott and Stiglitz (1986), (1990), Greenwald and Stiglitz (1986), and Hammond (1987). Several recent papers such as Geanakoplos and Polemarchakis (2004) and Bisin, et. al. (2001) showed in very general settings that economies with asymmetric informations are inefficient and argued for Pareto-improving anonymous taxes.

e.g., Golosov, Kocherlakota, and Tsyvinski 2003 or a review in Golosov, Tsyvinski, and Werning 2006 and Kocherlakota 2006), private information (skill shocks) is dynamic and separable from consumption. The inefficiency of the competitive equilibrium in Golosov and Tsyvinski (2006) arises because of the dynamic nature of the private shocks. In our model, private information does not change stochastically over time, and the inefficiency arises due to non-separability of shocks and consumption. The second difference is in the extent of the results that we obtain. Golosov and Tsyvinski (2006) and Bisin et. al. (2001) are able to identify only the direction of a *local* policy change that leads to a Pareto improvement. We characterize the *optimal* allocation in the presence of private markets and show that the optimal liquidity regulation implements the optimum SP^3. We derive an analytical solution for the interest rate associated with the optimal liquidity requirement in terms of an easily interpretable wedge depending on the specification of preferences and distribution of shocks. Moreover, we provide a complete closed form solution for the optimum for two important examples. Studying optimal rather than locally improving interventions is not only interesting from the theoretical point of view. Optimal interventions may achieve a significant improvement in welfare compared to the competitive equilibrium. For example, we show that in the case of Diamond and Dybvig (1983) the optimal liquidity requirement implements the unconstrained optimum.

Related is a recent paper by Albanesi (2006) that studies a model of entrepreneurship and financial assets. The focus of that paper is on an implementation of the optimal program with observable consumption as a competitive equilibrium with taxes in which agents can trade multiple assets. She derives a general result on differential asset taxation in such models.

In Diamond (1997), as in our paper, the optimal allocation is different from autarky. His result relies on the assumption that some consumers are exogenously restricted from participating in private markets. Unlike that paper, in our model all consumers can participate in markets. An elegant paper by Bisin and Rampini (2004) justifies an institution of bankruptcy in a model of non-exclusive contracts. In their work, borrowers (entrepreneurs) have an access to secondary markets. A possibility of default on these secondary contracts worsens return to hidden borrowing and lending and yields a Pareto improvement.

One justification for reserve requirements is found in the existence of deposit insurance. The rationale given is usually as follows: deposit insurance encourages risk taking behavior of intermediaries (see, e.g., Merton 1977) which can be controlled by requiring intermediaries to hold adequate levels of liquidity. In this argument, existence of one potentially suboptimal policy, deposit insurance, justifies necessity of another policy - reserve requirements. Typically, this literature, with the exception of Hellman, Murdock and Stiglitz (1998, 2000), does not consider optimal policy in the absence of deposit insurance. This literature also does not pose a friction or a market failure and does not find an optimal policy that can be deposit insurance, reserve requirement, some combination of those, or maybe neither. Our results on the optimality of reserve requirements do not rely on the existence of any other exogenously given policy.

3 Model

We consider a standard mechanism design model of financial intermediation similar to Diamond and Dybvig (1983) and to Allen and Gale (2004). The economy lasts three periods ($t = 0, 1, 2$). There are two assets (technologies) in the model. The short asset is a storage technology that returns one unit of consumption good at $t + 1$ for each unit invested at t. Investment in the long asset has to be done at $t = 0$ to yield $\hat{R}$ units of the consumption good at $t = 2$.

The economy is populated by a continuum of measure one of ex-ante identical agents, or investors. Suppose there are two types of agents denoted by $\theta \in \{0, 1\}$. At $t = 0$, all individuals are (ex-ante) identical and receive an endowment e. At $t = 1$, each consumer gets a draw of his type. With probability π he is an agent of type $\theta = 0$, and with probability $(1 - \pi)$ he is an agent of type $\theta = 1$. The fraction of agents of each type is then π and $(1 - \pi)$, respectively. Preferences of an agent of type θ are given by

$$U(c_1, c_2, \theta) = (1 - \theta)u(c_1) + \theta \rho u(c_1 + c_2),$$

where u is twice continuously differentiable, increasing, strictly concave, and satisfies Inada conditions $u'(0) = +\infty$ and $u'(+\infty) = 0$. Also, we assume as in Diamond and Dybvig (1983) that the coefficient of relative risk aversion is everywhere greater than 1

$$\frac{-cu''(c)}{u'(c)} \geq 1 \text{ for all } c \geq 0,$$

and that and $1 > \rho > \hat{R}^{-1}$.

Agents of type $\theta = 0$ need to consume in the first period – they are affected by liquidity shocks. Agents of type $\theta = 1$ are indifferent between consuming in the first and the second period. We use these preferences throughout the main body of the paper, and in the the last section consider a more general class of preferences pointing to somewhat specific properties of the Diamond-Dybvig setup.

A key informational friction is that types of agents are private, i.e., observable only by the agent himself but not by others.

We denote by$\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0,1\}}$ an *allocation* of consumption across consumers. An allocation is *feasible* if it satisfies:

$$\pi \left[c_1(0) + \frac{c_2(0)}{\hat{R}} \right] + (1 - \pi) \left[c_1(1) + \frac{c_2(1)}{\hat{R}} \right] \leq e. \tag{1}$$

We do not impose a sequential service constraint so there are no bank runs in our model. We also restrict our attention to pure strategies and consider symmetric equilibria.

4 Benchmark environment without private markets

In this section, we define and characterize a benchmark economy in which the only friction is unobservability of types. In this environment, agents are allocated consumption allocations depending on their types. Agents cannot engage in any unobservable transaction, and their consumption is therefore observable.

We start by defining a constrained efficient program which we call problem SP^2 or a "second best" problem:

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\{0,1\}}} \pi u(c_1(0)) + (1-\pi)\rho u(c_1(1)+c_2(1)) \tag{2}$$

s.t.

$$\pi\left(c_1(0)+\frac{c_2(0)}{\hat{R}}\right)+(1-\pi)\left(c_1(1)+\frac{c_2(1)}{\hat{R}}\right)\leq e, \tag{3}$$

$$u(c_1(0))\geq u(c_1(1)), \tag{4}$$

$$u(c_1(1)+c_2(1))\geq u(c_1(0)+c_2(0)). \tag{5}$$

The planner maximizes the expected utility of an agent subject to the feasibility constraint (3) and to two incentive compatibility constraints. Constraint (4) ensures that an agent of type $\theta = 0$ does not want to pretend to be an agent of type $\theta = 1$. Constraint (5) ensures that an agent of type $\theta = 1$ does not want to pretend to be an agent of type $\theta = 2$.

We can also define an unconstrained optimum that we call SP^1 in which there is no private information – the "first best" program. That program differs from the problem SP^2 as the incentive compatibility constraints (4) and (5) are omitted.

As noted by Diamond and Dybvig (1983), the incentive compatibility constraints are not binding at the optimum of (2). In other words, solutions to problems SP^1 and SP^2 coincide. This allocation is then characterized by

$$c_2(0)=c_1(1)=0, \tag{6}$$

$$\rho\hat{R}u'(c_2(1))=u'(c_1(0)), \tag{7}$$

$$\pi c_1(0)+(1-\pi)\frac{c_2(1)}{\hat{R}}=e. \tag{8}$$

To verify that this allocation satisfies incentive compatibility, we need only check that $c_2(1)\geq c_1(0)$, which follows from $\rho\hat{R}>1$.

