Regime Shifts and the Stability of Backward Looking Phillips Curves in Open Economies*

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February 2004

Abstract

In this paper we assess the stability of open economy backward looking Phillips curves estimated across two different exchange rate regimes. The time-series we deal with come from the simulation of a New-Keynesian hybrid model suited for performing monetary policy analysis. The statistical assessment we undertake is based on a standard Chow-test. Our results confirm Lindè (2001)'s finding on the low power of the Chow-test in small samples. However, we do not find strong statistical support for the quantitative relevance of the Lucas critique when the 'true' model of the economy is featured by low degrees of forwardness.

Keywords: Lucas Critique, rational expectations, backward looking Phillips curve, exchange rates, Chow-test.

JEL Classification: E17, E52, F41

*First draft: December 2003. I thank Giorgio Brunello, Fabio Canova, Nunzio Cappuccio, Carlo Favero, Raffaele Miniaci, Alessandro Missale, Carola Moreno, Roberto Perotti, Ferhan Salman, Frank Smets, Ulf Söderström, Paolo Surico, Guglielmo Weber, and participants to seminars held at Ente L. Einaudi and University of Padua for useful comments and suggestions. The hospitality of Ente Einaudi, where much of this research was conducted, is gratefully acknowledged. The usual disclaimers apply. Author's coordinates: Efrem Castelnuovo, Department of Economics, University of Padua, Via del Santo 33, 35123 Padova (PD), Italy. Phone: +39 049 827 4257, Fax: +39 049 827 4211. E-mail account: efrem.castelnuovo@unipd.it.

'Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of the series relevant to the decision makers, it follows that any change in policy will systematically alter the structure of econometric models.' (Robert E. Lucas Jr., 1976, p.41)

'[The] question of whether a particular model is structural is an empirical, not theoretical, one.' (Robert E. Lucas Jr. and Thomas J. Sargent, 1981, pp. 302-303).

1 Introduction

Since the publication of the seminal paper by Lucas (1976), many researchers have undertaken efforts toward the microfoundation of economic models to be employed for performing policy analysis. Indeed, one of the fields that has been intensely affected by this push toward microfoundation is the monetary one (see e.g. Rotemberg and Woodford 1997; McCallum and Nelson, 1999a,b). In fact, a different strand of the literature (e.g. Rudebusch and Svensson 1999,2002; Ball 1999,2000) has focussed on backward looking models, whose structure follows in spirit the VAR models popularized by Sims (1980). While being quite ad-hoc, monetary backward looking models fit the data quite satisfactorily, and do not seem to produce counterfactual dynamics. On the contrary, this drawback does affect the more theoretically appealing forward looking frameworks, as shown by Estrella and Fuhrer (2002).[1]

Evidently enough, backward looking models are right the policy tools criticized by Lucas (1976). Indeed, if agents are rational, reduced-form coefficients should theoretically be quite unstable across different policy regimes. In fact, as pointed out by Lucas and Sargent (1981), the relevance of the Lu-

[1] Estrella and Furher (2002) show e.g. that microfounded forward looking models build in a *positive* (and data consistent) correlation between the *level* of inflation and the real output, but a *negative* (and counterfactual) correlation between the *change* in inflation and real output. This can only occur if inflation first falls in response to an output contraction, and then jumps back to the steady state, a behavior that is at odds with the host of empirical evidence suggesting 'hump-shaped' responses to real and monetary shocks. As a solution to this problem, Estrella and Fuhrer (2002) suggest the use hybrid/backward looking models.

cas critique is fundamentally a *quantitative* issue. Then a natural question comes: Is the Lucas critique empirically relevant when backward looking monetary models are employed for policy purposes?

Some recent studies suggest a negative answer.[2] Interestingly enough, Lindè (2001) shows that the Chow (1960) test, usually employed in this kind of analysis for detecting some schedules' instability, is just a too low power test in small samples. Then, his claim is that researchers have not found the Lucas critique to be important just because of their choice of relying on such a statistical tool.

In this paper we aim at deepening this point. In particular, our goal is that of better understanding the relationship between the 'degree of forwardness' of the 'true' model of the economy and the power of the Chow test. In fact, the Chow test should detect a break in an estimated reduced-form schedule if this schedule's stability is harmed by the regime shift. But if a regime shift is such that its impact on the reduced-form coefficients is negligible - e.g. because the 'true' model of the economy is mainly featured by endogenous inertia - than backward looking models may turn out to be of interest for policy analysis.

To investigate this issue, we employ a hybrid open-economy model (i.e. a convex combinations of forward and backward-looking representations) to obtain simulated time-series for variables such as inflation, output gap, real exchange rate, and policy rates. We go for an analysis regarding open-economies because the impact of regime shifts on open-economy backward looking schedules has not been deeply investigated yet. In fact, we think of it as being an important issue, in the light of the enormous importance played by international markets for several countries all over the world (see e.g. Lane 2001; Sarno 2001). In performing our simulations, we allow for a regime shift, i.e. the shift from a 'controlled' nominal exchange rate volatility to a floating exchange rate framework. We use these simulated time-series

[2] See Rudebusch (2003) and Estrella and Fuhrer (2003). Notice that here we are referring to contributions that are very closely related to our object of investigation, i.e. the empirical relevance of the Lucas critique for *backward looking monetary policy* models. Of course, the quantitative importance of the Lucas critique has been subject to wide attention since 1976. For surveys in this sense, see Favero and Hendry (1992) and Ericsson and Irons (1995).

to estimate a backward looking schedule. Then, we check with a small and a large sample if the Chow test detects any shift in the estimated coefficients.

