Competition, Innovation and Growth with Limited Commitment*

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Abstract

We study how barriers to business start-up affect the investment in knowledge capital when contracts are not enforceable. Barriers to business start-up lower the competition for knowledge capital and, in absence of commitment, reduce the incentive to accumulate knowledge. As a result, countries with large barriers experience lower income and growth. Our results are consistent with cross-country evidence showing that the cost of business start-up is negatively correlated with the level and growth of per-capita income.

1 Introduction

It is widely recognized that sustained economic growth—especially, in advanced societies—requires investment in R&D, adoption of advanced technologies and innovation. A distinguished feature of modern technologies

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such as information and communication technologies, biotechnologies and nanotechnologies, is the importance of *skilled* human capital or *knowledge*. The ability of a society to innovate and grow is then dependent on how human capital is accumulated, organized and the returns shared among the participants in the innovation process. This is especially important when the parties who provide the financial needs—the investors—differ from those who acquire the innovation skills—innovators. For instance, whether a certain innovation project is funded depends on the ability of the investors to recover at least some of the returns of the project. At the same time, workers and managers—who carry out the project and are the actual innovators—must have the right incentives to accumulate the required skills or knowledge. This depends on the type of contractual arrangements that are feasible or enforceable. The goal of this paper is to study how free entry or competition affects the accumulation of innovation skills and the long-term growth of the economy when contracts are not perfectly enforceable.

The limited enforceability of contracts is double-sided. On the one hand, the innovator could put low effort in accumulating knowledge or quit the firm. This makes advance payments to the innovator unfeasible. On the other, the investor could replace the innovator and renege promises of payments. The main result of the paper is to show that, given the limited enforceability of contracts, free entry or competition increases the accumulation of knowledge and enhances the income and growth.

With free entry, the innovator can quit the current firm and being hired by a new firm. Competition for innovators creates an outside value for knowledge which is then used as a threat against the investor's attempt to renegotiate the promised payments. Under these conditions, the innovator has an incentive to accumulate more knowledge to keep the threat value high. Because innovations make existing capital obsolete, the higher accumulation of knowledge is inefficient at the firm level. In principle, the firm could prevent the over-accumulation of knowledge by making advance payments. However, this is not possible if contracts are not enforceable for the innovator. It becomes then crucial whether the firm can commit to future payments, conditional on the accumulation of knowledge. It is in the sense that the double-sided limited commitment plays a crucial role in our framework. We would like to point out that, although the higher accumulation of knowledge is inefficient at the firm level, it may still be efficient for the whole economy if there are 'spillovers'.

Our result differs from other papers that also study the accumulation of skills within the firm. In some of these models the investor has full control over the accumulation of skills or human capital. For example, in Acemoglu & Shimer (1999), is the employer that decides the amount of training. In this environment, greater mobility or outside opportunities hold-up the firm from investing in training because the workers capture a larger share of the firm's rents. In other models, such as the one studied in Acemoglu (1997), workers do control the accumulation of skills, but the main conclusion does not change: greater mobility worsens the hold-up problem and lead to lower accumulation of skills because workers are less likely to benefit from it. In our framework, instead, 'mobility' and 'competition' increase human capital (knowledge) investment.

Whether free entry enhances innovation has been a major topic of research and debate since Schumpeter's claim that, while product market competition could be detrimental for innovations, competition in the innovation sector enhances the rate of innovation. See, for example, Aghion & Howitt (1999). Most of the subsequent literature has focused on market structure and product market competition. In particular, on the ability to gain market shares and appropriate the returns to R&D, as in Aghion, Bloom, Blundell, Griffith, & Howitt (2005). More closely related to our work is Aghion, Blundell, Griffith, Howitt, & Prantl (2004). They show—both, theoretically and empirically—that 'firm entry' spurs innovation in technological advanced sectors as firms try to 'escape competition'. In contrast, we focus on the less studied dimension of 'human capital' competition. When contracts are not enforceable, competition for human capital becomes essential for growth because it keeps the market value of knowledge high and guarantees that innovators are rewarded for their innovation efforts. Free entry enhances the competition for human capital, and therefore, it stimulates growth.

Our results are consistent with the technological advances of the U.S. economy. For example, Bresnahan & Malerba (2002) argue that the lead of the US in the computer industry was possible thanks to a highly competitive environment, more prompt to stimulate new firms creation and innovations. For example, they claim that "The most important U.S. National institutions and policies supporting the emergence at this time [of the PC industry] were entirely non-directive: the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States". In the next section we also provide evidence that income and growth are negatively associated with the cost of business start-up in a cross-section of countries.

The paper relates to several strands of literature. First, the labor literature that studies the hold-up problem already discussed above (e.g., Acemoglu (1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). Second, the endogenous growth literature, starting with the pioneering work of Romer (1990, 1993), that studies the economics of ideas and the link between competition and innovations (e.g., Jones (1995), Aghion et al. (2004), Aghion et al. (2005) and, in particular, Aghion & Griffith (2005)). Third, the growth literature that, building on the work of economic historians (e.g., Mokyr (1990)), emphasizes the role of barriers to riches in slowing growth (Parente & Prescott (1990)). Forth, and foremost, the literature on dynamic contracts with enforcement constraints such as Marcet & Marimon (1992). Most of the contributions in this literature are not concerned with the issue of technology adoption and innovation (one exception is Kocherlachota (2001)) and they assume one-side commitment. Another typical assumption is that default or repudiation leads to market exclusion. In our framework, instead, the value of defaulting is the value of re-entering the market similar to Cooley, Marimon, & Quadrini (2004). It is this feature that makes 'competition' or 'free entry' relevant for growth. Furthermore, we show that limited commitment may enhance growth and, when there are spillovers, efficiency. This is in contrast to the prevailing result in the optimal contract literature where limited enforcement leads to lower investment.

The plan of the paper is as follows. In Section 2 we provide cross-country evidence on barriers to business start-up and macroeconomic performance. Section 3 describes the model. To facilitate the intuition for the theoretical results, Section 4 studies a simplified version of the model with only two periods. Section 5 generalizes it to the infinite horizon. Section 7 concludes.

2 Cross-country evidence

A recent publication from the World Bank (2005) provides data on the quality of the business environment for a cross-section of countries. Especially important for this study is the *Cost of Starting a Business*. This is the 'average pecuniary cost' needed to set-up a corporation in the country, expressed in percentage of the country per-capita income.

Figure 1 plots the level of per-capita income against this cost. Both variables are in log. The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant for the

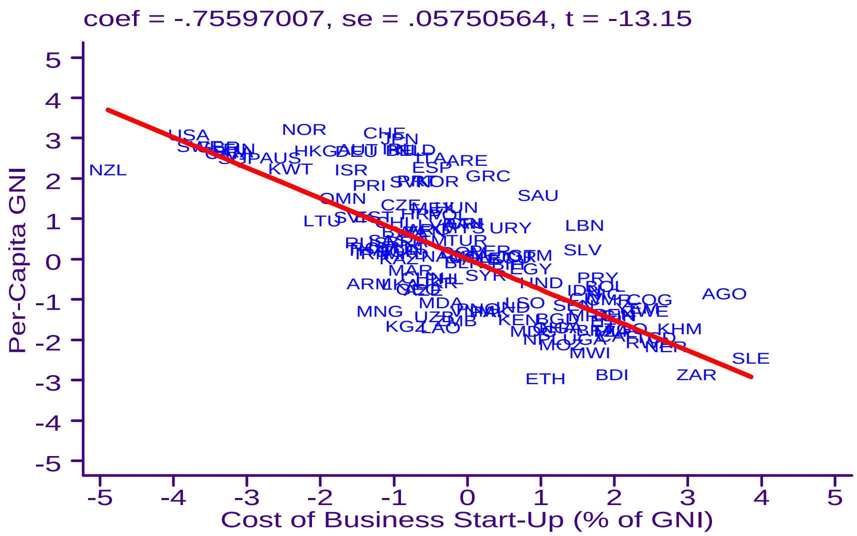

Figure 1: Cost of starting a business and level of development.

decision to start a business is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost is on average higher in advanced economies, the value of a new business is also higher.

As can be seen from Figure 1, there is a strong negative correlation between the cost of starting a business and the development of the country. A similar figure would result if we use alternative measures of the cost of business start-up, such as the 'number of bureaucratic procedures' or the 'number of days' needed to start a new business.