An important feature of the solution to (2) to note is that $c_1(0)>e$ and $c_2(1)<e\hat{R}$. The planner redistributes resources to consumers of type $\theta = 0$ who are given a higher present value of consumption than the value of their endowment. Late consumers, those with $\theta = 1$, receive consumption that is less than a present value of their endowment.

We can also define a competitive equilibrium problem in which there is a continuum of intermediaries providing insurance to agents. The intermediaries are subject to the same constraint as the social planner and do not observe the types of agents. We omit a formal definition here but notice that a version of the first welfare theorem would hold here as shown by Prescott and Townsend (1984) and Allen and Gale (2004): the competitive equilibrium allocations would coincide with the solution to the problem SP^2. The key to this result is that consumption is observable – agents cannot engage in unobservable trades.

5 Private markets

The allocations described in the previous section may not be achieved if agents can engage in transactions on markets. Allen (1985) and Jacklin (1987) were the first to point out that the possibility of such trades may restrict or even lead to a complete elimination of redistribution across agents. In this section, we first formally describe how to model unobservable consumption.[5] This formalization will be central to defining and characterizing both competitive equilibria and constrained efficient allocations with private markets.

We model unobservability of consumption using the setup of private markets as in Golosov and Tsyvinski (2007). Consider an environment in which all consumers have access to a market in which they can trade assets among themselves unobservably[6]. Formally, suppose that consumers are offered a menu of contracts $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$. We model private markets as an endowment economy where endowments are allocations that agents receive (possibly by misrepresenting their types). A consumer treats the contract and the equilibrium interest rate R on the private market as given and chooses his optimal reporting strategy θ' that determines his endowment of consumption $\left(c_1(\theta'), c_2(\theta')\right)$. Unlike in the environment without private markets, actual after-trade consumption (x_1, x_2) may differ from the consumption specified in the contract, since it is impossible to preclude a consumer from borrowing and lending the amount s on the private market[7]. The problem that a consumer solves is formally as follows. Given a menu of consumption allocations $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ and an interest rate R[8] an agent of type θ solves:

$$\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta) = \max_{x_1,x_2,s,\theta'} U(x_1, x_2; \theta), \tag{9}$$

subject to:

$$x_1 + s = c_1(\theta'), \tag{10}$$

$$x_2 = c_2(\theta') + Rs. \tag{11}$$

[5] An alternative interpretation of the assumption of the private markets is non-exclusivity by which we mean that it is impossible for an intermediary to observe or control transactions of a consumer with other intermediaries.

[6] All our analysis is easily extended to the case in which agents can trade not only among themselves but also with other intermediaries. This case would bring this model closer to an interpretation as an environment of non-exclusive contracts. Key assumption that allows us to extend our results to that case is that portfolios of the intermediary (investment in short and long assets) are observable while transactions with individual consumers are not observable. Our choice of modelling side trades as private markets allows us to economize on notation without affecting the substance of the results.

[7] It can be shown that a consumer trades only a risk free security (Golosov and Tsyvinski 2007).

[8] It is redundant to write allocations and reports depending on both $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ and R. Conditioning only on $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ would be sufficient. We choose to carry through conditioning on R to underscore the mechanism by which the choice of the profile of endowments determines the interest rate.

Let $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $s(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$ be a solution to problem (9).

We now formally define an equilibrium in the private market.

Definition 1 *An equilibrium in the private market given the profile of endowments* $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ *consists of an interest rate* R*; and, for each agent* θ*: allocations* $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$*, trades* $s(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$*, and choices of a reported type* $\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$ *such that*

(i) $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $s(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$ *are a solution to problem (9);*

(ii) the feasibility constraints on the private market are satisfied, for $\forall t = 1, 2$*:*

$$\begin{aligned} & \pi x_t(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0) + (1-\pi)\,\pi x_t(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1) \\ = \ & \pi c_t\left(\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0)\right) + (1-\pi)\,\pi c_t\left(\theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1)\right). \end{aligned} \tag{12}$$

We assume that for any menu of contract $\{c_1(\theta), c_2(\theta)\}$ that is offered there exists a unique equilibrium.

6 Competitive equilibrium with private markets CE^3

In this section, we formally describe a competitive equilibrium in which risk sharing is hindered by the possibility of agents to engage in unobservable trades on the private markets.

Consider a market with a continuum of intermediaries. We assume throughout the paper that all activities at an intermediary level are observable. In period 0, before the realization of idiosyncratic shocks, consumers deposit their initial endowment with the intermediary. An intermediary agrees to provide a menu of consumption allocations $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$. In the presence of private markets, intermediaries need to take into account, in addition to unobservable types, that consumers are able to engage in transactions in the private market. The contracts are offered competitively, and there is free entry for intermediaries. Therefore, consumers sign a contract with the intermediary that promises the highest ex-ante expected utility. We denote the equilibrium utility for a consumer by $\underline{U}$.

We assume that intermediaries can trade bonds b among themselves. Without aggregate uncertainty the market for trades among intermediaries is very simple, and we describe it in this section as it is useful for later extensions to the case of aggregate uncertainty. We denote by q the price of a bond b in period $t = 1$ that pays one unit of consumption good in period 2. All intermediaries take this price as given. They also pay dividends d_1, d_2 to its owners.[9]

It is important to note that intermediaries take the interest rate on the private market R as given. The maximization problem of the intermediary that faces an intertemporal price q, an

[9] Since intermediaries make zero profits in equilibrum, we do not formally specify how these dividends are distributed.

interest rate on the private market R, and a reservation utility $\underline{U}$ is

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\{0,1\}},(d_1,d_2),b} d_1 + qd_2 + qb - b/\hat{R} \tag{13}$$

s.t.

$$\pi\left(c_1(0)+\frac{c_2(0)}{\hat{R}}\right)+(1-\pi)\left(c_1(1)+\frac{c_2(1)}{\hat{R}}\right)+d_1+d_2/\hat{R}+qb-b/\hat{R}\le e, \tag{14}$$

$$\theta = \theta'(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta),\ \forall\theta, \tag{15}$$

$$\pi\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 0) + (1-\pi)\,\pi\tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, 1) \ge \underline{U}\ . \tag{16}$$

The first constraint in the intermediary's problem is the budget constraint. The second constraint is incentive compatibility that states that, given the profile of consumptions $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ and the possibility to borrow or lend at an interest rate R, consumers choose to truthfully reveal their types, i.e. the true type θ is a solution to the problem (9). The last constraint states that the intermediary cannot offer a contract which delivers a lower expected utility than the equilibrium utility $\underline{U}$ from the contracts offered by other intermediaries. In equilibrium, all intermediaries act identically and make zero profits. The definition of the competitive equilibrium is then as follows.

Definition 2 *A competitive equilibrium with private markets, CE^3, is a set of allocations $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$, a price q, dividends $\{d_1, d_2\}$, bond trades b, utility $\underline{U}$, and the corresponding interest rate on the private market R such that*

(i) intermediaries choose $\{\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, \{d_1, d_2\}, b\}$ to solve problem (13) taking q, R, and $\underline{U}$ as given;

(ii) consumers choose the contract of an intermediary that offers them the highest ex-ante utility;

(iii) the aggregate feasibility constraint (1) holds;

(iv) the private market, given the menus $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$, is in an equilibrium of Definition 1, and R is an equilibrium price on the private market;

(v) firms make zero profits;

(vi) bonds markets clear: $b = 0$.

It is easy to see that the interest rates on the markets for trades among intermediaries must be equal to the return on the production technology, so that $1/q = \hat{R}$. We now present a straightforward lemma that the incentive compatibility constraint (15) takes the form of equalizing present value of intertemporal allocations across periods. The proof is simple. If the present values are not equated across types, an agent would pretend to claim a type that gives a higher present value of allocations and engage in trades on the private markets to achieve desired consumption allocations.