To make our point, we concentrate on a backward looking version of the Phillips curve. The choice of the Phillips curve comes as natural when exploring the monetary policy territory. In fact, the theoretical analysis and the quantitative assessment of the relationship between inflation and output gap (or inflation and unemployment) is one of the key-issue for researchers and monetary policymakers. While on the one hand the expectations augmented New-Keynesian Phillips curve has become the workhorse model of modern research on monetary policy (e.g. Clarida, Galì, and Gertler 1999), on the other hand researchers such as Mankiw (2001) and Estrella and Fuhrer (2002) have shown that this model is just at odds with the facts. In particular, the failure of the New Keynesian Phillips curve refer to its predictions concerning i) disinflationary booms caused by fully credible disinflations, ii) a low-autocorrelated inflation rate, and iii) an immediate and one-shot reaction to a monetary policy shock. Therefore, the attention on a backward looking Phillips curve seems to be well-placed.

Basically, the 'simulation-and-estimation' methodology we employ here is that proposed by Taylor (1989). Taylor (1989) points toward an assessment of the stability of several VAR-type schedules in an open-economy framework. By implementing the above described strategy with an estimated open-economy model, Taylor (1989) reaches the conclusion that the Lucas critique's importance is not quantitatively overwhelming.

Similarly to Taylor's contribution, we consider a shift from a 'controlled' exchange rate regime to a 'floating' one, whose importance is evident in the light of both historical evidence and the discussion on the choice of the optimal exchange rate regime (e.g. Corden 2002). Contrarily to Taylor (1989), we do allow for imperfect exchange rate pass-through (so capturing the insights coming from Campa and Goldberg, 2002) and non-standard exchange rate expectations' formation (as suggested by Frankel and Froot, 1987), in order to handle a satisfactory model from the goodness-of-fit viewpoint. More importantly, we do employ a statistical criterium, i.e. the Chow (1960) breakpoint test, to assess the stability of the estimated coefficients of our reduced-form Phillips curve.

Our results confirm Lindé (2001)'s finding on the low-power of the Chow test in small samples. Indeed, this might be a reason why some researchers have not detected the statistical relevance of the Lucas critique so far. However, some qualifications are needed. In fact, according to the set-up at hand, the (in)stability of the estimated coefficients is connected to the 'forwardness' of the 'true' model of the economy. An economy characterized by a strong endogenous persistence may be featured by a small impact of a certain regime shift on the coefficients of a reduced-form equation. In fact, we find that a large part of the spectrum of possible 'degrees of forwardness' does not seem to signal any instability of the estimated schedule. Then, the Lucas critique might not necessarily have an overwhelming empirical relevance. This is good news for the reliability of the monetary policy analyses performed with backward looking models, e.g. the assessment of different policy rules (e.g. Ball 1999,2000), or the assessment of the sacrifice ratio (e.g. Leitemo and Røste, 2003).

The paper is structured as follows. In Section 2 we present the small macro model we employ to produce the simulated time-series of interest. In Section 3 we offer a very simple example to explain why a regime shift may harm the stability of backward-looking models' coefficients. Section 4 offers an explanation of the steps we implement to perform our econometric exercise. In Section 5 we present our findings, whose robustness is discussed in Section 6. Section 7 concludes. In the Technical Appendix we discuss i) how to reduce the open-economy model to an AR(1)-type law-of-motion in order to treat it with standard techniques, and ii) alternative possible formalizations of the 'controlled' exchange rate regime. References follow.

2 A simple open-economy macro-model

We present here the model we employ for performing our numerical simulations. In our open-economy framework, the Phillips curve and the IS schedule defining the paths of the domestic inflation rate and the output gap read as follows:

$$\pi_{t+1} = \mu_\pi \pi^e_{t+2} + (1 - \mu_\pi)\pi_t + \alpha_y y_t + \alpha_q q^e_{t+1} + u_{t+1} \tag{1}$$

$$y_{t+1} = \mu_y y^e_{t+2} + (1-\mu_y)y_t - \beta_r(i_t - \pi^e_{t+1}) + \beta_q q_t + \beta_y y^*_t + v_{t+1} \qquad (2)$$

where π_t is the four-quarter domestic inflation rate (i.e. $\pi_t \equiv 400(p_t - p_{t-1})$, $p_t \equiv logP_t$, with P_t being the price index of the economy), y_t is the output gap (i.e. 100 times the log-difference between the real GDP and a measure of potential output), q_t is the real exchange rate, i_t is the short-term nominal interest rate controlled by the Central Bank, x^e_{t+j} stands for $E_t x_{t+j}$, u_t and v_t are iid processes with zero mean and standard deviations σ_u and σ_v, and y^*_t is the foreign output gap (as, in general, starred variables refer to foreign variables).[3]

Equations (1) and (2) are fairly in line with those in Svensson (2000) and Leitemo and Söderström (2003).[4] In particular, equation (1) determines the domestic inflation rate as a function of the expected inflation rate, the lagged one, and the lagged values of the output gap and the real exchange rate. This is an open economy version of a hybrid Phillips curve, in which the inflation rate is pre-determined one period, it is endogenously inertial (e.g. may be due to wage contracting, see e.g. Fuhrer and Moore, 1995, or to indexation of those prices that are not re-optimized in a given period, as in Christiano *et al*, 2003, and in Smets and Wouters, 2003), and it also takes into account the effect of expected costs of imported intermediate inputs. The 'cost push shock' u_{t+1} is justified by the time-varying markup of monopolistically competitive firms, as in Smets and Wouters (2003). Equation (2) defines the path of the output gap, which is caused by expectations on future output gap's realizations as well as past values (the latter finding its rationale in habit formation, see e.g. Fuhrer, 2000), the ex-ante real interest rate, and the

[3] In this model the same measure of the output gap (detrended output) enters both the Phillips curve and the IS equation. While this is a common choice in the empirical literature we are interested into, it must be acknowledged that some authors disagree; see e.g. McCallum and Nelson (1999a) for a model with a measure of detrended output in the IS curve, and of output gap (i.e. detrended actual output minus detrended potential output) in the inflation equation.