The costs of business start-up is also negatively related to economic growth. Figure 2 correlates the average growth rate in per-capita GDP in the last five years of available data (1999-2003) to the cost of starting a business. The correlation is negative and statistically significant, showing that countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years to compute the average growth rate.

To further investigate the relation between the cost of business start-up and growth, we regress the five years average growth in per-capita GDP to the cost of business start-up and the 1998 per-capita GDP to control for the initial level of development. The estimation results, with t-statistics in

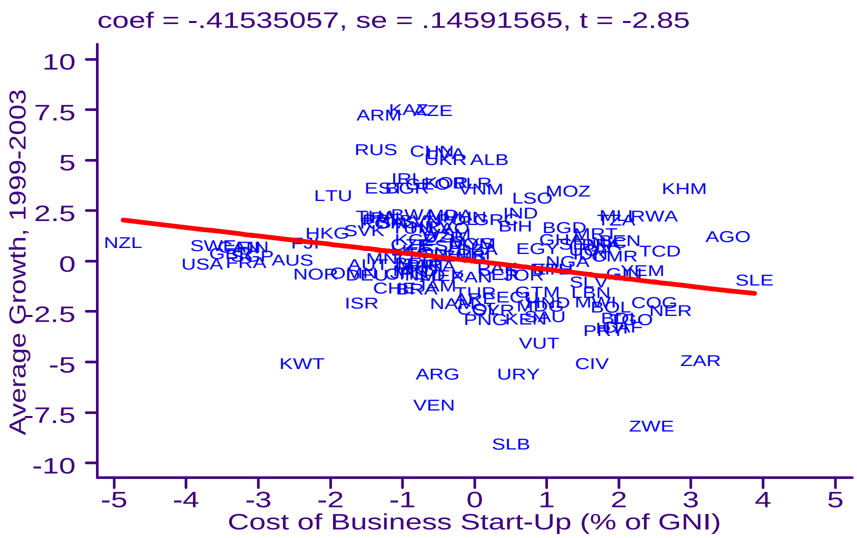

Figure 2: Cost of starting a business and economic growth.

parenthesis, are reported in the top section of Table 1. They show that the cost of business start-up is negatively associated with growth even if we control for the level of economic development.

To show that these findings are not an artifact of normalizing the cost of business start-up by the level of per-capita income, the bottom section of Table 1 repeats the same regression estimation but using dollar values, in log, of the cost of business start-up. Again, the cost of business start-up is statistical significant with a negative sign. In this regression, however, the initial per-capita GDP is no longer significant.

To summarize, the general picture portrayed by the analysis of this section is that the economic development and growth of a country is negatively associated with the cost of starting a business. In the following sections we present a model that is consistent with these findings.

3 The model

There are two types of agents in the economy: a continuum of 'entrepreneurs' of total mass $m > 1$ and a continuum of 'innovators' of total mass 1. Therefore, innovators are in short supply relatively to the number of entrepreneurs. Innovators maximize the lifetime utility $\sum_{t=0}^{\infty} \beta^t c_t$, where c_t is consumption. Entrepreneurs maximize the utility $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$, where c_t is also con-

Table 1: Cost of business start-up and growth.

		Constant	*Initial Per-Capita GDP*	*Cost of Business Start-Up*
(a)	Coefficients	11.93	-0.86	-0.87
	t-Statistics	*(3.84)*	*(-2.83)*	*(-4.05)*
	R-square	0.110		
	N. of countries	136		
(b)	Coefficients	4.65	0.25	-0.76
	t-Statistics	*(2.53)*	*(1.23)*	*(-3.61)*
	R-square	0.093		
	N. of countries	131		

NOTES: Dependent variable is the average annual growth rate in per-capita GDP for the five year period 1999-2003. Initial Per-Capita GDP is the log of per-capita GDP in 1998. In panel (a) the costs of business start-up is in percentage of the per-capital Gross National Income as reported in *Doing Business in 2004*. In panel (b) is the dollar value of this cost. Both measures of the cost of business start-up are in logs.

sumption and e_t is the effort to accumulate human capital (knowledge) as specified below.

Innovators do not save. This simplifying assumption should be interpreted as an approximation to the case in which innovators discount more heavily than entrepreneurs. The risk neutrality of entrepreneurs implies that the equilibrium interest rate is equal to the intertemporal discount rate, that is, $r = 1/\beta - 1$.

Firms are owned by entrepreneurs who need the management and innovation skills of innovators. The production function is:

$$y_t = z_t^{1-\alpha} k_t^{\alpha}$$

where z_t is the level of technology and k_t is the capital chosen at time $t-1$.

The variable z_t changes over time as the firm adopts new technologies. The key assumption is that the implementation of more advanced technologies requires higher knowledge. An innovator with knowledge h_t has the ability to install or implement any technology $z_t \leq h_t$.

Innovators are needed not only for the introduction and implementation of a new technology but also to run the firm once the technology has been installed. We should think as if a period is divided in two subperiods. The technology is introduced in the first sub-period and run in the second sub-period. However, once the technology has been implemented, any innovator can run the firm, independently of his knowledge.

The investment in knowledge, $h_{t+1}-h_t$, requires effort from the innovator. The required effort depends on the economy-wide level of knowledge H_t, due to leakage or spillover effects, and it is determined by:

$$e_t = \varphi(H_t; h_t, h_{t+1})$$

The function φ is homogeneous of degree 1, strictly decreasing in H_t and h_t, strictly increasing in h_{t+1}, and satisfies $\varphi(H_t; h_t, h_t) = 0$. We can give different interpretations to H_t. It can be interpreted as the average level of knowledge in the country. In this case the paper provides a theory of cross-country growth differentials. Alternatively, H_t can be interpreted as the world-wide level of knowledge. This is external to each individual country. In this case we would have a theory of cross-country income differentials. From now on we adopt the first interpretation but it should be clear that the analysis can be easily adjusted to provide a theory of cross-country income differences.

Physical capital is technology-specific: when the firm innovates, only part of the physical capital is usable with the new technology. Furthermore, capital obsolescence increases with the degree of innovation. This is formalized by assuming that the depreciation rate increases with the size of the innovation, that is,

$$\delta_t = \delta \cdot \left(\frac{z_{t+1} - z_t}{z_t} \right)$$

Because of capital obsolescence, there is an asymmetry between *incumbent* and *new* firms. Because new firms are still uncommitted to any previous investment, they have greater incentive to innovate.

The competitive structure of the model is as follows. In each period there is a walrasian market for innovators who can move freely from one firm to the other. The market opens twice: before and after the accumulation of knowledge. Both incumbent and new firms can participate in this market. However, the presence of barriers to entry may limit the effective presence of new firms. There are different ways to model barriers to entry. Here we adopt a simple formulation and assume that new firms incur a deadweight loss proportional to the initial level of knowledge. Given h_{t+1} the initial knowledge chosen by a new firm, the entry cost is $\tau \cdot h_{t+1}$. Our goal is to study how this cost affects the equilibrium when contracts are not enforceable.[1]

The last assumption is that firms remain productive with probability p. Whether a firm survives is revealed after the investment in knowledge. This guarantees that, in the second stage of each period, the mass of innovators is larger than incumbent (surviving) firms. This is important for the characterization of the equilibrium as we will emphasize below. Here we want to stress that what is important is not the value of p but the fact that it is strictly smaller than 1. Given that, we assume that p is very close to 1 and, in the characterization of the individual problems, we will ignore it. The explicit consideration of p will only make the notation more complex but it would not change the results.

4 Two-period model

To gauge some intuitions about the key properties of the model, it would be convenient to consider first a simplified version of the model with only two periods: period zero and period one. The state variables of the firm at the beginning of period zero are h_0 and k_0. After making the investment decisions, h_1 and k_1, the firm generates output $y_1 = z_1^{1-\alpha} k_1^{\alpha}$ in period one. Because $z_1 = h_1$, the output can also be written as $y_1 = h_1^{1-\alpha} k_1^{\alpha}$. In this simple version of the model we assume that physical capital fully depreciates after production. The innovator receives a payment from the firm (compensation) at the end of period zero, after the choice of h_1. Payments before the choice of h_1 are not feasible because of the limited enforcement of contracts for the innovator, while allowing for additional payments in period 1 does not change the results. For the analysis of this section we also assume that there is no discounting and the effort cost does not depend on the economy-wide knowledge H. The leakage or spillover effect is not relevant when there are

[1] Alternative assumptions would have similar implications. For example, we could assume that the cost is proportional to the initial capital k_{t+1} or to the initial output $Ah_{t+1}^{1-\alpha} k_{t+1}^{\alpha}$ or to the discounted flows of outputs. None would change the results but would make the analysis more complex. The assumption of proportionality guarantees that the equilibrium impact of τ is continuous while the impact of a fixed cost would be discontinuous, that is, it would impact on the economy only after it has reached the prohibitive level.

only two periods.