Lemma 1 *An allocation of consumptions satisfies incentive compatibility constraint (15) iff*

$$c_1(\theta) + \frac{c_2(\theta)}{R} = c_1(\theta') + \frac{c_2(\theta')}{R} \text{ for any } \theta,\ \theta'. \tag{17}$$

Proof. In text above. ■

Let us rewrite the problem of the intermediary in a more manageable form by considering its dual, simplifying incentive compatibility constraint using Lemma 1, and using the fact that $d_1 = d_2 = b = 0$ to reduce to:

$$\max_{c_1, c_2} \pi u(c_1(0)) + (1-\pi)\rho u(c_1(1) + c_2(1)), \tag{18}$$

s.t. (17) and

$$\pi \left(c_1(0) + \frac{c_2(0)}{\hat{R}} \right) + (1-\pi) \left(c_1(1) + \frac{c_2(1)}{\hat{R}} \right) \leq e. \tag{19}$$

We now argue that $R = \hat{R}$, otherwise, arbitrage opportunities are created. For example, suppose that $R < \hat{R}$, i.e., an interest rate on the private market is lower than $\hat{R}$. An intermediary then chooses to invest only in the long asset (respectively, paying $(\pi c_2(0) + (1-\pi) c_2(1)) = e\hat{R}$ in period $t = 2$) and sets investment in the short asset to be equal to zero (respectively, paying $(\pi c_2(0) + (1-\pi) c_2(1)) = 0$ in period $t = 1$). Consumers then can borrow on the private market at the interest rate R that is lower than the technological rate of return $\hat{R}$ available to the intermediary. Therefore, $R < \hat{R}$ cannot be an equilibrium interest rate. Analogously, we rule out $R > \hat{R}$. The only price that can be an equilibrium price is $R = \hat{R}$ so that intermediaries do not engage in arbitrage. We can summarize this reasoning in the following proposition.

Proposition 1 ***(Absence of redistribution without regulations)*** *Let R^* denote equilibrium price on the private market corresponding to the competitive equilibrium in Definition 2. Then $R^* = \hat{R}$. The only allocation that competitive markets can achieve in such an economy is an allocation in which the present values of endowments evaluated at $\hat{R}$ are equated across different types:*

$$c_1(\theta) + \frac{c_2(\theta)}{\hat{R}} = c_1(\theta') + \frac{c_2(\theta')}{\hat{R}} \text{ for any } \theta, \theta'.$$

Moreover,

$$\begin{aligned} c_2(0) &= c_1(1) = 0, \\ c_1(0) &= e, c_2(1) = e\hat{R}. \end{aligned}$$

Proof. In text above. ■

The arbitrage among competitive intermediaries forces the equilibrium interest rate on the private market to be equal to the return on savings $\hat{R}$. Then, there is absence of redistribution across consumers as in Jacklin (1987) and Allen and Gale (2004). The present value of endowments are equated across consumers of different types.

Intuitively, the reason that the competitive equilibrium achieves only an autarcic allocation is an externality. Intermediaries do not take into account how the contracts offered to its investors affect the return on trades and thus incentives to reveal information truthfully for consumers of other intermediaries. Individual intermediaries can not internalize this effect. Competition between different intermediaries implies that interest rates at which consumers trade are equated to $\hat{R}$. The interpretation of our result is different from Jacklin (1987) and Allen and Gale (2004) as we describe it as an externality that we show in the next section can be corrected by a government intervention.

7 Constrained efficient allocation with private markets

In this section, we define and characterize the constrained efficient problem with private markets. We call such program SP^3 or the "third best" program. Consider a social planner that cannot observe or shut down trades on private markets and cannot observe agents' types. The difference with the problem SP^2 is that, in addition to the private information faced by SP^2, planner SP^3 faces constraints that agents may trade on the private market. The social planner SP^3 chooses the allocations $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ that maximize the ex ante utility of consumers. The revelation principle shows that, without loss of generality, the social planner can offer a contract $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ so that all consumers choose to report their types truthfully to the planner and do not trade on the private market.

Formally, a constrained efficient allocation $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ is a solution to the problem SP^3 given by:

$$\max_{\{c_1(\theta),c_2(\theta)\}_{\theta\in\{0,1\}}} \pi U(c_1(0), c_2(0); 0) + (1-\pi) U(c_1(1), c_2(1); 1), \tag{20}$$

s.t.

$$\pi\left(c_1(0) + c_2(0)/\hat{R}\right) + (1-\pi)\left(c_1(1) + c_2(1)/\hat{R}\right) \le e, \tag{21}$$

$$u(c_1(\theta), c_2(\theta); \theta) \ge \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R; \theta),\ \forall\theta, \tag{22}$$

and

$$\begin{aligned} c_1(\theta) &= x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta), \\ c_2(\theta) &= x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta),\ \forall\theta, \end{aligned} \tag{23}$$

where $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}, R, \theta)$, and R constitute an equilibrium on the private market, given the profile of endowments $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ according to Definition 1.

We now show that choosing consumption allocations in the constrained efficient problem (20) is equivalent to a problem of a planner choosing an interest rate R on the private market and allocating the same income (present value of consumption allocations), I, to agents of different types. Intuitively, we argued above that the interest rate on the private markets depend on the relative amount of aggregate consumption provided by the planner in the first period versus the aggregate amount of consumption provided in the second period. The incentive compatibility constraint presents itself as a requirement that the same present value of resources I is allocated across agents of different types. Therefore, the planner has only two instruments: an income I and interest rate R.

Formally, we proceed as follows. Let:

$$V(I,R;\theta) = \max_{x_1,x_2} U(x_1,x_2;\theta) \tag{24}$$

subject to

$$x_1 + \frac{x_2}{R} \leq I, \tag{25}$$

be the ex-post indirect utility of an agent of type θ if her income is I, and the interest rate on the private market is R. Denote by $x_1^u(I,R;\theta)$ and $x_2^u(I,R;\theta)$ solutions (uncompensated demands) to this problem.

Consider the problem of a social planner who chooses the interest rate R and income I to maximize the expected indirect utility of agents subject to feasibility constraints.

$$\max_{I,R} \pi V(I,R;0) + (1-\pi)\, V(I,R;1) \tag{26}$$

subject to

$$\pi\left(x_1^u(I,R;0) + \frac{x_2^u(I,R;0)}{\hat{R}}\right) + (1-\pi)\left(x_1^u(I,R;1) + \frac{x_2^u(I,R;1)}{\hat{R}}\right) \leq e, \tag{27}$$

where $x_1^u(I,R;\theta), x_2^u(I,R;\theta)$ are defined above.

We now prove the equivalence of the problem (20) and the problem (26).

Lemma 2 *Let I^* and R^* be solutions to (26), and $\{x_1^u(I^*,R^*;\theta), x_2^u(I^*,R^*;\theta)\}_{\theta\in\{0,1\}}$ be solutions to (24) given I^* and R^*. Let $\{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}$ be solutions to (20). Then*

$$\{x_1^u(I^*,R^*;\theta), x_2^u(I^*,R^*;\theta)\}_{\theta\in\{0,1\}} = \{c_1(\theta), c_2(\theta)\}_{\theta\in\{0,1\}}.$$

Proof. In the appendix. ■

The above lemma is useful as it reduces the dimension of the problem as the planner chooses only two variables: interest rate R and income I. Using this lemma, we can now provide a characterization of the constrained efficient program.