[4] Svensson (2000) shows that equations similar to (1) and (2) in the text may be derived from first principles. However, we do not want to push this point here, given that we are just interested in having a model producing plausible simulated patterns of the time-series of interest.

real exchange rate, which proxies the increased foreign demand for domestic goods driven by exchange rate depreciation. The stochastic component of the aggregate demand curve, i.e. v_{t+1}, may be interpreted as a preference shock (Smets and Wouters, 2003).[5]

The nominal exchange rate s_t is a key-element in our analysis.[6] In our model, its temporal evolution is driven by the following hybrid stochastic version of the uncovered interest parity (UIP) condition:

$$i_t = i_t^* + \mu_s s_{t+1}^e + (1-\mu_s)[(1+\phi)s_t - \phi s_{t-1}] - s_t + \varphi_t \tag{3}$$

where the risk-premium φ_t follows an AR(1) process with root ρ_ψ and stochastic term ψ_t (with zero mean and standard deviation equal to σ_ψ), i.e.[7]

$$\varphi_t = \rho_\psi \varphi_{t-1} + \psi_t \tag{4}$$

Clearly, when $\mu_s = 1$, eq. (3) is a standard stochastic UIP condition. However, there is a certain evidence of backward-lookingness in the exchange rate expectations' formation (e.g. Frankel and Froot, 1987,1990).[8] We model this possibility by allowing for the lagged nominal exchange rate s_{t-1} to play an active role in the exchange rate determination; this happens when $\mu_s < 1$. In this case, the parameter $\phi \,\epsilon\, [-1, 1]$ shapes the exchange rate dynamics.[9] In particular, when ϕ assumes positive values the exchange rate formation is formalized as a 'chartist' scheme, suggesting for the lagged

[5] Note that the steady state value of the real exchange rate q_t in this model is equal to zero. Then, the natural rate hypothesis holds.

[6] In this paper we line up with the stardard definition of the log-nominal exchange rate, i.e. $s_t \equiv \log\left(\frac{Domestic\_currency}{Foreign\_currency}\right)$, so that an *increase* in s_t stands for *depreciation.*

[7] The assumption for φ_t to follow an AR(1) process is mainly made to capture the commonly observed persistence of the risk-premium, and it is quite common in the open-economy monetary policy literature (see e.g. McCallum and Nelson, 1999b; Svensson, 2000; Leitemo and Söderström, 2003).

[8] Frankel and Froot (1987) concentrate on three departures from rational expectations, i.e. distributed lags, adaptive expectations, and regressive expectations. All these three models turn out to be supported by the data. We work with the distributed lags model because it allows us to easily switch to the 'chartist' formalization, as explained in the text.

[9] Notice that ϕ might also assume values lower than -1. The domain indicated in the text is just that investigated in our exercise.

exchange rate s_{t-1} a positive impact on the current one. By contrast, when $-1 \leq \phi < 0$, then we are dealing with a 'distributed lags' structure, with the lagged interest rate s_{t-1} having a negative effect on the current one.[10] Notice that when $\mu_s = 0$ and $\phi = 1$, the UIP equation (3) assumes the backward looking flavor that Debelle and Wilkinson (2002) attribute to it.

The real exchange rate q_t is defined as:

$$q_t = s_t + p_t^* - p_t \tag{5}$$

Eq. (5) implies that the relationship between the domestic and foreign inflation rate and the real exchange rate is given by

$$q_t = q_{t-1} + \Delta s_t + \pi_t^* - \pi_t \tag{6}$$

As indicated above, one of the arguments (potentially) of interest for the central banker is the CPI inflation rate π_t^{CPI}, which is defined as

$$\pi_t^{CPI} = (1 - \chi)\pi_t + \chi\pi_t^M \tag{7}$$

where χ is the weight of imported goods in the aggregate consumption basket, and π_t^M stands for imported inflation. Following Leitemo and Söderström (2003), we define the imported price level p_t^M as follows:

$$p_t^M = (1 - \theta)p_{t-1}^M + \theta(p_t^* + s_t) \tag{8}$$

Importantly, the parameter θ allows for the possibility of deviating from the law of one price *in the short-run.* In fact, if $0 \leq \theta < 1$, then the imported price level does not immediately fully adjust after that a shock has hit the foreign inflation rate or the nominal exchange rate. This price stickiness is intended to capture the imperfection of the exchange rate pass-through observed in the real world, imperfection that tend to be much less important

[10] Notice that the two models (i.e. chartist and distributed lags) have very different implications for the path of the exchange rate expectations. In fact, while the former suggests an inelastic (i.e. less than proportionate, then stabilizing) reaction of the expectations with respect to a variation of the current exchange rate, the latter implies right the opposite.

in the long run, as shown in Campa and Goldberg (2002).[11] The strategy of modelling price stickiness and not pricing-to-market is also followed by Smets and Wouters (2002) and Lindé *et al* (2003).

Following equation (8), the short-run deviation from the law of one price is reflected in the imported inflation rate as follows:

$$\pi_t^M = (1-\theta)\pi_{t-1}^M + \theta(\pi_t^* + \Delta s_t) \tag{9}$$

Then, equations (6), (7), and (9) suggest the following link between real exchange rate and CPI inflation:[12]

$$\pi_t^{CPI} = (1-\chi)\pi_t + \chi[(1-\theta)\pi_{t-1}^M + \theta(\pi_t + \Delta q_t)] \tag{10}$$

In an open-economy framework, optimal monetary policy is set by taking into account both domestic constraints and stimuli coming from the Rest-Of-the-World (ROW henceforth). We represent the foreign economy *a la* Svensson (2000). In particular, we assume that ROW follows a Taylor rule, i.e.

$$i_t^* = (1-\rho_{i*})(f_\pi^* \pi_t^* + f_y^* y_t^*) + \rho_{i*} i_{t-1}^* + \zeta_t^* \tag{11}$$

where f_π^* and f_y^* are the coefficients respectively associated to foreign inflation and foreign output gap, ρ_{i*} is the interest rate smoothing coefficient, while ζ_t^* is a zero-mean white noise process with variance σ_ζ^*.[13] To catch the persistence typically observed in macro data, π_t^* and y_t^* are defined as AR(1) processes, i.e.