The timing of the model can be summarized as follows: The firm starts period zero with initial states h_0 and k_0. At this stage the innovator decides whether to stay or quit the firm. If he quits, he can be hired by an incumbent firm or a new firm (funded by a new entrepreneur). If the innovator decides to stay, he will choose the new level of knowledge h_1 and implement the technology $z_1 = h_1$. The entrepreneur provides the funds to accumulate the new physical capital k_1. After the investment decision has been made, the entrepreneur pays w_0 to the innovator. At this stage the innovator can still quit, but he cannot change the level of knowledge h_1. The entrepreneur is the residual claimant of the firm's output.

4.1 Equilibrium with entrepreneur's commitment

To show the importance of contract enforcement, we will first characterize the equilibrium under the assumption that the entrepreneur commits to the contract. This will facilitate the characterization of the equilibrium without commitment.

When the entrepreneur commits to the long-term contract, all variables are chosen at the beginning of the first period to maximize the total surplus. Let $D(h_0)$ be the repudiation value before choosing h_1 and $\widehat{D}(h_1)$ the repudiation value after choosing h_1. These functions are endogenous and will be derived below as the value that the innovator would get by quitting the firm and re-entering the market. From now on we will use the *hat* sign to denote all functions that are defined *after* the investment in knowledge. The participation of the innovator requires that the value of staying is greater than the repudiation value before and after the knowledge investment, that is,

$$\begin{aligned} w_0 - \varphi(h_0, h_1) &\geq D(h_0) \\ w_0 &\geq \widehat{D}(h_1) \end{aligned}$$

The first is the participation constraint before choosing h_1 and the second is the participation constraint after the choice of h_1. We will show below that, if the first constraint is satisfied, the second is also satisfied. Therefore, in the derivation of the optimal policy we can neglect the second constraint and write the optimization problem as follows:

$$\max_{h_1, k_1, w_0} \left\{ -\varphi(h_0, h_1) - k_1 + \left[1 - \delta \cdot \left(\frac{h_1 - h_0}{h_0}\right)\right] k_0 + A h_1^{1-\alpha} k_1^{\alpha} \right\} \quad (1)$$

s.t.

$$w_0 - \varphi(h_0, h_1) \geq D(h_0)$$

$$-w_0 - k_1 + \left[1 - \delta \cdot \left(\frac{h_1 - h_0}{h_0}\right)\right] k_0 + A h_1^{1-\alpha} k_1^{\alpha} \geq 0$$

where the second constraint is the participation constraint for the entrepreneur. In writing the problem we have substituted $z_1 = h_1$ in the production and depreciation function.

From the maximization problem it can be verified that the investment choices (in knowledge and physical capital) are independent of the choice of the innovator's payment w_0. The value of w_0 is determined by the split of the surplus which we specify below. The only constraint is that the payment does not violate the two participation constraints.

To determine the repudiation value before the choice of h_1, we have to solve for the optimal investment when the innovator quits the current firm. The innovator could be hired by an incumbent or a new firm, whoever makes the best offer. However, an incumbent firm will never offer more than a new firm. Therefore, it becomes relevant to determine the offer made by a new firm. This is derived by solving the contractual problem of a new firm, that is:

$$S(h_0) = \max_{h_1, k_1, w_0} \left\{ -\varphi(h_0, h_1) - \tau h_1 - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \right\} \tag{2}$$

s.t.

$$w_0 - \varphi(h_0, h_1) \geq D(h_0)$$

$$-w_0 - \tau h_1 - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \geq 0$$

The problem of a new firm differs from the problem solved by an incumbent firm in two respects. First, a new firm does not have any initial physical capital, and therefore, it does not incur any obsolescence of capital. Second, a new firm has to pay the entry cost τh_1. It is still the case, however, that the choices of h_1 and k_1 are independent of w_0.

Because of competition, the innovator gets the whole surplus $S(h_0)$. This implies that $D(h_0) = S(h_0)$. Therefore, if the innovator stays with the incumbent firm, the payment w_0, net of the effort cost, $\varphi(h_0, h_1)$, must be at least as large as $S(h_0)$. Formally, the participation constraint in problem (1) becomes:

$$w_0 - \varphi(h_0, h_1) \geq S(h_0) \tag{3}$$

Problems (1) and (2) show the different incentive to invest for an incumbent versus a new firm. On the one hand, new firms do not have any physical capital and innovations do not generate capital obsolescence. Hence, they have a greater incentive to innovate than incumbent firms. On the other, they must pay the entry cost τh_1, which discourages knowledge and capital accumulation. This is clearly shown by the first order conditions for h_1 in problems (1) and (2), that is,

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right) \tag{4}$$

$$(1-\alpha)\left(\frac{k_1}{h_1}\right)^{\alpha} = \varphi_{h_1}(h_0, h_1) + \tau \tag{5}$$

where the subscripts denote derivatives.

Condition (4) is for incumbent firms and condition (5) is for new firms. The left-hand-side terms are the marginal productivity of knowledge. The right-hand-side terms are the marginal costs. The marginal cost for an incumbent firm is the effort cost incurred by the innovator plus the obsolescence of physical capital. For a new firm the obsolescence cost is replaced by the entry cost. Using the first order condition for the choice of physical capital, which is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms, the above first order conditions can be rewritten as:

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right) \tag{6}$$

$$(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} = \varphi_{h_1}(h_0, h_1) + \tau \tag{7}$$

These two conditions clearly show that, when the entry cost is zero (free entry), new firms innovate more than incumbents. However, the innovation incentive of new firms declines as the entry cost increases.

Let h^{Old} be the optimal knowledge investment of an incumbent firm and h^{New} the optimal knowledge investment of a new firm. The following proposition formalizes the above results.

Proposition 1 *The knowledge investment of a new firm h^{New} is strictly decreasing in the entry cost τ and there exists $\bar{\tau} > 0$ such that $h^{New} = h^{Old}$.*

Proof 1 *This follows directly from conditions (6) and (7).* *Q.E.D.*

In the equilibrium with entrepreneur's commitment, there will not be any entrance of new firms at the beginning of the period and the investment in knowledge is $h_1 = h^{Old}$. The potential entrance of new firms only affects the minimum payment received by an innovator. In the second stage of the period there will be the entrance of new firms because some incumbents become unproductive. However, the initial knowledge of innovators cannot be changed at this stage. The next step is to show that this is not an equilibrium when contracts are not enforceable (limited commitment from the entrepreneur).

4.2 Equilibrium with double-side limited commitment

We want to show first that, after the accumulation of knowledge, the entrepreneur has an incentive to renegotiate the optimal contract. Because the innovator can always be hired by a new firm, the value received by an incumbent firm must be at least as large as the surplus generated by a new firm. The surplus of new firms, however, changes before and after the investment in knowledge, which creates the condition for renegotiation.

Let's derive first the surplus generated by a new firm after the investment in knowledge. This is given by:

$$\widehat{S}(h_1) \;=\; \max_{k_1, w_0} \Big\{ -\tau h_1 - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \Big\} \tag{8}$$

s.t.

$$w_0 \geq \widehat{D}(h_1)$$

$$-w_0 - \tau h_1 - k_1 + A h_1^{1-\alpha} k_1^{\alpha} \geq 0$$

Because of competition, the innovator gets the whole surplus, that is, $\widehat{D}(h_1) = \widehat{S}(h_1) = w_0$, and the participation constraint, after the investment in knowledge, becomes $w_0 \geq \widehat{S}(h_1)$.

If the innovator stays with the firm and accumulate h^{Old}, the innovator will renegotiate the promised payments if they are greater than $\widehat{S}(h^{Old})$, that is, greater than the payment that the innovator would obtain by quitting the firm with $h_1 = h^{Old}$. In this case the total utility enjoyed by the innovator would be:

$$-\varphi(h_0, h^{Old}) + \widehat{S}(h^{Old})$$

If the innovator quits the firm at the beginning of the period, he will get the whole surplus $S(h_0)$ generated by the new firm, which is equal to:

$$\begin{aligned} S(h_0) &= \max_h \Big\{ -\varphi(h_0, h) + \widehat{S}(h) \Big\} \\ &= -\varphi(h_0, h^{New}) + \widehat{S}(h^{New}) \end{aligned}$$

Because $h^{New} \neq h^{Old}$, the value of quitting at the beginning of the period, $S(h_0)$, is greater than the value of staying. Therefore, the innovator will quit the firm at the beginning of the period unless the entrepreneur agrees to the same knowledge investment and innovation chosen by a new entrant firm, that is, $h^{Old} = h^{New}$. In this way the innovator keeps the repudiation value high and prevents the entrepreneur from renegotiating. This leads to the following proposition.