Theorem 1 *Solutions to the problem SP^3 and problem SP^1 coincide. Moreover, the interest rate R^* on the private market corresponding to the solution of SP^3 is such that $R^* \in (1, \rho\hat{R}]$. If $u(c) = \log(c)$, then $R^* = \rho\hat{R}$.*

Proof. Choose (I^*, R^*) to solve the following system of equations:

$$u'(I) = \rho\hat{R}u'(RI), \tag{28}$$
$$I = \frac{e}{\pi + (1-\pi)\frac{R}{\hat{R}}}. \tag{29}$$

In particular, R^* is a solution to

$$\frac{u'\left(\frac{e}{\pi+(1-\pi)\frac{R}{\hat{R}}}\right)}{u'\left(\frac{Re}{\pi+(1-\pi)\frac{R}{\hat{R}}}\right)} - \rho\hat{R} = 0. \tag{30}$$

Denote by $f(R)$ the left hand side of (30). Note that $f(R)$ is increasing and $f(1) = 1 - \rho\hat{R} < 0$, so that the solution to (30) involves $R > 1$. Since the coefficient of relative risk aversion is everywhere greater than 1, we have

$$\frac{u'\left(\frac{e}{\pi+(1-\pi)\frac{R}{\hat{R}}}\right)}{Ru'\left(\frac{Re}{\pi+(1-\pi)\frac{R}{\hat{R}}}\right)} \geq 1$$

which implies that $f(\rho\hat{R}) \geq 0$, with equality if $u(c) = \log(c)$. This show that $R^* \in (1, \rho\hat{R}]$.

Note that an agent of type $\theta = 1$ if $R \geq 1$ consumes only in the second period:

$$V(I, R; 1) = \rho u(RI),\ x_1^u(I, R; 1) = 0,\ \text{and } x_2^u(I, R; 1) = RI. \tag{31}$$

An agent of type $\theta = 0$ consumes all its income in the first period:

$$V(I, R; 0) = u(I),\ x_1^u(I, R; 0) = I,\ \text{and } x_2^u(I, R; 0) = 0. \tag{32}$$

It is then immediate that (I^*, R^*) defined in (28) and (29) satisfy the first order conditions of problem (26). By Lemma 2, these are the solution to the constrained efficient problem SP^3 defined in (20). Comparing the first order conditions of the problem SP^1 (6)-(8) and the (31)-(32), evaluated at I^* and R^*, we conclude that solutions to SP^1 and SP^3 coincide. ■

This theorem shows one of the central results of the paper that the social planner, even in the presence of hidden trades, can achieve allocations superior to the ones achieved by the competitive markets. Moreover, we fully characterize the constrained efficient allocation and show that for the case of Diamond-Dybvig preferences it coincides with the unconstrained, full information optimum. The intuition for the result is that a change in the interest rate affects the incentive compatibility of agents. Consider a relevant deviation in the model. An agent of type $\theta = 1$ wants to claim to be an agent of $\theta = 0$ and then save the allocation $c_1(0)$ at the private market interest rate R. An interest rate on the private markets $R^* < \hat{R}$ reduces the profitability of this deviation. In the case of Diamond-Dybvig preferences, the effects of interest rates are such that they allow perfect screening of types and allows to achieve no only the constrained efficient allocation SP^3 but also the unconstrained allocation SP^1. Note that the manipulation of equilibrium rate by the planner is indirect and happens through the general equilibrium effect of changing the profile of endowments. Remember that the planner cannot observe trades on the private market. However, the planner can increase the amount of investment in the short asset (amount of allocations paid in the first period) and correspondingly reduce the amount of investment in the long asset (amount of allocations paid in the second period). Lemma 2 showed that such manipulation of endowments induces the change in the interest rate on trades in private markets.

More generally, a change in the interest rate leads to a relative redistribution of resources from the first to the second period which may benefit an agent who derives a higher marginal utility of income and lead to an improvement in the ex-ante welfare. Recall that the unobservability of agents' types and possibility of trades require that agents of various types receive the same present value of consumption evaluated at the private market interest rate R:

$$c_1(\theta) + \frac{c_2(\theta)}{R} = c_1(\theta') + \frac{c_2(\theta')}{R}.$$

For a given level of the present value of consumption, a regulator has a policy instrument – changing the interest rate. Therefore, the amount of resources evaluated at the real rate of return may differ across agents

$$c_1(\theta) + \frac{c_2(\theta)}{\hat{R}} \neq c_1(\theta') + \frac{c_2(\theta')}{\hat{R}}.$$

In our case, the changes in the interest rate transfer resources to the agent $\theta = 0$ affected by a liquidity shock.

What do we conclude from Theorem 1? Firms in the competitive equilibrium cannot provide any redistribution across agents while the planner can achieve the unconstrained optimum. In the next section we show how imposition of a regulation on the side of financial intermediaries in a competitive equilibrium can implement the constrained efficient allocation.

8 Implementing constrained efficient allocations – liquidity requirements

In this section we show that there exists an intervention – *a liquidity floor* – that implements the constrained efficient allocation SP^3.

A liquidity requirement is a constraint imposed on all intermediaries, i.e., a constraint on the problem (13) that requires that investment in the short asset (payments to the consumers in the first period) for any intermediary should be higher than a level i

$$\pi c_1(0) + (1-\pi) c_1(1) \geq i. \tag{33}$$

An attractive feature of the liquidity requirement is that it does not require a regulator to observe individual contracts $c_1(\theta)$ – only an aggregate portfolio allocation of the intermediaries needs to be observed.

We now intuitively describe the effects that a binding liquidity requirement has on the interest rate on private markets. Let $\hat{X}$ be the investment in the short asset that arises in a competitive equilibrium without in Definition 2. Suppose that a liquidity floor i is set higher than the amount of aggregate liquidity provided by competitive markets:

$$i > \hat{X}.$$

When a liquidity floor is imposed, the aggregate endowment in the private markets in the first period equal to i rather than $\hat{\imath}$. Recall that private trading markets in which agents participate after receiving their allocation from the intermediaries are an endowment economy. The liquidity floor increases the first period aggregate endowment in the private market (and, correspondingly, decreases the second period endowment) and, therefore, has a general equilibrium effect in indirectly lowering the interest rate R such that $R < \hat{R}$.

The mechanism by which a liquidity requirement affects the interest rate on the private markets is a key to understanding the main idea behind how our model works. In the absence of regulations, it is impossible for the interest rate R on the private market to differ from $\hat{R}$. As we showed in Proposition 1, an intermediary would engage in arbitrage and would not internalize possible adverse effects that such arbitrage has on the provision of incentives and risk-sharing in the economy. The liquidity requirement puts a limit on the minimal first period payments an intermediary can make and limits arbitrage by intermediaries. Competitive markets lack this additional instrument because of arbitrage and the fact that each individual firm cannot set the interest rate. A regulator, however, can affect the interest rate and achieve allocations better than autarcic allocations achieved by the markets. A properly chosen liquidity floor influences the interest rate and implements the constrained efficient allocation.

Proposition 2 *Let the liquidity floor i^* defined in (33) be given by*

$$i^* = \pi I^*,$$

where I^* is a solution to (28) and (29). Then competitive equilibrium allocations specified in Definition (2) coincide with the constrained efficient allocation SP^3.