[11] Campa and Goldberg (2002) investigate exchange rate pass-through short-run and long-run elasticities on a sample of 25 OECD countries for the period 1975-1999. It turns out that in the short-run the law of one price is rejected in 22 out of 25 cases, but in the long-run 16 cases out of 25 support an elasticity statistically equivalent to one.

[12] It is immediate to see that, when θ is equal to one (i.e. when the law of one price holds), then the CPI inflation rate is defined as in the standard perfect exchange rate pass-through case, i.e. $\pi_t^{CPI} = \pi_t + \chi\Delta q_t$.

[13] We shape the foreing Taylor rule with *interest rate smoothing* and a *white noise* process for the policy shock to attribute to the decision makers' willingness the source of the commonly observed short-term nominal interest rate persistence. For some contributions on the relative importance of the lagged interest rate vs. the (potentially serially correlated) policy shock in causing the policy rate path, see Rudebusch (2002a), English, Nelson, and Sack (2003), and Castelnuovo (2003a).

$$\pi^*_{t+1} = \rho^*_\pi \pi^*_t + u^*_{t+1} \tag{12}$$

$$y^*_{t+1} = \rho^*_y y^*_t + v^*_{t+1} \tag{13}$$

with σ^*_u and σ^*_v being the variances featuring the two processes.

The last bit of the model describes the monetary authorities' behavior. In our framework, the Central Banker controls the short-term nominal interest rate i_t. His aim is that of minimizing the volatility of the arguments of his loss function. Of course, different monetary policy regimes go hand-in-hand with different penalty function. The following generic loss function captures the different regimes we will analyze in our exercise:[14]

$$\begin{aligned} E(L_t) &= \lambda_{\Delta s} Var(\Delta s_t) + \lambda_{\pi^{CPI}} Var(\pi^{CPI}_t) \\ &\quad + \lambda_y Var(y_t) + \lambda_{\Delta i} irsmooth \end{aligned} \tag{14}$$

where $irsmooth \equiv Var(\Delta i_t)$.[15]

2.1 A note on the Loss function

The loss function (14) deserves some explanations. The weights $\{\lambda_{\Delta s}, \lambda_{\pi^{CPI}}, \lambda_y\}$ are structural parameters of our framework, i.e. the preferences of the Central Banker over the targeted arguments (Svensson, 1999), and identify the regime shift we will work with.

[14] In fact, the CB solves an *intertemporal* problem featured by the following loss function: $E_t \sum_{\tau=0}^{\infty} \delta^\tau \left(\sum_{i=1}^{n} x^2_{i,t+\tau} \right)$, where x_i is one of the n arguments targeted by the monetary authorities. As shown by Rudebusch and Svensson (1999), the conditional expectation presented here tends to the unconditional expectation discussed in the text for $\delta \rightharpoonup 1$. In this study, we fix the discount factor δ to be equal to .99, a standard choice given the quarterly frequency assumed for our model.

[15] The interest rate smoothing argument is technically introduced in the loss function in order to avoid counterfactual extreme fluctuations of the short-term nominal interest rate. As discussed by Sack and Wieland (2000), Srour (2001), and Castelnuovo (2003b), the so-formalized interest rate smoothing is intended to be a 'catch-all' approximating features of the real-world central bank's problem such as e.g. uncertainties, financial market sensitivity to interest rate movements, optimal gradualism, and learning. In performing our simulations, we will consider for the interest rate smoothing argument a relative weight $= 0.2$, as in Rudebusch and Svensson (2002).

As mentioned already, we aim at mimicking a shift from a 'fixed/limited flexibility' to a 'managed floating/flexible' exchange rate regime, a shift that concerned a certain number of countries in the past decades, as testified by Reinhart and Rogoff (2002);[16] a few examples are collected in Table 1. Notice that in most of the selected cases Reinhart and Rogoff (2002) do not find evidence of a fixed exchange rate regime; this is why we will go for a 'controlled exchange rate volatility' scenario. Since the debate on the best exchange rate scheme is still on-going,[17] it seems to be interesting to know if backward looking models estimated by employing data coming from samples including such a break may be reliably used for policy purposes.

[**Table 1 about here**]

To maintain the convenient linear-quadratic structure of the problem, our strategy is that of simulating the 'controlled' nominal exchange rate regime by implementing the following targeting:

$$E(L_t) = Var(\Delta s_t) + \lambda_{\Delta i} irsmooth \tag{15}$$

This is a targeting of the stationary difference existing between today's and yesterday's nominal exchange rate. Given the equation involving the

[16] Reinhart and Rogoff (2002) develop a novel classification of the exchange rate regimes historically observed based on a new data-set on marked-determined parallel exchange rates. Their classification is in stark contrast with the traditional IMF Annual Report on Exchange Rate Arrangements and Exchange Restrictions (based on countries' self-declarations), the contrast coming from the non-consideration (by the IMF classification) of the role of dual markets. Reinhart and Rogoff (2002)'s conclusions are (quite synthetically) summarized here. i) Dual or multiple rates/parallel markets have played a pretty large role for most of the open economies all around the World. ii) For many countries, it is difficult to detect *any* change in exchange rate behavior between a period of 'fixed' exchange rate and a period of 'floating' regime, since de facto many official 'pegs' just masks cases of limited flexibility, and the opposite also holds. iii) the most popular exchange rate regimes in the last 30 years have been 'pegs' (33%) and 'crawling pegs/narrow crawling pegs' (26%). iv) significant differences in business cycles across exchange rate arrangements *do* show up when back doors/dual markets are taken in adequate account. Our exercise on the exchange rate regime break is thought for describing the situation of countries such as those listed in Table 1.