Proposition 2 *Without entrepreneur's commitment, the investment in knowledge is $h_1 = h^{New}$.*

According to the proposition, incumbent firms accumulate the same level of knowledge as new firms. Because h^{New} is decreasing in τ (see Proposition 1), the accumulation of knowledge decreases with the cost of entry. Therefore, higher are the barriers to entry, and lower is the accumulation of knowledge.

There are several points in the proof of the proposition that should be emphasized. First, the renegotiation threat of the firm after the accumulation of knowledge is credible because the firm can always replace the current innovator with some other innovator. This could be either an innovator still employed by an incumbent (surviving) firm, or an innovator who separated from an exiting firm. Because innovators are in the long side of the market, they only get the reservation value, which is the value received from a new firm.

How would the results change if $p = 1$ and the number of innovators is equal to the number of incumbent firms? We would have multiple equilibria. Each firm would renegotiate if all other firms renegotiate. But each individual firm would not renegotiate if all other firms do not renegotiate. The assumption of a positive probability of exit, although small, eliminates this multiplicity.

Another point to clarify is the role played by the assumption that knowledge is only necessary for the introduction of technologies but not for managing a firm after their implementation (although the management still need to be done by an innovator). This assumption eliminates the following problem. Suppose that we are in an equilibrium with $h_1 = h^{New} \neq h^{Old}$. Under this equilibrium, an individual firm may actually be able to make a credible payment promise to the innovator and convince him to accumulate h^{Old}. Because in the second stage this is the only innovator with knowledge h^{Old} (all others have accumulated h^{New}), there is no innovator in the market that the firm can hire to replace him. Consequently, the firm will be unable to renegotiate.[2] This implies that $h_1 = h^{New}$ cannot be an equilibrium. This problem does not arise if the firm can use any innovator to run the firm once the technology has been adopted.

To summarize, free entry or competition leads to greater investment in knowledge. The greater investment is not individually efficient due to the creative destruction of physical capital. Without externalities, this is also inefficient at the social level. However, the presence of splillovers in the accumulation of knowledge may make higher investment socially desirable. We will re-introduce the splillovers in the analysis of the infinite horizon model.

[2]This is true independently of whether h^{New} is greater or smaller than h^{Old}. If $h^{New} < h^{Old}$, then all the replacements have lower knowledge and unable to run the more advance technology. If $h^{New} > h^{Old}$, then the deviating firm could use innovators with higher knowledge. But these innovators must be paid more because their reservation value (when employed by a new firm) is higher.

5 The infinite horizon model

In this section we study the general model with infinitely lived agents. To use a more compact notation, we define the gross output function, inclusive of undepreciated capital, as follows:

$$\pi(h_t, k_t, h_{t+1}) = \left[1 - \delta \cdot \left(\frac{h_{t+1} - h_t}{h_t}\right)\right] k_t + A h_t^{1-\alpha} k_t^{\alpha}$$

In writing this expression, we assume that the firm uses the best technology adoptable by the innovator, that is, $z_t = h_t$. It is easy to show that the choice of $z_t < h_t$ is never optimal.

We first characterize the equilibrium when the entrepreneur commits to the contract and then we turn to the case of limited enforcement.

5.1 Equilibrium with entrepreneur's commitment

Let $D_t(h_t)$ be the repudiation value for the innovator at the beginning of the period, before investing in knowledge. This is the value that the innovator with knowledge h_t would get from quitting an incumbent firm. Furthermore, let $\widehat{D}_t(h_{t+1})$ be the value of quitting after choosing the knowledge investment, and therefore, after exercising effort. At this point the stock of knowledge is h_{t+1}. These functions also depend on the aggregate states of the economy, which explains the time subscript. For the moment we take these two functions as given.

Consider the optimization problem solved by a new firm that hires an innovator with knowledge capital h_t at the beginning of period t. This is before the innovator makes the new investment in knowledge. Because with competition the innovator gets the whole surplus, it will be convenient to characterize the optimal contract by maximizing the value for the innovator, subject to the enforceability and participation constraints.[3] The optimization problem can be written as:

$$V_t(h_t) = \max_{\{w_s, k_{s+1}, h_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} \Big[w_s - \varphi(H_s; h_s, h_{s+1}) \Big] \tag{9}$$

subject to

[3] Alternatively, we could maximize the whole surplus as we did in the analysis of the two-period model. Of course, this would give the same results.

$$\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]\geq D_s(h_s), \qquad \text{for } s\geq t \tag{10}$$

$$w_s+\sum_{j=s+1}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big]\geq \widehat{D}_s(h_{s+1}), \quad \text{for } s\geq t \tag{11}$$

$$-\tau h_{t+1}-w_t-k_{t+1}+\sum_{s=t+1}^{\infty}\beta^{s-t}\Big[\pi(h_s,k_s,h_{s+1})-w_s-k_{s+1}\Big]\geq 0 \tag{12}$$

The objective is the discounted flow of utilities for the innovator. In each period he receives the payment w_s, which is subject to a non-negativity constraint, and faces the disutility from effort $\varphi(H_s;h_s,h_{s+1})$. Constraints (10) and (11) are the enforcement constraints. Starting at time $t+1$, the innovator could quit at the beginning of the period, before choosing the investment in knowledge. In this case the repudiation value is $D_s(h_s)$. After choosing the knowledge investment, the value of quitting becomes $\widehat{D}_s(h_{s+1})$. The last constraint is the participation constraint for the entrepreneur or break-even condition. This simply says that the value of the contract for the entrepreneur cannot be negative.

For an innovator who gets hired *after* investing in knowledge, the value of the contract is:

$$\widehat{V}_t(h_{t+1}) \;=\; \max_{\{w_s,\,k_{s+1},\,h_{s+2}\}_{s=t}^{\infty}}\left\{w_t+\sum_{s=t+1}^{\infty}\beta^{s-t}\Big[w_s-\varphi(H_s;h_s,h_{s+1})\Big]\right\}$$

subject to (10), (11) and (12)

The key difference respect to problem (9) is that the effort to accumulate knowledge has already been exercised by the innovator and h_{t+1} is given at this point. Consequently, the current return for the innovator is only w_t. This also explains why the choice of knowledge starts at $t+2$.

Given the definitions of $V_t(h_t)$ and $\widehat{V}_t(h_{t+1})$, it is easy to see that these two functions are related as follows:

$$V_t(h_t)=\max_{h_{t+1}}\Big\{-\varphi(H_t;h_t,h_{t+1})+\widehat{V}_t(h_{t+1})\Big\} \tag{13}$$

The above optimization problems assume that the repudiation functions $D_t(h_t)$ and $\widehat{D}_t(h_{t+1})$ are known. But these functions are endogenous because they depend on the value functions $V_t(h_t)$ and $\widehat{V}_t(h_{t+1})$. More specifically, the repudiation value is equal to the value obtained by moving to a new firm, that is, $D_s(h_s) = V_s(h_s)$ and $\widehat{D}_s(h_{s+1}) = \widehat{V}_s(h_{s+1})$.

Before proceeding we establish the following property:

Lemma 1 *Constraint (11) is always satisfied if constraint (10) is satisfied.*

Proof 1 *See Appendix A.*

Hence, in characterizing the solution of the optimal contract with entrepreneur's commitment, we can ignore the enforcement constraint (11). This constraint becomes relevant when the entrepreneur does not commit.