The proof is omitted as it is a simple check that this allocation induces the same interest rate as R^*. This proposition is important as it specifies a simple regulation that implements the optimum. Note that this regulation does not shut down private markets, rather it affects the investments and holding of assets by financial intermediaries. In general, finding implementation of the constrained efficient allocations is a difficult task in an environment where trades are possible. An abstract treatment of a related problem is given in Bisin, et.al. (2001) who show that, in a general class of environments with anonymous markets, taxes can achieve Pareto improvement. The difference with our setup is that they do not define the constrained efficient problem SP^3 but rather show that a local linear tax can improve upon the market allocation. Golosov and Tsyvinski (2007) study a dynamic model of optimal taxation and do define the optimal program similar to our SP^3. They also show that a linear tax on savings locally improves upon the competitive equilibrium allocation.

In their model, in fact, it can be shown that a linear tax (locally optimal policy intervention) would not implement the problem SP^3. The question of the implementation of the problem SP^3 in that model, or more generally in a dynamic Mirrlees model with private trades, is still an open question. The result of Proposition 2 shows that one can find an implementation of the constrained efficient allocation in a model with side trades.

9 General preferences

The analysis of the previous sections characterized a constrained efficient allocation in the presence of private markets and showed that a liquidity floor implements the optimal allocation for a case of Diamond-Dybvig preferences. Moreover, the constrained efficient allocations with and without private markets coincided. In this section, we consider a more general specification of preferences. We show that the form of preferences matter for the form of the optimal regulation. Also, the ability of agents to engage in trades may lead to constrained efficient allocations inferior to those without trades. We then provide an analytical characterization of this more general model.

9.1 Setup

In this section, we consider a more general setup of a model of financial intermediation. There is now a continuum of possible types. We denote the preference shock by $\theta \in \Theta = [\theta_L, \theta_H]$, $\theta_L < \theta_H$. At $t = 1$, each consumer gets an i.i.d. draw of his type. The probability distribution of being an investor of type θ is denoted by $F(\theta)$. We assume that the "law of large numbers" holds, and that the cross-sectional distribution of types is the same as the probability distribution F. One can, therefore, interpret $F(\theta)$ as the number of agents of type below θ. Investor's preferences are represented by a utility function $u(c_1, c_2; \theta)$, where c_t denotes consumption at date $t = 1$, 2. The utility function $u(\cdot; \theta)$ is assumed to be concave, increasing, and continuous for every type θ. We also assume a single crossing property.

Assumption 1 (Single crossing): $\frac{\partial}{\partial \theta}\left(\frac{\partial u/\partial c_2}{\partial u/\partial c_1}\right) > 0$.

Specifically, we focus on three types of preferences: discount factor shocks, liquidity shocks, and valuation-neutral shocks. Let $\hat{u}(\cdot)$ be concave, increasing, and continuous.

Example 1 *Discount factor shocks:* $u(c_1, c_2; \theta) = \hat{u}(c_1) + \theta \hat{u}(c_2)$.

The first feature of these preferences is that an agent with a higher θ shock has a higher marginal utility of consumption in the second period. The second feature of these preferences is that an agent with higher θ has higher lifetime marginal utility of income.

Example 2 *Liquidity shocks:* $u(c_1, c_2; \theta) = \frac{1}{\theta} \hat{u}(c_1) + \hat{u}(c_2)$.

In this case, low θ is a shock increases marginal utility of consumption in period. Similar to the case of the discount shocks, the second feature of these preferences is that an agent with lower θ

has a higher lifetime marginal utility of income than an agent with lower θ.[10] This preferences are a straightforward generalization of the Diamond-Dybvig setup.

Example 3 *Valuation-neutral shocks: Let* $\hat{u}(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$ *and*

$$u(c_1, c_2; \theta) = \frac{1-\theta}{\left[\theta^{1/\sigma} + (1-\theta)^{1/\sigma} \hat{R}^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}} \hat{u}(c_1) + \frac{\theta}{\left[\theta^{1/\sigma} + (1-\theta)^{1/\sigma} \hat{R}^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}} \hat{u}(c_2). \quad (34)$$

If $\hat{u}(c) = \log(c)$*, then*

$$u(c_1, c_2; \theta) = (1-\theta)\, \hat{u}(c_1) + \theta \hat{u}(c_2).$$

In this case, agents differ in marginal utility of consumption across periods, but the second feature of the preferences that we described above is absent here, and all agents have the same marginal value of income. Note that in the case of the log utility, there is no need to normalize preferences by $\hat{R}$, and valuation-neutral preferences do not depend on technology.

9.2 Characterization

Many of the definitions and results of the previous sections immediately apply in this setup, and we omit the formalism in the section for the cases where results derived above directly generalize. We refer the reader to the working paper version for more detailed analysis. Specifically, the definition of the equilibrium in private markets given in Definition 1 immediately extends to the more generalized setup of this section. Let R be the interest rate on the private market. The analysis of the competitive equilibrium with private markets in this environment is a direct extension of Definition 2. As in Proposition 1, we conclude that competitive equilibria provide no redistribution across types, and that the interest rate R on private markets is equal to $\hat{R}$.

We can now extend the definition of the constrained efficient problem with private markets, SP^3 :

$$\max_{\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}} \int u(c_1(\theta), c_2(\theta); \theta)\, dF(\theta), \quad (35)$$

s.t.

$$\int \left(c_1(\theta) + c_2(\theta)/\hat{R}\right) dF(\theta) \leq e, \quad (36)$$

$$u(c_1(\theta), c_2(\theta); \theta) \geq \tilde{V}(\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}, R; \theta), \forall \theta, \theta', \quad (37)$$

[10] A natural question arises whether uility specification of liquidity shocks $\frac{1}{\theta}\hat{u}(c_1) + \hat{u}(c_2)$ is a renormalization of the discount shocks $\hat{u}(c_1) + \theta\hat{u}(c_2)$, and that by dividing utility in the case of discount shocks by θ we would arrive to the model with liquidity shocks. It is true that both of preferences have the same marginal rates of substitutions. However, the preferences are different in the direction of marginal utility of income. In the case of liquidity shocks, it is low θ that gives an agent a higher marginal utility of income. In the case of dicount factor shocks, it is exactly the opposite – high θ leads to high lifetime marginal utility of income.

and

$$c_1(\theta) = x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}, R, \theta), \tag{38}$$
$$c_2(\theta) = x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}, R, \theta), \forall\theta,$$

where $x_1(\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}, R, \theta)$, $x_2(\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}, R, \theta)$, and R constitute an equilibrium on the private market, given the profile of endowments $\{c_1(\theta), c_2(\theta)\}_{\theta\in\Theta}$.

As a next step we define a relevant notion of the liquidity requirement. Here, it is more general and can take a form of wither liquidity floor or a liquidity cap. Formally, a liquidity requirement is a constraint imposed on all intermediaries, that requires that investment in the short asset for any intermediary should be higher (lower) than a level i:

$$\left(\int c_1(\theta)\, dF(\theta)\right) \begin{matrix}\geq \\ \leq\end{matrix}\; i. \tag{39}$$

[11]

We call a liquidity requirement a *liquidity cap* if (39) is imposed with less or equal sign. A liquidity cap stipulates the maximal amount of the short asset that an intermediary can hold. We call a liquidity requirement a *liquidity floor* if (39) is imposed with a greater or equal sign.

We now make the following technical assumption.

Assumption 2. *For all* (I, R),

$$\text{Cov}\left\{x^u_{2,I}(I, R; \theta), \frac{x^u_2(I, R; \theta)}{R^2}\right\} > 0. \tag{40}$$

The lemma that follows shows two natural cases in which Assumption 2 holds.