[17] For a book regarding this discussion, see Corden (2002). Much information on the fixed vs. flexible exchange rate regime may also be found in Nouriel Roubini's web-page, i.e. http://www.stern.nyu.edu/globalmacro/.

real exchange rate (6) and the UIP (3), this calls for an optimal policy rule that takes into account both domestic elements and, above all, foreign variables such as the foreign policy rate and the risk-premium. In this sense, we believe our approximation of a controlled exchange rate regime may be considered as being fairly satisfactory.[18] After the regime shift, we assume that the monetary authorities will follow a CPI inflation targeting strategy (Svensson, 2000), identified by the following penalty function:[19]

$$E(L_t) = \lambda_{\pi^{CPI}} Var(\pi_t^{CPI}) + \lambda_y Var(y_t) + \lambda_{\Delta i} irsmooth \tag{16}$$

2.2 A note on the solution of the Central Banker's problem

In computing the solution of the Central Banker's problem, we assume that the central banker does not own any commitment technology. Although somewhat extreme, this assumption seems to be quite reasonable for describing the inflation targeting framework widely adopted all over the world by many central banks for some time now, as discussed by Bernanke and Mishkin (1997). It is possible to show (see Söderlind, 1999, and the Technical appendix of this paper for more details) that the solution of the monetary authorities' problem in this context is given by the following feedback rule:

$$i_t = f x_{1t} \tag{17}$$

where f is a $(1x9)$ vector whose elements are complicated convolutions of the policymakers' preferences and the parameters of the economy, while x_{1t} is the $(9x1)$ vector of pre-determined variables of the problem, i.e. $x_{1t} = \begin{bmatrix} \pi_t & y_t & \varphi_t & q_{t-1} & \pi_{t-1}^M & i_t^* & \pi_t^* & y_t^* & i_{t-1} \end{bmatrix}'$. This endogenous policy rule closes the model.

[18] A discussion on possible alternative ways of shaping a 'controlled' exchange rate regime is offered in the Appendix.

[19] Notice that in this work we are considering monetary policy shifts that are intimately related to the *open* economy dimension of the model. In other words, in a closed economy set up the targeting schemes we are discussing here could just not be implemented. Then, we are *not* going to claim that a regime shift having no consequences on the estimated reduced-form coefficients in a closed economy set-up might indeed have an impact when we open up the economy.

Our model (1)-(13) and (17) is quite flexible, and it can be easily forced to assume a pure forward looking fashion (i.e. $\mu_\pi = \mu_y = \mu_s = 1$) or a pure backward looking structure (i.e. $\mu_\pi = \mu_y = \mu_s = 0$). We think of it as representing a fair compromise between highly stylized formalizations of the open economy framework (e.g. Ball, 1999,2000) and more complex, fully-fledged ones (e.g. Smets and Wouters, 2002). Although not fully in line with representations whose beauty is due to theoretical coherence coming from microfoundation (e.g. Galì and Monacelli, 2003), this model is actually quite close to one of them, i.e. Lindé, Nessén, and Söderström (2003).

2.3 Model parametrization

The benchmark parametrization used in our exercise is largely borrowed from the existing literature dealing with dynamic stochastic monetary modeling. In fact, we make no attempt of estimating the model; instead, we select plausible parameters values in order to provide a first assessment on what it might be the relevance of the Lucas critique in an open-economy environment. In particular, the domestic economy is almost fully parametrized on the basis on the paper by Leitemo and Söderström (2003).[20] As far as the degree of forwardness of the UIP condition is concerned, we set $\mu_s = .7$, i.e. slightly larger than the degree of forwardness of the Phillips curve $\mu_\pi = .5$ (the latter value being in line with Fuhrer and Moore, 1995) and that of the IS equation $\mu_\pi = .3$ (as in Fuhrer, 2000). We make this choice to recognize to the nominal exchange rate its feature of 'forward looking determined asset price' (Svensson, 2000). Still about the UIP condition, our benchmark choice is that of considering the distributed lags scheme, whose coefficient is - somehow arbitrarily - set as $\phi = -.5$. Moreover, we set the exchange rate pass-through coefficient $\theta = .5$, in line with many of the point estimates in Campa and Goldberg (2002). The foreign economy is parametrized as in Svensson (2000). As a sole exception, we enriched the ROW Taylor rule with the interest rate smoothing parameter $\rho_{i*} = .75$, pretty much in line with the value estimated by Clarida, Galì, and Gertler (2000) for the US.

[20] Since Leitemo and Söderström (2003) work with a quarterly inflation rate, while we work with a four-quarter inflation rate, we re-scaled their Phillips curve coefficients by multiplying them by 4.

All the parameters of the benchmark model are collected in Table 2.

The models is thought for representing quarterly dynamics. All the variables are in log-deviations with respect to their steady states, which are normalized to zero. The timing of the model goes as follows: 1) private agents form their expectations; 2) CB sets the policy rate; 3) shocks strike the economy. As testified by its impulse response functions, the model is quite appealing from an empirical viewpoint, i.e. it is a tool capable to replicate the observed persistence of the variables it describes, a key requirement for running sensible policy experiments (as discussed in Estrella and Fuhrer, 2002) and for performing our exercise. Given the similarity between our impulse response functions and those reported in Svensson (2000) and Leitemo and Söderström (2003), we refer to those contributions for a detailed comment.

[**Figures 1-2 about here**]

Table 3 collects the coefficients of the optimal rules (17) conditioned to the regime shift above described and the model parameters as in Table 2. As expected, huge differences exist between optimal rules associated to different targeting schemes. In particular, the Central Banker attributes a large importance to the elements entering the UIP condition in the Pre-Shift phase, and mainly to domestic elements in the Post-Shift period. Notably, the shift also leads to a higher optimal interest rate smoothing, given the higher concern toward inflation stabilization under discretion (Woodford, 2003).