5.1.1 Stationary transformation and first order conditions

Because the economy may experience persistent growth, it will be convenient to rescale all growing variables to make the problem stationary. Define $g_t = H_{t+1}/H_t$, $\tilde{h}_t = h_t/H_t$, $\tilde{k}_t = k_t/H_t$, $\tilde{w}_t = w_t/H_t$. Because the effort cost $\varphi(H_t; h_t, h_{t+1})$ and the gross output $\pi(h_t, k_t, h_{t+1})$ are homogeneous of degree 1, these two functions can be rewritten as:

$$\begin{aligned} \varphi(H_t; h_t, h_{t+1}) &= \varphi(\tilde{h}_t, g_t\tilde{h}_{t+1}) \cdot H_t \\ \pi(h_t, k_t, h_{t+1}) &= \pi(\tilde{h}_t, \tilde{k}_t, g_t\tilde{h}_{t+1}) \cdot H_t \end{aligned}$$

We then have the following lemma:

Lemma 2 *The functions $V_t(h_t)$, $D_t(h_t)$ and $\widehat{D}_t(h_{t+1})$ take the form:*

$$\begin{aligned} V_t(h_t) &= V_t(\tilde{h}_t) \cdot H_t \\ D_t(h_t) &= D_t(\tilde{h}_t) \cdot H_t \\ \widehat{D}_t(h_{t+1}) &= \widehat{D}_t(g_t\tilde{h}_{t+1}) \cdot H_t \end{aligned}$$

Proof 2 *See Appendix B.*

Given this lemma, we can use the aggregate stock of knowledge H_t—which grows over time—as a scaling factor for all growing variables. The optimization problem can be rewritten as:

$$V_t(\tilde{h}_t) = \max_{\{\tilde{w}_s, \tilde{k}_{s+1}, \tilde{h}_{s+1}\}_{s=t}^{\infty}} \left\{ \sum_{s=t}^{\infty} \beta_{t,s} \Big[\tilde{w}_s - \varphi(\tilde{h}_s, g_s \tilde{h}_{s+1}) \Big] \right\} \tag{14}$$

subject to

$$\sum_{j=s}^{\infty} \beta_{s,j} \Big[\tilde{w}_j - \varphi(\tilde{h}_j, g_j \tilde{h}_{j+1}) \Big] \geq D_s(\tilde{h}_s), \qquad \text{for } s \geq t \tag{15}$$

$$-\tau g_t h_{t+1} - \tilde{w}_t - g_t \tilde{k}_{t+1} + \sum_{s=t+1}^{\infty} \beta_{t,s} \Big[\pi(\tilde{h}_s, \tilde{k}_s, g_s \tilde{h}_{s+1}) - \tilde{w}_s - g_s \tilde{k}_{s+1} \Big] \geq 0 \tag{16}$$

where $\beta_{t,s} = \Big(\Pi_{j=t}^{s-1} \beta g_j \Big)$.

In general, the optimization problem depends on the future evolution of the economy-wide knowledge H_t, which in turn depends on the distribution of firms over the normalized knowledge and physical capital. In the analysis that follows, however, we concentrate on the balanced growth equilibrium where the distribution of firms remains constant and the average knowledge grows at the constant rate g. Under these conditions, the discount factor becomes $\beta_{t,s} = (g\beta)^{s-t}$ and we can ignore the time subscript in all value functions.

To characterize the solution, it will be convenient to use a transformation of problem (14). Appendix C shows that in the balanced growth path the optimization problem can be reformulated as follows:

$$\min_{\mu' \geq \mu_0} \max_{\tilde{w} \geq 0, \tilde{h}', \tilde{k}'} \Bigg\{ -\tau g \tilde{h}' - \tilde{w} - g \tilde{k}' + \mu' \Big[\tilde{w} - \varphi(\tilde{h}_0, g \tilde{h}') \Big] - (\mu' - \mu_0) D(\tilde{h}_0) + g\beta W(\mu', \tilde{h}', \tilde{k}') \Bigg\} \tag{17}$$

where μ_0 is the inverse of the Lagrange multiplier associated with the participation constraint (16) and the function W is defined recursively as:

$$W(\mu, \tilde{h}, \tilde{k}) = \min_{\mu' \geq \mu} \max_{\tilde{w} \geq 0, \tilde{h}', \tilde{k}'} \Bigg\{ \pi(\tilde{h}, \tilde{k}, g\tilde{h}') - \tilde{w} - g\tilde{k}' + \mu' \Big[\tilde{w} - \varphi(\tilde{h}, g\tilde{h}') \Big]$$

$$-(\mu' - \mu)D(\tilde{h}) + g\beta W(\mu', \tilde{h}', \tilde{k}')\Big\} \quad (18)$$

Problem (17) is the problem solved by a new firm that hires an innovator with (normalized) knowledge $\tilde{h}_0$. After starting the firm and choosing the first period investment, the problem becomes recursive as written in (18). Therefore, (18) is the problem solved by an incumbent firm. The variable μ can be interpreted as the weight that a hypothetical planner gives to the innovator. The weight given to the entrepreneur is 1. Over time the planner increases the weight μ to make sure that the innovator does not quit the firm, until $\mu = 1$. Higher is the initial weight μ_0 and higher is the initial value of the contract for the innovator, and therefore, lower is the value for the entrepreneur. The value of μ_0 is determined such the breaking even condition for the entrepreneur, that is, constraint (16) satisfied with the equality sign. See Marcet & Marimon (1997) for details about the use of the saddle-point formulation.

The solution to problem (17) is characterized by the first order conditions:

$$D(\tilde{h}_0) \leq \tilde{w} - \varphi(\tilde{h}_0, g\tilde{h}') + g\beta D(\tilde{h}') \quad (19)$$

$$\mu' \leq 1 \quad (20)$$

$$\tau + \mu'\varphi_2(\tilde{h}_0, g\tilde{h}') = \beta W_2(\mu', \tilde{h}', \tilde{k}') \quad (21)$$

$$\beta\pi_2(\tilde{h}', \tilde{k}', g\tilde{h}') = 1 \quad (22)$$

and the solution to problem (18) by the first order conditions:

$$D(\tilde{h}) \leq \tilde{w} - \varphi(\tilde{h}, g\tilde{h}') + g\beta D(\tilde{h}') \quad (23)$$

$$\mu' \leq 1 \quad (24)$$

$$-\pi_3(\tilde{h}, \tilde{k}, g\tilde{h}') + \mu'\varphi_2(\tilde{h}, g\tilde{h}') = \beta W_2(\mu', \tilde{h}', \tilde{k}') \quad (25)$$

$$\beta\pi_2(\tilde{h}', \tilde{k}', g'\tilde{h}'') = 1 \quad (26)$$

Here we use subscripts to denote the derivative with respect to the particular argument. In conditions (19) and (23) the inequality constraints are strict if $\mu' > \mu$, while in conditions (20) and (24) the inequalities are strict if the innovator's payments are zero, that is, $\tilde{w} = 0$. The envelope term is:

$$W_2(\mu, \tilde{h}, \tilde{k}) = \pi_1(\tilde{h}, \tilde{k}, g\tilde{h}') - \mu'\varphi_1(\tilde{h}, g\tilde{h}') - (\mu' - \mu)D_1(\tilde{h})$$

5.1.2 Equilibrium

We can now provide a formal definition of a balanced growth equilibrium.

Definition 1 *A Balanced Growth Equilibrium with entrepreneur's commitment is defined as:* `(a)` *Decision rules* $\mu^0(\mu,\tilde{h})$, $d^0(\mu,\tilde{h})$, $h^0(\mu,\tilde{h})$, $k^0(\mu,\tilde{h})$ *for new firms;* `(b)` *Decision rules for incumbent firms* $\mu(\mu,\tilde{h})$, $d(\mu,\tilde{h})$, $h(\mu,\tilde{h})$, $k(\mu,\tilde{h})$*;* `(c)` *Initial state* $\mu_0(\tilde{h})$ *for new firms;* `(d)` *A repudiation function* $D(\tilde{h})$ *and a value function* $V(\tilde{h})$*;* `(e)` *A distribution of firms* $M(\mu,\tilde{h},\tilde{k})$ *and a growth rate* g^**. Such that:* `(i)` *The decision rules* $\mu^0(\mu,\tilde{h})$, $d^0(\mu,\tilde{h})$, $h^0(\mu,\tilde{h})$, $k^0(\mu,\tilde{h})$ *solve problem (17);* `(ii)` *The decision rules* $\mu(\mu,\tilde{h})$, $d(\mu,\tilde{h})$, $h(\mu,\tilde{h})$, $k(\mu,\tilde{h})$ *solve problem (18);* `(iii)` *The initial state* $\mu_0(\tilde{h})$ *is such that new entrepreneurs break even;* `(iv)` *The repudiation function is the value of starting a new firm,* $D(\tilde{h}) = V(\tilde{h})$*;* `(v)` *The distribution* $M(\mu,\tilde{h},\tilde{k})$ *remains constant over time and the economy grows at rate* g^**.*

The key object of a steady state equilibrium is the constancy of the distribution $M(\mu,\tilde{h},\tilde{k})$. With positive spillovers, the accumulation of knowledge is cheaper for agents with $\tilde{h} < 1$ and more costly for agents with $\tilde{h} > 1$. This implies that the steady state distribution of knowledge is characterized by a unit mass of agents at $\tilde{h} = 1$, that is, all firms have the average knowledge.