Lemma 3 *Assumption* 2 *holds under the conditions that follow.*

1. *The function* $\hat{u}(\cdot)$ *is homothetic.*

2. *The variance of the shocks is small. Consider a family of distributions* $\{F^\gamma\}$ *indexed by* $1 \geq \gamma \geq 0$ *with support in* $[\theta_L, \theta_H]$. *Suppose that* $F^\gamma(\theta, z)$ *is continuous in* (θ, γ). *Suppose that* $\lim_{\gamma\to 0} \sigma_{F^\gamma} = 0$ *where* σ_{F^γ} *is the variance of* F^γ. *Then there exists* $1 > \bar{\gamma} > 0$ *such that for all* $\bar{\gamma} \geq \gamma \geq 0$, *assumption* 2 *holds.*

Proof. In the appendix. ■

The theorem that follows characterizes the constrained efficient allocation SP^3 and determines the form of the liquidity adequacy requirement that implements such allocation for different form of these more general preferences.

[11] Formally, we would define a problem that is a generalization of the problem of an intermediary (13) and impose the liquidity requirement as an additional constraint.

Theorem 2 *Suppose Assumption 2 holds. Let R^* be the interest rate on the private market associated with the solution to the constrained efficient problem SP^3.*

1. *If preferences are of the discount shock form as in example* 1, $R^* > \hat{R}$. *Competitive markets are inefficient, and the liquidity requirement that implements SP^3 is a liquidity cap.*

2. *If preferences are of liquidity shock form as in example* 2, $R^* < \hat{R}$. *Competitive markets are inefficient, and the liquidity requirement that implements SP^3 is a liquidity floor.*

3. *If preferences are valuation-neutral shocks, then $R^* = \hat{R}$. No regulations are needed. Moreover, the solution to the optimal problem SP^2 without private markets, constrained efficient solution with private markets SP^3, and the solution to the competitive equilibrium with private markets CE^3 coincide.*

Proof. In the appendix. ■

Theorem 2 provides a characterization of the constrained efficient allocation and the form of liquidity adequacy requirement implementing it depending on the nature of the preferences that agents have. The proof of a theorem also derives characterization of the interest rate R^* in terms of an easily interpretable wedge.

The intuition for the theorem parallels the discussion of the Theorem 1. In the case of liquidity shocks, the planner wants to allocate higher present value of resources to agents with lower θ as they have higher marginal lifetime utility of income. An agent with $\theta' > \theta$ wants to engage in a double deviation: pretend to be an agent with a lower type θ and save on the private market. An interest rate $R < \hat{R}$ discourages such deviation and relaxes the incentive compatibility constraint. In the case of discount factor shocks, the direction of the deviation is reverse: pretend to be an agent of higher θ and borrow on the private market. Increasing the interest rate on the private market $R > \hat{R}$ discourages such deviations.[12]

The case of Diamond-Dybvig preferences is conceptually close to the case of liquidity shocks preferences in this section. The direction of the relevant deviation and the ability to affect them with the lower interest rate directly generalize to the more general preferences. The important difference between these two cases is that, in general, solutions to constrained efficient problem with trades, SP^3, and constrained efficient problem without trades, SP^2, (or problem without private information, SP^1) do not coincide. A case of Diamond-Dybvig preferences is one such case in which all three constrained efficient problems coincide. The reason for that is that agents of type $\theta = 0$ and $\theta = 1$ have very different marginal rates of substitution. In the case of $\theta = 0$, the marginal rate of substitution between period $t = 1$ and $t = 2$ is zero, and in case of $\theta = 1$, the marginal rate of substitution is one. Such differences in preferences the social planner can perfectly

[12] The liquidity cap may appear a somewhat unrealistic requirement. One can however argue that it mimics an implicit subsidy to investment. Such subsidies are prevalent, especially, in the context of encouraging small businesses (for example, through Small Business Administration in the USA). One can show a parallel between agents in our model experiencing liquidity shocks and a model in which entrepreneurs face investment risk shocks.

screen agents of various types. In general, this is not the case and the solutions to constrained efficient programs would not coincide.

The technical reason for the failure of the welfare theorem that we proved in Theorem 2 is an externality that each financial intermediary faces. When intermediaries allocate consumption to agents, they do not take into account how such allocations affect interest rates on the private market. We can see that the interest rate enters the production set of each intermediaries in equation (15). An imposition of a liquidity regulation and a corresponding change in the private market interest rate partially corrects such externality and leads to improvement in welfare. This externality can be called a "pecuniary externality" as it operates through price (interest rate) on the private markets. The reason why such an externality has effects on the welfare is because our environment is that of private information.[13]

We now provide a simple case for which a closed form solution is available. Consider the case of liquidity shocks and assume that utility takes form:

$$u\left(c_1, c_2; \theta\right) = \frac{1}{\theta} c_1 + \log c_2.^{14}$$

Assume that the distribution of shocks $F(\theta)$ is log-normal with with (μ, σ). Then one can show that the optimal interest rate is given by:

$$R^* = \hat{R} e^{-\sigma^2}.$$

This closed form solution provides us with an additional insight. As the variance of the shocks σ increases and the informational friction increases, the optimal interest rate decreases.

10 Extensions

In this section we consider three extensions of the model described above: introducing aggregate shocks, modelling an environment with idiosyncratic shocks to financial intermediaries, and considering direct access to intertemporal technology by agents.

10.1 Aggregate shocks

It is easy to extend the model to the case in which the economy experiences aggregate shocks to e and $\hat{R}$ that are known in period $t = 0$. Suppose that there are N aggregate states $\eta = \{1, 2, .., N\}$ and the state is observable. We denote the probability of these states occurring as $\mu(\eta)$. We notice that it is technologically impossible for the society to transfer resources across aggregate states.

[13] Allen and Gale (2004) results of whether intermediaries underprovide or overprovide liquidity compared to the optimum depends on the degree of risk aversion of consumers. Our result on private intermediaries underproviding or overproviding liquidity does not depend on the degree of risk aversion and depends only on the structure of shocks.

[14] These preferences do not strictly fall in our specification of liquidity shocks preferences, $u(c_1, c_2; \theta) = \frac{1}{\theta}\hat{u}(c_1) + \hat{u}(c_2)$ because utility functions of the first and second period consumption differ. We use these preferences nevertheless as they admit an easy closed form solution.

Therefore, the problem with aggregate shocks can be reduced to solving N independent problems described in case without aggregate shocks and is, essentially, a comparative statics exercise with respect to the aggregate shock.

10.2 Idiosyncratic shocks to intermediaries and interbank markets

In this section we discuss an extension of the model to the case in which intermediaries experience idiosyncratic observable shocks. We show that if there are complete interbank markets then this model reduces to the case described in previous sections in which all intermediaries are identical. The intuition for this result is simple: in period 0, intermediaries can trade bonds with the payoff contingent on the shocks realized in period 1. We illustrate the result on the case without aggregate uncertainty in which intermediaries face return shocks.

Formally, we proceed as follows. At time $t = 1$, an intermediary can face a rate of return shock $n \in \{1, ..., N\}$ with probability η^n under which the return on the long asset is $\hat{R}(n)$. We assume that there is no aggregate uncertainty and that

$$\sum_n \eta^n \hat{R}(i) = \hat{R},$$

$$\sum_n \eta^n = 1.$$

At time 0 there are interbank markets in which intermediaries trade N Arrow securities. The price of each security is q^n. The security pays 1 if state n occurs and 0 otherwise. Prices q^n are determined by a market clearing condition. It is immediate to see that intermediaries choose to fully insure themselves at $t = 0$ against idiosyncratic shocks. The problem of each intermediary then reduces to the case of no idiosyncratic shocks described above.