[**Tables 2-3 about here**]

3 Why may the Lucas critique affect backward looking models?

Before moving to the description of the exercise, we present a simple example that explains why the use of backward-looking models may be misleading.[21] Consider a model like this:

[21] This example is borrowed from Lindé (2001), page 998.

$$x_t = E_t \sum_{j=0}^{\infty} m_{t+j} + \varepsilon_t \tag{18}$$

$$m_t = \phi m_{t-1} + v_t \tag{19}$$

where x_t is the variable targeted by the policy makers, m_t is the policy variable, ε_t and v_t are white noise exogenous shocks, and $|\phi| < 1$. In this model, agents form expectations on the future path of the policy variable m_t; the equilibrium value of the target-variable x_t is right a function of these expectations. By plugging (19) into (18) and imposing rational expectations, we obtain the following equation:

$$x_t = \gamma_m m_t + \varepsilon_t \tag{20}$$

where $\gamma_m \equiv \frac{1}{1-\phi}$. Then, if the econometrician estimates γ_m in (20) and use this model to perform policy simulations based on alternative paths $\{m_{t+j}\}_{j=0}^{\infty}$ (which is to say, based on alternative values of ϕ), he will miss the link existing between new policies $\{m_{t+j}\}_{j=0}^{\infty}$ and the corresponding new values for γ_m.

As stressed by Lucas and Sargent (1981), the relevance of the Lucas critique is an *empirical* issue. Referring to the above illustrated example, if γ_m is barely (i.e. statistically) constant even if ϕ varies over time, then we cannot rule out the possibility of performing policy exercises on the basis of the Lucas critique.[22]

Indeed, this assessment is the goal of our exercise. The object of our test is the following open-economy version of the Phillips curve:

$$\pi_t = \sum_{i=1}^{4} \left(\gamma_{\pi i}\pi_{t-i} + \gamma_{yi} y_{t-i} + \gamma_{qi} q_{t-i}\right) + \xi_t^{\pi} \tag{21}$$

Eq. (21) embeds all and no more than the variables present in the theoretical Phillips curve (1), and it is intended to capture it in a backward looking fashion. In fact, it is nothing but an open-economy version of the one proposed by Rudebusch and Svensson (1999,2002) for the US. Notably,

[22] In this case, m_t is said to be superexogenous for γ_m.

with adequate restrictions on the coefficients γ_s, this reduced-form equation collapses to the one by Ball (1999, 2000).[23]

Then, is eq. (21) stable across the two different regimes at stake? In the next Section we investigate this issue.

4 Steps for assessing the empirical relevance of the Critique

In our exercise we consider different parametrizations of our 'true' model of the economy. In particular, we take into account all the combinations of these batteries of forward looking degrees: $(\mu_\pi, \mu_y) = \{(.3, .1), (.5, .3), (.8, .8)\}$, $\mu_s = \{.4, .7, .9\}$. The first battery is that employed by Rudebusch (2003) in his study on the Lucas critique, while the second one is intended to explore the consequence of having different degrees of forward lookingness in the UIP condition (3).

Then, our algorithm to assess the importance of the Lucas critique goes as follows:

1. We simulate the 'structural' model of the economy for $I + T$ periods *under the null of absence of regime shifts*. In doing so, we draw $I + T$ times from a zero-mean normal distribution per each shock hitting the economy, i.e. u_t, v_t, ψ_t, u_t^*, v_t^*, ζ_t^*, shocks whose variances are indicated in Table 2. Notice that the first $I = 100$ periods are simulated in order to obtain a stochastic vector of initial values for the model, and are just discarded before implementing Step 2.

2. With the whole sample of simulated data (sample whose size is equal to T), we OLS estimate the 'reduced form' coefficients of the backward looking Phillips curve (21). Then, we compute the F-statistical value related to the Chow (1960) breakpoint test. To do that, we consider subsamples of equal size $T_1 = T_2 = \frac{T}{2}$.[24] Notice that here we are

[23] Ball (1999,2000)'s Phillips curve reads as follows: $\pi_t = \pi_{t-1} + \alpha y_{t-1} - \gamma(q_{t-1} - q_{t-2}) + \eta$. To be precise, in those papers y_t stands for the log of real output.

[24] To compute the F-statistic, we adopt the following formula (k stands for the number of estimated coefficients): $\frac{(\hat{\sigma}_T^2 - \hat{\sigma}_{T1}^2 - \hat{\sigma}_{T2}^2)/k}{(\hat{\sigma}_{T1}^2 + \hat{\sigma}_{T2}^2)/(T-2k)} \sim F(k, T-2k)$ under the null of stability.

applying the Chow test to detect the break occurring at a *known* date, given that we perfectly know the date of the break.[25] We perform our exercise with both $T = 200$ and $T = 500$; we do this in order not to obtain misleading indications from the Chow-test, whose power is low in small samples (Lindé, 2001);

3. We repeat Steps 1-2 $N = 5,000$ times. Once done so, we compute the F-critical value for the Chow test, so obtaining the corrected-per-sample-size critical value of the test (we go for a 5% statistical confidence);

4. We implement Step 1 *allowing for the above described regime-shift* at $t = \frac{T}{2}$;

5. We implement Step 2. Once done so, we compare the F-statistical value with the F-critical value computed in Step 3. Notice that if the statistical value is larger/smaller than the critical one, the null of stability is rejected/non rejected;

6. We repeat Steps 4-6 $N = 5,000$ times. Then, we count how many times we rejected the null of stability (in Step 5). This *rejection rate* is a 'p-value' indicating the probability of *rejecting* the null of stability of the estimated backward looking Phillips curve at a 5% level of statistical confidence. If this rejection-rate is larger/smaller than .05, then the estimated backward looking schedules should be judged as being potentially unstable/stable, and the Lucas critique turns out to be empirically relevant/non-relevant. This is so because, if the Null holds true, just a 5% should show up (due to the level of confidence we selected). Then, a higher value would signal that the Null is statistically rejected.[26]

[25] For a note on the Chow test vs. alternative ones when the break-date is *unknown* see Hansen (2001).