We can now characterize the balanced growth equilibrium using the first order conditions derived above. Because all firms have been active for a long period of time, they will have $\mu = \mu' = 1$. Furthermore, their normalized knowledge has converged to $\tilde{h} = 1$, the normalized capital has converged to some constant value $\tilde{k}^*$, and they grow at the constant rate g^*. The steady state values of $\tilde{k}^*$ and g^* are determined by the conditions:

$$-\pi_3(1,\tilde{k}^*,g^*) + \varphi_2(1,g) = \beta\Big[\pi_1(1,\tilde{k}^*,g^*) - \varphi_1(1,g^*)\Big]$$

$$\beta\pi_2(1,\tilde{k}^*,g^*) = 1$$

After solving for $\tilde{k}^*$ and g^*, we can solve for the payment $\tilde{w}^*$. This requires us to solve for the whole transition dynamics of new entrant firms as characterized by the first order conditions (19)-(26). Even though we limit the analysis to the balanced growth path, the repudiation value of an innovator is given by the surplus generated by a new firm. But to solve for this surplus, we need to solve for the whole transition.[4]

[4]If we interpret H_t as the world-wide level of knowledge which evolves exogenously, then g^* is exogenous and $\tilde{h}^*$ is different from 1. In this case equations (27) and (27) would

5.2 Equilibrium without commitment

Without commitment, the entrepreneur could renegotiate the payments promised to the innovator with the long-term contract. In particular, the entrepreneur will renegotiate the contract when the present value of promised payments exceeds the repudiation value of the innovator.

We have shown in the previous section that the contractual problem with entrepreneur's commitment is subject to the following enforcement constraints for the innovator:

$$\sum_{j=s}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq D_s(h_s)$$

$$w_s + \sum_{j=s+1}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] = \widehat{D}_s(h_{s+1})$$

which must hold for any $s \geq t$. The first constraint is before the investment in knowledge while the second is after the investment.

We have also shown that the second constraint is never binding (see Lemma 1). This implies that, after the innovator has chosen the knowledge investment, the discounted value of promised payments is bigger than the repudiation value. Therefore, the entrepreneur has an incentive to renegotiate down the promised payments. Because of renegotiation, this constraint cannot be satisfied with the inequality sign. The optimization problem can still be written as in (9) but with the second enforcement constraint satisfied with equality.

Remembering that $D_s(h_s) = V_s(h_s)$ and $\widehat{D}_s(h_{s+1}) = \widehat{V}_s(h_{s+1})$, we can rewrite the enforcement constraints as follows:

$$\sum_{j=s}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] \geq V_s(h_s) \tag{27}$$

$$\sum_{j=s}^{\infty} \beta^{j-s}\Big[w_j - \varphi(H_j; h_j, h_{j+1})\Big] = -\varphi(H_s; h_s, h_{s+1}) + \widehat{V}_s(h_{s+1}) \tag{28}$$

In the previous section we have also shown that the two value functions are related as follows (see equation (13)):

$$V_t(h_t) = \max_h \Big\{ -\varphi(H_t; h_t, h) + \widehat{V}_t(h)\Big\}$$

determine $\tilde{h}^*$ and $\tilde{k}^*$.

The solution to this problem, denoted by h^{New}, is the investment in knowledge chosen by a new firm. This could be different from the knowledge investment chosen by an incumbent firm, h^{Old}. If this is the case, we will have that $V_t(h_t) > -\varphi(H_t; h_t, h^{Old}) + \widehat{V}_t(h^{Old})$. But then constraints (27) and (28) cannot be both satisfied. Therefore, the only feasible solution when the entrepreneur does not commit to the long-term contract is the one for which incumbent firms choose the same investment level as new firms, that is, $h^{Old} = h^{New}$. We summarize this in the following proposition:

Proposition 3 *Without entrepreneur's commitment, the knowledge investment chosen by an incumbent firm is equal to the knowledge investment chosen by a new firm.*

Proof 2 *Implicit in the analysis above.*

As for the two-period model, this result has a simple intuition. Because the entrepreneur can renegotiate the promised payments after the investment in knowledge, the innovator would not stay with the firm unless the entrepreneur agrees to the same level of knowledge chosen by a new firm. In this way, the innovator keeps the outside value high and prevents the entrepreneur from renegotiating.

Denote by $\tilde{h}_{s+1} = f_s(\tilde{h}_s)$ the normalized knowledge investment of a new firm created at time $s \geq t$. The optimization problem for a new firm created at time t can be written as:

$$V_t(\tilde{h}_t) = \max_{\tilde{h}_{t+1}, \{\tilde{w}_s, \tilde{k}_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} \Big[\tilde{w}_s - \varphi(\tilde{h}_s, g_s \tilde{h}_{s+1})\Big] \tag{29}$$

subject to

$$\sum_{j=s}^{\infty} \beta^{j-s} \Big[\tilde{w}_j - \varphi(\tilde{h}_j, g_j \tilde{h}_{j+1})\Big] \geq D_s(\tilde{h}_s), \qquad \text{for } s \geq t$$

$$-\tau g_t \tilde{h}_{t+1} - \tilde{w}_t - g_t \tilde{k}_{t+1} + \sum_{s=t+1}^{\infty} \beta^{s-t} \Big[\pi(\tilde{h}_s, \tilde{k}_s, g_s \tilde{h}_{s+1}) - \tilde{w}_s - g_s \tilde{k}_{s+1}\Big] \geq 0$$

$$\tilde{h}_{s+1} = f_s(\tilde{h}_s), \qquad \text{for } s \geq t+1$$

After the first period, the investment in knowledge is determined by the function $f_s(\tilde{h}_s)$, which is taken as given by the firm.

Following the same steps of Appendix C, we can show that in a balanced growth path, problem (29) can be reformulated as follows:

$$\min_{\mu'\geq\mu_0}\max_{\tilde{w}\geq 0,\tilde{h}',\tilde{k}'}\Bigg\{-\tau g\tilde{h}'-\tilde{w}-g\tilde{k}'+\mu'\Big[\tilde{w}-\varphi(\tilde{h}_0,g\tilde{h}')\Big] -(\mu'-\mu_0)D(\tilde{h}_0)+g\beta W(\mu',\tilde{h}',\tilde{k}')\Bigg\} \tag{30}$$

where μ_0 is the inverse of the Lagrange multiplier associated with the participation constraint for the entrepreneur and the function W is defined as:

$$W(\mu,\tilde{h},\tilde{k}) = \min_{\mu'\geq\mu}\max_{\tilde{w}\geq 0,\tilde{k}'}\Bigg\{\pi\Big(\tilde{h},\tilde{k},gf(\tilde{h})\Big)-\tilde{w}-g\tilde{k}'+\mu'\Big[\tilde{w}-\varphi\Big(\tilde{h},gf(\tilde{h})\Big)\Big] -(\mu'-\mu)D(\tilde{h})+g\beta W\Big(\mu',f(\tilde{h}),\tilde{k}'\Big)\Bigg\} \tag{31}$$

As in the case with entrepreneur's commitment, the problem can be divided in two parts: the problem solved by new firms and the problem solved by incumbent firms. For new firms the problem is similar to the case of commitment. For incumbent firms, instead, the investment in knowledge is not chosen optimally but it is given by the policy function $f(\tilde{h})$. Obviously, the optimal value of $\tilde{h}'$ chosen by a new firm in problem (30) also depends on the investment policy that the firm will follow in the future, that is, $f(\tilde{h})$. Therefore, we have to solve a non-trivial fixed point problem. This is in addition to the fixed point problem in the determination of the repudiation function $D(\tilde{h})$.