10.3 Direct access to technology

Another variation of our setup would be the case in which some agents have access to technology that yields $\hat{R}$ directly without the need for financial intermediaries while other agents need an intermediary to access the technology. If we modified our assumption that all activities at the intermediary level is observable and instead supposed that a regulator could observe aggregate amount of investment in the technology yielding $\hat{R}$, then our results would also hold. The constrained optimum in that model would be implemented by a linear tax on returns to investment of those who can access the technology and by liquidity adequacy requirement on the financial intermediaries serving liquidity needs of agents who can not access the technology.

11 Appendix

11.1 Proof of Lemma 2

Note that the incentive compatibility constraint can be rewritten as

$$U(c_1(\theta), c_2(\theta); \theta) \geq V(c_1(\theta') + \frac{c_2(\theta')}{R}, R; \theta) \text{ for all } \theta, \theta',$$

and in turn

$$U(c_1(\theta), c_2(\theta); \theta) \geq V(\max_{\theta'} c_1(\theta') + \frac{c_2(\theta')}{R}, R; \theta).$$

This is equivalent to

$$\begin{aligned} c_1(\theta) + \frac{c_2(\theta)}{R} &= \max_{\theta'} c_1(\theta') + \frac{c_2(\theta')}{R}, \\ c_1(\theta) &= x_1^u(c_1(\theta) + \frac{c_2(\theta)}{R}, R; \theta) \text{ and } c_2(\theta) = x_2^u(c_1(\theta) + \frac{c_2(\theta)}{R}, R; \theta). \end{aligned}$$

Therefore, allocations that satisfy the incentive compatibility constraints are entirely characterized by the following: there exists an equilibrium income level I such that for all θ,

$$c_1(\theta) = x_1^u(I, R; \theta) \text{ and } c_2(\theta) = x_2^u(I, R; \theta)$$

The level of utility achieved ex post by an agent of type θ is then given by $V(I, R; \theta)$. This proves the theorem.

11.2 Proof of Lemma 3

Part 1. If preferences are homothetic, then

$$\frac{I x_{2,I}^u(I, R; \theta)}{x_2^u(I, R; \theta)} = 1. \tag{41}$$

This implies, first, that date 2 consumption is a normal good, so that $x_{2,I}^u(I, R; \theta) > 0$. Second, it implies that $\text{Cov}\left\{x_{2,I}^u(I, R; \theta), \frac{x_2^u(I,R;\theta)}{R^2}\right\} \geq 0$, which in turn implies assumption 2.

Part 2. Let K be a compact set containing the optimal values of (R, I) for $1 \geq \gamma \geq 0$. Then there exists γ such that for all $(R, I) \in K$

$$\left|\text{Cov}\left\{x_{2,I}^u(I, R; \theta), \frac{x_2^u(I, R; \theta)}{R^2}\right\}\right| < M\sigma_{F^\gamma}^2. \tag{42}$$

The Lemma follows.

11.3 Proof of Theorem

We first note that the solution to problem (35) is equivalent to the solution of

$$\max_{I,R} \int V(I, R; \theta) dF(\theta) \tag{43}$$

subject to

$$\int \left\{ x_1^u(I, R; \theta) + \frac{x_2^u(I, R; \theta)}{\hat{R}} \right\} dF(\theta) \leq e. \tag{44}$$

We now analyze two key first order conditions that characterize problem (43). Consider the first order condition of this program with respect to income I:

$$\int \left[V_I(I, R; \theta) - \lambda \left\{ x_{1,I}^u(I, R; \theta) + \frac{x_{2,I}^u(I, R; \theta)}{\hat{R}} \right\} \right] dF_\theta = 0, \tag{45}$$

and the first order condition for the interest rate R :

$$\int \left\{ V_R(I, R; \theta) - \lambda \left[x_{1,R}^u(I, R; \theta) + \frac{x_{2,R}^u(I, R; \theta)}{\hat{R}} \right] \right\} dF_\theta = 0, \tag{46}$$

where we denote by λ a multiplier on (44), by $x_{1,I}^u$ and $x_{2,I}^u$ the derivatives of the uncompensated demands with respect to I, and by $x_{1,R}^u$ and $x_{2,R}^u$ derivatives of uncompensated demands with respect to R.

We manipulate these conditions to obtain a characterization of the optimal wedge between the interest rate on the private market and the return on savings. As these manipulations are purely algebraic and use basic properties of the indirect utility functions, we refer the interested reader to the working paper version of the paper. Specifically, let I^* and R^* be solutions to the problem (43). Let $x_2^c(I, R; \theta)$ be compensated demand in the problem (24), and $x_{2,R}^c(I, R; \theta)$ denote its derivative with respect to R. Then R^* satisfies

$$\frac{1}{\hat{R}} - \frac{1}{R^*} = \frac{\frac{1}{\lambda} \operatorname{Cov} \left\{ V_I(I^*, R^*; \theta), \frac{x_2^u(I^*, R^*; \theta)}{R^{*2}} \right\}}{\int x_{2,R}^c(I^*, R^*; \theta) dF_\theta + \operatorname{Cov} \left\{ x_{2,I}^u(I^*, R^*; \theta), \frac{x_2^u(I^*, R^*; \theta)}{R^{*2}} \right\}}, \tag{47}$$

Formula (47) characterizes the optimal wedge between the interest rate R^* and $\hat{R}$ in terms of easily interpretable parameters such as indirect utility functions, uncompensated and compensated demands, and the properties of the distribution of shocks.

Now we turn our attention to the denominator of (47). It is clear that $x_{2,R}^c(I, R; \theta) > 0$ – a standard property of compensated demand functions. However, the sign of $\operatorname{Cov} \left\{ x_{2,I}^u(I, R; \theta), \frac{x_2^u(I, R; \theta)}{R^2} \right\}$ is a priori indeterminate, even under the assumption that $V_I(I, R; \theta) > 0$ or $V_{I,\theta}(I, R; \theta) < 0$. Using Lemma 3 we determine the sign of the denominator

$$\int x_{2,R}^c(I, R; \theta) dF_\theta + \operatorname{Cov} \left\{ x_{2,I}^u(I, R; \theta), \frac{x_2^u(I, R; \theta)}{R^2} \right\} > 0. \tag{48}$$

Assumption 1 ensures that $\frac{\partial}{\partial\theta}\left(\frac{x_2^u(I,R;\theta)}{R^2}\right) > 0$. Then we conclude that if $V_{I,\theta}(I,R^*;\theta) > 0$ for all θ, then $\text{Cov}\left\{V_I(I^*,R^*;\theta), \frac{x_2^u(I^*,R^*;\theta)}{R^{*2}}\right\} > 0$; if $V_{I,\theta}(I,R^*;\theta) < 0$ for all θ, then $\text{Cov}\left\{V_I(I^*,R^*;\theta), \frac{x_2^u(I^*,R^*;\theta)}{R^{*2}}\right\} < 0$.

We now show a lemma that determines how $V_I(I,R^*;\theta)$ depends on preferences.