[26] Since we are working with an empirical F-distribution, standard deviations for the computed rejection-rates should also be taken into account. However, the standard deviation is equal to $\frac{rej.rate(1-rej.rate)}{\sqrt{N}}$, that is to say that the maximum value it can assume is .003536 (when the rejection-rate = .5). In claiming that an estimated equation is non-stable, we will consider large rejection-rates so to take into account the uncertainty surrounding them.

We now turn to the analysis of our results.

5 Findings

Our results are collected in Table 4.[27] This Table display the figures corresponding to the adjusted R^2 statistic and the 'p-values' (i.e. rejection-rates) computed as explained in the previous section. As said above, a rejection-rate larger that .05 indicates that the estimated coefficients are not stable. However, we will be more demanding than that, and we will search for higher values, somehow in the spirit of Rudebusch (2003).

Some interesting results seem to come out. First, as long as the 'true' model of the economy is featured by low degrees of forwardness of the AD-AS schedules, the estimated Phillips curve shows a quite appreciable stability. In particular, with the pair $(\mu_\pi, \mu_y) = (.3, .1)$ it is pretty hard to detect a strong evidence of instability. Things change when the 'true' model of the economy assumes a more forward looking fashion. However, the estimated curve (21) turns out to be clearly unstable only when pretty large degrees of forwardness are taken into account. Interestingly, those high values are not necessarily the most interesting ones, at least given what some contributions on the US teach us about those parameters.[28] Not surprisingly, we register a negative correlation linking the 'degrees of forwardness' of the economy and the $\bar{R}^2$ statistic.

The exercise we run allows us to reach also another conclusion regarding the power of the test we employed. Take as an example the model featured by the triple $(\mu_\pi, \mu_y, \mu_s) = (.5, .3, .4)$. If we limited ourselves on a short-sample analysis, we would be quite doubtful on the choice of rejecting the stability of the reduced-form Phillips curve. In fact, the larger sample result is much more in favor of a rejection of the Null of stability. Indeed, Lindè (2001)'s point concerning the low-power of the Chow-test in small samples emerges

[27] In order to save space, we do not present in the paper the volatilities of the main economic variables computed under the various regimes and models considered in our exercise. The Matlab codes to compute these figures are available upon request.

[28] See e.g. Fuhrer (1997) for the Phillips curve, and Fuhrer and Rudebusch (2003) for the IS equation. However, there is a hot debate about the value of the 'degree of forwardness' to be attached to the American Phillips curve. For a small survey, see Rudebusch (2002b).

also in this analysis. Nevertheless, as long as we think of 500 observations as forming a large sample, we may state that *not* in all the cases we analyzed the statistical importance of the Lucas critique turns out to be supported by the data.

[**Table 4 about here**]

6 Robustness checks

Of course, our qualitative findings may be affected by some of the choices we made when setting up the 'true' model of the economy, when deciding which reduced form to estimate, and so on. Indeed, we performed some checks to verify the robustness of our results. First, we replicated all the simulations/estimations previously presented with a lower weight for the 'CPI Quasi-Strict Inflation Targeting' regime, i.e. we took into account a value for $\lambda_y = .2$. Another check we performed was that of estimating a richer version of equation (21). In particular, we added four lags of both the domestic and the foreign short-term interest rate. Then, we implemented a grid-check having as protagonists some key-parameters, such as α_y (linking the output gap to the inflation rate in the Phillips curve 1), β_r (interest rate sensitivity of the economy), ϕ (responsible of the 'choice' and intensity of the distributed lags vs. chartist schemes in the UIP condition 34), and θ (indicating the degree 'imperfection' of the exchange rate pass-through).[29] Finally, we also considered a longer sample, i.e. $T = 1,000$, for our 'large-sample' analysis. All in all, the results commented above turn out to be fairly robust to these perturbations.[30]

7 Conclusions

We set up a small scale open economy dynamic stochastic model that allows for imperfect exchange rate pass-through and endogenous persistence

[29] Values investigated (single departures with respect to the benchmark model): $\alpha_y \in \{.15, .2, .25\}$, $\beta_r \in \{.1, .15, .2\}$, $\phi \in \{-1, -.5, 0, .5, 1\}$, $\theta \in \{0, .2, .4, .6, .8, 1\}$.

[30] For sake of brevity, we do not display here the results of our robustness checks. However, the Matlab codes for replicating them are available upon request.

in inflation, output gap, and nominal exchange rate. With this model, we simulated a regime shift, i.e. from 'controlled' to floating exchange rate. With our simulated data, we estimated a reduced-form equation, i.e. a backward looking Phillips curve, in order to evaluate its stability under such a regime shift. With this 'simulation-and-estimation' approach, we obtained some interesting results. i) Overall, our results do not support the empirical relevance of the Lucas critique for this formulation of the Phillips curve, at least when we do not assume large 'degrees of forwardness' for our 'true' model of the economy. This is good-news for policy analysis based on backward-looking models, above all when 'milder' policy shifts - like fairly small variations of a Taylor rule's coefficients - are taken into account (as in Ball 1999,2000), or for assessments such that of the sacrifice ratio (as in e.g. Leitemo and Røste, 2003). ii) The more forward the 'true' model of the economy is, the higher is the probability of rejecting the null of stability of the estimated Phillips curve. This finding calls for a serious quantification of these key-parameters. iii) Our analysis confirms Lindé (2001)'s evidence against the power of the Chow-breakpoint test in small samples. iv) The impact of different processes for the nominal exchange rate formation is not clear, and deserves further investigation.