The first order conditions for problem (30) are given by (19)-(22) and for problem (31) they are given by (23), (24) and (26). Notice that condition (25) is no longer relevant because incumbent firms take the policy for the accumulation of knowledge as given. As a result, the envelope term W_2 is now equal to:

$$W_2(\mu,\tilde{h},\tilde{k}) = \pi_1\Big(\tilde{h},\tilde{k},gf(\tilde{h})\Big)-\mu'\varphi_1\Big(\tilde{h},gf(\tilde{h})\Big)-(\mu'-\mu)D_1(\tilde{h}) + gf_1(\tilde{h})\pi_3\Big(\tilde{h},\tilde{k},gf(\tilde{h})\Big)$$

5.2.1 Equilibrium

We can now define the balanced growth equilibrium for this economy.

Definition 2 *A Balanced Growth Equilibrium without entrepreneur's commitment is defined as:* (a) *Decision rules* $\mu^0(\mu,\tilde{h})$, $d^0(\mu,\tilde{h})$, $h^0(\mu,\tilde{h})$, $k^0(\mu,\tilde{h})$ *for new firms;* (b) *Decision rules for incumbent firms* $\mu(\mu,\tilde{h})$, $d(\mu,\tilde{h})$, $k(\mu,\tilde{h})$*;* (c) *An investment rule for incumbent firms* $f(\tilde{h})$*;* (d) *Initial state* $\mu_0(\tilde{h})$ *for new firms;* (e) *A repudiation function* $D(\tilde{h})$ *and a value function* $V(\tilde{h})$*;* (f) *A distribution of firms* $M(\mu,\tilde{h},\tilde{k})$*. Such that:* (i) *The decision rules* $\mu^0(\mu,\tilde{h})$, $d^0(\mu,\tilde{h})$, $h^0(\mu,\tilde{h})$, $k^0(\mu,\tilde{h})$ *solve problem (30);* (ii) *The decision rules* $\mu(\mu,\tilde{h})$, $d(\mu,\tilde{h})$, $k(\mu,\tilde{h})$ *solve problem (31);* (iii) *The investment rule of incumbent firms is equal to the investment policy of new firms,* $f(\tilde{h}) = h^0(\mu_0(\tilde{h}),\tilde{h})$*;* (iv) *The initial state* $\mu_0(\tilde{h})$ *is such that entrepreneurs break even;* (v) *The repudiation function is the value of starting a new firm,* $D(\tilde{h}) = V(\tilde{h})$*;* (vi) *The distribution* $M(\mu,\tilde{h},\tilde{k})$ *remains constant over time.*

The definition is similar to the one provided for the case of commitment with the exception of condition (iii). This condition imposes that the knowledge investment chosen by an incumbent firm is equal to the investment chosen by a new firm with the same initial states.

The next step is to show that, because of capital obsolescence for incumbent firms, the knowledge investment chosen by a new firm is higher than the level preferred by an incumbent firm, and this leads to faster steady state growth.

Substituting the envelope term derived in the previous section in equation (21), the first order condition for the investment in knowledge of new firms with $\tilde{h}=1$ can be written as:

$$\tau + \mu'\varphi_2(1, g^*\tilde{h}') = \beta\Big[\pi_1\Big(\tilde{h}',\tilde{k}',g^*f(\tilde{h}')\Big) - \mu''\varphi_1\Big(\tilde{h}',g^*f(\tilde{h}')\Big) - (\mu''-\mu')D_1(\tilde{h}') + g^*f_1(\tilde{h}')\pi_3\Big(\tilde{h}',\tilde{k}',g^*f(\tilde{h}')\Big)\Big] \quad (32)$$

Because incumbent firms innovate at the same rate as new firms, this is also the condition that determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether the lack of commitment from the entrepreneur leads to faster or slower growth, we have to compare this condition with the condition that determines the optimal investment in knowledge when the entrepreneur is able to commit to the long-term contract. This is condition (25).

Let g^E be the balanced growth rate in the economy with contract enforcement and g^{LE} the balanced growth rate in the economy without contract enforcement. The following proposition characterizes the balanced growth equilibrium.

Proposition 4 *The balanced growth rate g^{LE} is strictly decreasing in τ while g^E does not depend on τ. There exists $\bar{\tau} > 0$ such that $g^{LE} > g^E$ for $\tau < \bar{\tau}$ and $g^{LE} < g^E$ for $\tau > \bar{\tau}$.*

Proof 3 *See Appendix D.*

6 Numerical example

In this section we present a numerical example. The goal of this section is not to provide a formal calibration exercise but simply to show, with a numerical example, the properties of the model emphasized in the previous sections. We start with the assignment of the parameter values and the specification of functional forms.

The discount factor is $\beta = 0.95$ and the effort cost function is derived from the accumulation equation for the stock of knowledge. Let's assume that the individual knowledge evolves according to:

$$h_{t+1} = h_t + H_t^{\rho} e_t^{1-\rho}$$

where H_t is the average knowledge in the economy and e_t is the effort to accumulate knowledge. The parameter ρ captures the importance of leakage or spillover effects, to which we assign the value of 0.4. By inverting we get the effort cost function:

$$e_t = \varphi(H_t; h_t, h_{t+1}) = \left(\frac{h_{t+1} - h_t}{H_t^{\rho}}\right)^{\frac{1}{1-\rho}}$$

The depreciation of capital is specified as:

$$\delta_t = \delta \cdot \left(\frac{z_{t+1} - z_t}{z_t}\right)$$

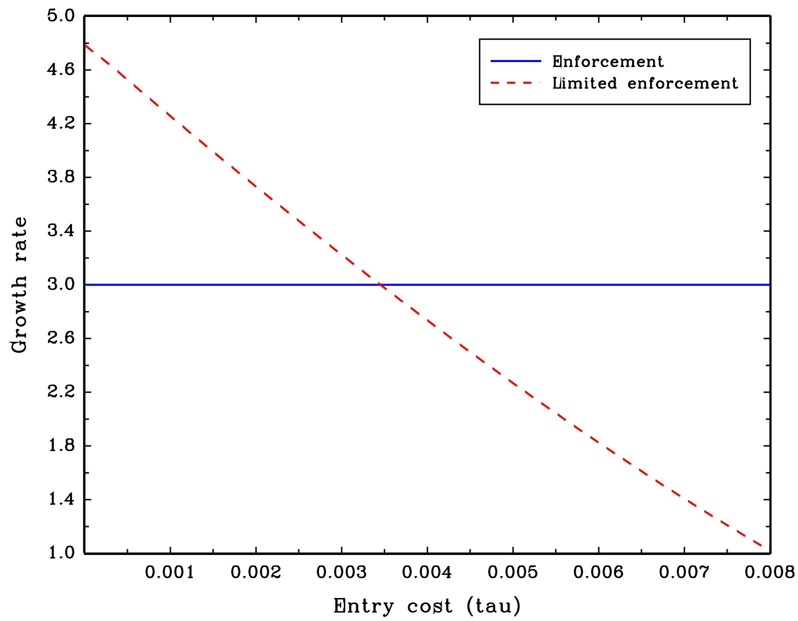

Figure 3: Steady state growth rates for different entry costs.

with $\delta = 0.1$. The production function takes the form $Ah^{1-\alpha}k^{\alpha}$. After setting $\alpha = 0.8$, the value of A is chosen to have a steady state growth rate of 3% when contracts are enforceable. The required value is $A = 0.0652$.

After the assignment of the parameter values, we solved the model for different entry costs τ. The steady state growth rates are plotted in Figure 3. As can be seen from the figure, barriers to entry do not affect the steady state growth when contracts are enforceable. With limited enforceability, instead, the equilibrium growth rate decreases monotonically with the entry cost.

7 Conclusion

Modern technologies are highly complementary to skilled labor. This implies that the adoption of these technologies requires the accumulation of innovation skills or knowledge from workers and managers. In absence of a commitment devise or enforcement for innovators and entrepreneurs, under-accumulation of skills may result. However, we have shown that limited enforcement alone is not sufficient to impair the accumulation of knowledge capital and the long-term growth. With competition—that is, free entry of new firms competing for human or knowledge capital—the economy would experience faster growth even if contracts are not enforceable. Business entry increases the demand of knowledge capital and creates an outside opportunity for those accumulating knowledge. It is this outside opportunity that guarantees the agent the reward for accumulating knowledge when contracts are not enforceable.

Our paper provides a theoretical foundation for the empirical finding that the cost of starting a business is negatively associated to the level of development and growth of a country.