Lemma 4 *$V_{I,\theta}(I,R^*;\theta) > 0$ for all θ, if preferences are discount factor shocks as in example* 1*. $V_{I,\theta}(I,R^*;\theta) < 0$ for all θ, if preferences are liquidity shocks as in example* 2*; if preferences are valuation-neutral shocks as in example* 3*, $V_{I,\theta}(I,R^*;\theta) = 0$ for all θ.*

Proof. We have

$$V(I,R;\theta) = \max_{x_1,x_2} u(x_1,x_2;\theta)$$

subject to

$$x_1 + \frac{x_2}{R} \leq I.$$

Suppose first that preferences are given by $u(x_1) + \theta u(x_2)$. Then, substituting x_1 using the budget constraint, we can rewrite this problem as

$$V(I,R;\theta) = \max_{x_2} \hat{u}\left(I - \frac{x_2}{R}\right) + \theta\hat{u}(x_2).$$

By the Envelope theorem, we have

$$V_\theta(I,R;\theta) = \hat{u}(x_2^u(I,R,\theta)).$$

Hence,

$$V_{I,\theta} = \hat{u}'(x_2^u(I,R,\theta))x_{2,I}^u(I,R,\theta) > 0.$$

Suppose now that preferences are given by $\frac{1}{\theta}u(x_1) + u(x_2)$. By the Envelope theorem, we have

$$V_\theta(I,R;\theta) = -\frac{1}{\theta^2}\hat{u}(x_1^u(I,R,\theta)).$$

Hence

$$V_{I,\theta} = \frac{-1}{\theta^2}\hat{u}'(x_2^u(I,R,\theta))x_{1,I}^u(I,R,\theta) < 0.$$

For the value neutral preferences $V_{I,\theta} = 0$.

This proves the Lemma. ■

Applying Lemma 4 to formula (47) finishes the proof of the theorem.

References

[1] Albanesi, S., 2006, Optimal Taxation of Entrepreneurial Capital with Private Information, NBER WP 12419 and CEPR DP 5647.

[2] Allen, F., 1985, "Repeated Principal-Agent Relationships with Lending and Borrowing," Economics Letters, 17, 27-31.

[3] Allen, G. and Gale, D., 2003, Reserve Regulation: In Search of a Rationale, in *Economics for an Imperfect World: Essays in Honor of Joseph Stiglitz* edited by R. Arnott, B. Greenwald, R. Kanbur and B. Nalebuff, Cambridge, MA: MIT Press, 83-109

[4] Allen, G. and Gale, D., 2004, Financial Intermediaries and Markets, *Econometrica*, 72(4), 1023-1061.

[5] Arnott, R. and Stiglitz, J.E., 1986, Moral Hazard and Optimal Commodity Taxation, *Journal of Public Economics* 29, 1-24

[6] Arnott, R. and Stiglitz, J.E., 1990, The Welfare Economics of Moral Hazard. In H. Louberge ed. ed., *Information and Insurance: Essays in Memory of Karl H. Borch*, Norwell, MA: Kluwer, 91-121

[7] Atkeson, Andrew and Lucas, Robert E, Jr, 1992, "On Efficient Distribution with Private Information," *Review of Economic Studies*, Blackwell Publishing, vol. 59(3), pages 427-53, July.

[8] Bisin, A., J. D. Geanakoplos, P. Gottardi, E. Minelli, and H. Polemarchakis, 2001, Markets and Contracts, *Journal of Mathematical Economics*, forthcoming.

[9] Bisin, A. and Rampini, A., 2006,"Exclusive Contracts and the Institution of Bankruptcy", *Economic Theory,* 27 (2006), 277-304.

[10] Bhattacharya, S and D. Gale, 1987, Preference Shocks, Liquidity and Central Bank Policy, in W Barnett and K Singleton (eds), *New approaches to monetary economics*, Cambridge University Press, New York.

[11] Caballero, R. J. and Krishnamurthy, A., 2003, Smoothing Sudden Stops, forthcoming in *Journal of Economic Theory.*

[12] Chiappori, Pierre-Andre, Macho, Ines, Rey, Patrick and Salanie, Bernard, 1994., "Repeated moral hazard: The role of memory, commitment, and the access to credit markets," European Economic Review, Elsevier, vol. 38(8), pages 1527-1553, October.

[13] Diamond, D., 1997, Liquidity, Banks and Markets, *Journal of Political Economy*, 105, 928-956

[14] Diamond, D. and Dybvig, P., 1983, Bank Runs, Liquidity, and Deposit Insurance, *Journal of Political Economy*, 91, 401-419

[15] Freixas, X. and J-C Rochet.,1997, *Microeconomics of Banking*, Cambridge, Mass.: MIT.

[16] Geanakoplos, J. and H. Polemarchakis, 1986, Existence, Regularity, and Constrained Suboptimality of Competitive Allocations When the Asset Market Is Incomplete, in Walter Heller, Ross Starr, and David Starrett eds. Essays in honor of Kenneth J. Arrow: Volume 3, *Uncertainty, information, and communication.* Cambridge, New York and Sydney: Cambridge University Press, 65-95. Also reprinted in Debreu, Gerard, ed. (1996). *General equilibrium theory.* Volume 2. Elgar Reference Collection. International Library of Critical Writings in Economics, no. 67. Cheltenham, U.K.: Elgar; distributed by Ashgate, Brookfield, Vt. 67-97.

[17] Golosov, M. and Tsyvinski, A., 2007, Optimal Taxation with Endogenous Insurance Markets, forthcoming in *Quarterly Journal of Economics.*

[18] Golosov, M., Kocherlakota, N. and Tsyvinski, A., 2003. Optimal Indirect and Capital Taxation. *Review of Economic Studies* 70, 569-587

[19] Golosov, Tsyvinski, A, and I. Werning, 2006, New Dynamic Public Finance: A User's Guide, forthcoming in *NBER Macroeconomics Annual.*

[20] Gorton, Gary and A. Winton, 2002, Financial Intermediation, National Bureau for Economic Research (Cambridge, MA) Working Paper 8928.

[21] Greenwald, B.C. and Stiglitz, J.E., 1986, Externalities in Economies with Imperfect Information and Incomplete Markets, *Quarterly Journal of Economics*, 101, 229-264

[22] Guesnerie, R., 1998, *A Contribution to the Pure Theory of Taxation.* Cambridge University Press.

[23] Hammond, Peter J, 1987, Markets as Constraints: Multilateral Incentive Compatibility in Continuum Economies, *Review of Economic Studies*, Blackwell Publishing, vol. 54(3), pages 399-412, July.

[24] Hellmann, T., K. Murdock and J. Stiglitz (1998). Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Liquidity Requirements Enough?, Working Paper 1466R, Graduate School of Business, Stanford University.

[25] Hellmann, T., K. Murdock and J. Stiglitz, 2000, Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are liquidity Requirements Enough?, *American Economic Review,* 90, 147-165.

[26] Hellwig, M., 1994, Liquidity Provision, Banking, and the Allocation of Interest Rate Risk, *European Economic Review,* 38, 1363-1389

[27] Hellwig, M., 1998, Banks, Markets, and the Allocation of Risks in an Economy, *Journal of Institutional and Theoretical Economics,* 154, 328-345

[28] Holmstrom, B. and J. Tirole, "Private and Public Supply of Liquidity" , *Journal of Political Economy* , Vol. 106.

[29] Jacklin, C.J., 1987, Demand Deposits, Trading Restrictions, and Risk Sharing, in *Contractual Arrangements for Intertemporal Trade,* ed. E. Prescott and N. Wallace, Minneapolis: University of Minnesota Press, 26-47.

[30] Kocherlakota, N., 2006, "Advances in Dynamic Optimal Taxation," *Advances in Economics and Econometrics, Theory and Applications: Ninth World Congress of the Econometric Society.*

[31] Lorenzoni, G., 2006, Liquidity Crises and Efficiency, MIT mimeo.

[32] Merton, R., 1977, An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory, *Journal of Banking and Finance* 1, 3-11.

[33] Prescott, E. and R. Townsend, 1984, Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard, *Econometrica* 52, 21-45.

[34] von Thadden, E.-L., 1999, Liquidity Creation through Banks and Markets: Multiple Insurance and Limited Market Access, *European Economic Review*, 43, 991-1006.