A Proof of Lemma 1

Conditions (10) and (11) can be rewritten as:

$$\begin{aligned}
\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big] &\geq D_s(k_s)\\
\sum_{j=s}^{\infty}\beta^{j-s}\Big[w_j-\varphi(H_j;h_j,h_{j+1})\Big] &\geq -\varphi(H_s;h_s,h_{s+1})+\widehat{D}_s(h_{s+1})
\end{aligned}$$

Therefore, to show that the second constraint is satisfied when the first constraint is satisfied, it is enough to show that $D_s(h_s)\geq-\varphi(H_s;h_s,h_{s+1})+\widehat{D}_s(h_{s+1})$ for any value of h_{s+1}. From the definition of the repudiation values we have that $D_s(h_s)=\max_h\{-\varphi(H_s;h_s,h)+\widehat{D}_s(h)\}$. This is at least as big as $-\varphi(H_s;h_s,h_{s+1})+\widehat{D}_s(h_{s+1})$ for any h_{s+1}. *Q.E.D.*

B Proof of Lemma 2

We use a guess and verify procedure. Suppose that $\widehat{D}_t(h_{t+1})=\widehat{D}_t(g_t\tilde{h}_{t+1})\cdot H_t$. Obviously $D_t(h_t)$ takes a similar form:

$$\begin{aligned}
D_t(h_t) &= \max_{g_t}\Big\{\varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})\cdot H_t+\widehat{D}_t(g_t\tilde{h}_{t+1})\cdot H_t\Big\}\\
&= \max_{g_t}\Big\{\varphi(\tilde{h}_t,g_t\tilde{h}_{t+1})+\widehat{D}_t(g_t\tilde{h}_{t+1})\Big\}\cdot H_t\\
&= D_t(\tilde{h}_t)\cdot H_t
\end{aligned}$$

We want to show next that $V_t(h_t)=V_t(\tilde{h}_t)\cdot H_t$. This can be easily proved by normalizing all variables by H_t in problem (9). After normalizing, the optimization is over $\{\tilde{w}_s,\tilde{k}_{s+1},\tilde{h}_{s+1}\}_{s=t}^{\infty}$. Simple inspection of the normalized problem proves our claim. *Q.E.D.*

C Saddle-point formulation

Consider problem (14). Given γ_s the Lagrange multiplier associated with the enforcement constraint (15) and λ_t the Lagrange multiplier associated with the enforcement constraint (16), the Lagrangian can be written as:

$$\begin{aligned}
\mathcal{L} &= \sum_{s=t}^{\infty}\beta_{t,s}\Big[\tilde{w}_s-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]\\
&+ \sum_{s=t}^{\infty}\beta_{t,s}\gamma_s\left\{\sum_{j=s}^{\infty}\beta_{s,j}\Big[\tilde{w}_j-\varphi(\tilde{h}_j,g_j\tilde{h}_{j+1})\Big]-D_s(\tilde{h}_s)\right\}
\end{aligned}$$

$$+ \quad \lambda_t \left\{ -\tilde{w}_t - \tau g_t \tilde{h}_{t+1} - g_t \tilde{k}_{t+1} + \sum_{s=t+1}^{\infty} \beta_{t,s} \Big[\pi(\tilde{h}_s, \tilde{k}_s, g_s \tilde{h}_{s+1}) - \tilde{w}_s - g_s \tilde{k}_{s+1} \Big] \right\}$$

Define $\tilde{\mu}_s$ recursively as follows: $\tilde{\mu}_{s+1} = \tilde{\mu}_s + \gamma_s$, with $\tilde{\mu}_t = 0$. Using this variable and rearranging terms, the Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= \sum_{s=t}^{\infty} \beta_{t,s} \left\{ (1 + \tilde{\mu}_{s+1}) \Big[\tilde{w}_s - \varphi(\tilde{h}_s, g_s \tilde{h}_{s+1}) \Big] - (\tilde{\mu}_{s+1} - \tilde{\mu}_s) D_s(\tilde{h}_s) \right\} \\ &+ \lambda_t \left\{ -\tilde{w}_t - \tau g_t \tilde{h}_{t+1} - g_t \tilde{k}_{t+1} + \sum_{s=t+1}^{\infty} \beta_{t,s} \Big[\pi(\tilde{h}_s, \tilde{k}_s, g_s \tilde{h}_{s+1}) - \tilde{w}_s - g_s \tilde{k}_{s+1} \Big] \right\} \end{aligned}$$

Define $\mu_s = (1 + \tilde{\mu}_s)/\lambda_t$ for $s \geq t+1$ with $\mu_t = 1/\lambda_t$. Substituting we get:

$$\begin{aligned} \mathcal{L} &= \lambda_t \Bigg\{ - \tilde{w}_t - \tau g_t \tilde{h}_{t+1} - g_t \tilde{k}_{t+1} + \mu_{t+1} \Big[\tilde{w}_t - \varphi(\tilde{h}_t, g_t \tilde{h}_{t+1}) \Big] - (\mu_{t+1} - \mu_t) D_t(\tilde{h}_t) \\ &+ \sum_{s=t+1}^{\infty} \beta_{t,s} \Big[\pi(\tilde{h}_s, \tilde{k}_s, g_s \tilde{h}_{s+1}) - \tilde{w}_s - g_s \tilde{k}_{s+1} + \mu_{s+1} \Big[\tilde{w}_s - \varphi(\tilde{h}_s, g_s \tilde{h}_{s+1}) \Big] \\ &\qquad - (\mu_{s+1} - \mu_s) D_s(\tilde{h}_s) \Big] \Bigg\} \end{aligned}$$

Dividing by λ_t and looking at the special case of a balanced growth path in which the stock of aggregate knowledge grows at the constant rate g, the problem can be rewritten as:

$$\begin{aligned} \min_{\mu_{t+1} \geq \mu_t} \max_{\substack{\tilde{w}_t \geq 0, \\ \tilde{h}_{t+1}, \tilde{k}_{t+1}}} \Bigg\{ & - \tilde{w}_t - \tau g_t \tilde{h}_{t+1} - g \tilde{k}_{t+1} + \mu_{t+1} \Big[\tilde{w}_t - \varphi(\tilde{h}_t, g \tilde{h}_{t+1}) \Big] \\ & - (\mu_{t+1} - \mu_t) D_t(\tilde{h}_t) + g\beta W(\mu_{t+1}, \tilde{h}_{t+1}, \tilde{k}_{t+1}) \Bigg\} \end{aligned}$$

for given μ_t and with the function W defined recursively as follows:

$$\begin{aligned} W(\mu, \tilde{h}, \tilde{k}) &= \min_{\mu' \geq \mu} \max_{\substack{\tilde{w} \geq 0, \\ \tilde{h}', \tilde{k}'}} \Bigg\{ \pi(\tilde{h}, \tilde{k}, g\tilde{h}') - \tilde{w} - g\tilde{k}' + \mu' \Big[\tilde{w} - \varphi(\tilde{h}, g\tilde{h}') \Big] \\ &\qquad - (\mu' - \mu) D(\tilde{h}) + g\beta W(\mu', \tilde{h}', \tilde{k}') \Bigg\} \end{aligned}$$

The initial state μ_t is determined such that the participation constraint for the entrepreneur is satisfied.

D Proof of Proposition 4

All firms are alike in the balanced growth path. Therefore, $\tilde{h} = 1$. If $\mu_0 = 1$, the first order condition for the accumulation of knowledge, equation (32), can be written as:

$$\tau + \varphi_2(1, g) = \beta\Big[\pi_1(1, \tilde{k}, g) - \varphi_1(1, g)\Big] + g\beta f_1(1)\pi_3(1, \tilde{k}, g)$$

With commitment, the first order condition for the accumulation of knowledge is given by equation (25), which in the steady state becomes:

$$\varphi_2(1, g) = \beta\Big[\pi_1(1, \tilde{k}, g) - \varphi_1(1, g)\Big] + \pi_3(1, \tilde{k}, g)$$

Because $\pi_3(1, \tilde{k}', g) < 0$ and $g\beta f_1(1) < 1$, the marginal cost from innovating $\varphi_2(1, g)$ in the first condition must be greater than in the second condition. Therefore, if $\tau = 0$ the economy without commitment must grow at a rate higher than in the economy with commitment. As we increase τ, however, the marginal benefit will decrease in the economy without commitment. This implies that the growth rate g^* decreases with τ. It is obvious that for a sufficiently high value of τ the growth rate in the economy without commitment is lower than in the economy with commitment.

Notice that, without obsolescence in physical capital, $\pi_3(1, \tilde{k}, g) = 0$. Therefore, if $\tau = 0$ the two conditions above are indistinguishable, which implies that the commitment of the entrepreneur does not affect the long-term growth. *Q.E.D.*

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