Of course, the flip-coin of our findings leads us to state that if the 'true' model of the economy is prevalently characterized by forward-looking agents, then backward-looking models might turn out to be severely unstable. In this case, should these models be discarded when performing such analysis? Interestingly, nothing guarantees that forward looking models would show a superior stability, as shown by Estrella and Fuhrer (2003). Therefore, we interpret the evidence in this and other papers on the Lucas critique as a call for monetary policy analyses based on a large variety of different models, an exercise already done in some occasions, e.g. the NBER conference on Taylor rules whose contributions are collected in Taylor (1999).

Notice that the results reached in our study are strictly related to the magnitude of the shifts we considered. In fact, tougher shifts might lead to different conclusions on the stability of the Phillips curve. We think of this as being a possible extension of this work. Other extensions also come as natural. First, the estimation of a whole closed system of equations seems to be warranted. The Phillips curve is not the only protagonist of the monetary policy transmission mechanism; then, the stability of other schedules is equally important for a complete assessment of backward looking models. Then, an analysis country-by-country on the importance of the Lucas critique should be performed. This would imply the estimation of the system e.g. via ML/Bayesian methods, as in Ireland (2001) or Smets and Wouters (2002), and would allow us to draw idiosyncratic conclusions concerning the stability of the estimated reduced form equations in a given country. Moreover, the stability of models with time-varying parameters could be assessed. This line of research has been already undertaken by Cogley and Sargent (2003) and Primiceri (2003), and promises to be quite fruitful. Finally, some authors have employed hybrid models like the one employed in our simulations. A stability check concerning this type of models for open-economies seems to be warranted, and it is already in our agenda.

Appendix

In this Appendix we i) explain how to set up the problem in order to arrive to a standard optimal stochastic regulator problem, and ii) offer a discussion on alternative ways to formalize a 'controlled exchange rate regime' with respect to that implemented in the paper.

7.1 Appendix i): The standard stochastic regulator problem

In this technical appendix we explain how to re-express the model presented in the text in its state-space form. To do so, we exploit the following equivalences coming from the assumption of rational expectations:

$$\pi_{t+1} = \pi^e_{t+1} + u_{t+1} \tag{22}$$

$$y_{t+1} = y^e_{t+1} + v_{t+1} \tag{23}$$

Then, we can re-write our economic system as follows:

$$\mu_\pi \pi^e_{t+2} + \alpha_q q^e_{t+1} = \pi^e_{t+1} - (1-\mu_\pi)\pi_t - \alpha_y y_t \tag{24}$$

$$\mu_y y^e_{t+2} = y^e_{t+1} - (1-\mu_y)y_t + \beta_r(i_t - \pi^e_{t+1}) - \beta_q q_t - \beta_y y^*_t \tag{25}$$

$$\mu_s s_{t+1} = i_t - i^*_t - (1-\mu_s)[(1+\phi)s_t - \phi s_{t-1}] + s_t - \varphi_t \tag{26}$$

$$\varphi_{t+1} = \rho_\varphi \varphi_t + \psi_{t+1} \tag{27}$$

$$\pi^M_t = (1-\theta)\pi^M_{t-1} + \theta(\pi^*_t + s_t - s_{t-1}) \tag{28}$$

$$i^*_t = (1-\rho_{i*})(f_{\pi*}\pi^*_t + f_{y*}y^*_t) + \rho_{i*} i^*_{t-1} + \zeta^*_t \tag{29}$$

$$\pi^*_{t+1} = \rho_{\pi*}\pi^*_t + u^*_{t+1} \tag{30}$$

$$y^*_{t+1} = \rho_{y*} y^*_t + v^*_{t+1} \tag{31}$$

Notice that it is possible to get rid of the nominal exchange rate in equation (26) by exploiting the following expressions (coming from eq. (5) and the definition of inflation rate (i.e. $\pi_t \equiv p_t - p_{t-1}$, $p_t = \log(P_t)$, $P_t =$ price level at time t):

$$\begin{aligned}
s^e_{t+1} &= q^e_{t+1} - p^{*e}_{t+1} + p^e_{t+1} \\
p^{*e}_{t+1} &= \pi^{*e}_{t+1} + p^*_t \\
&= \pi^{*e}_{t+1} + \pi^*_t + p^*_{t-1} \\
&= (1 + \rho^*_\pi)\pi^*_t + p^*_{t-1} \\
p^e_{t+1} &= \pi^e_{t+1} + p_t \\
&= \pi^e_{t+1} + \pi_t + p_{t-1}
\end{aligned}$$

Then, it is possible to rewrite (26) as follows:

$$\begin{aligned}
i_t &= i^*_t + \mu_s s^e_{t+1} + (1-\mu_s)[(1+\phi)s_t - \phi s_{t-1}] - s_t + \varphi_t \qquad (32) \\
&= i^*_t + \mu_s(q^e_{t+1} - p^{*e}_{t+1} + p^e_{t+1}) \\
&\quad +(1-\mu_s)[(1+\phi)(q_t - p^*_t + p_t) - \phi(q_{t-1} - p^*_{t-1} + p_{t-1})] \\
&\quad -q_t + p^*_t - p_t + \varphi_t \\
&= i^*_t + \mu_s[q^e_{t+1} - (1+\rho^*_\pi)\pi^*_t - p^*_{t-1} + \pi^e_{t+1} + \pi_t + p_{t-1}] \\
&\quad +(1-\mu_s)[(1+\phi)(q_t - \pi^*_t - p^*_{t-1} + \pi_t + p_{t-1}) \\
&\quad -\phi(q_{t-1} - p^*_{t-1} + p_{t-1})] - q_t + \pi^*_t + p^*_{t-1} - \pi_t - p_{t-1} + \varphi_t
\end{aligned}$$

Notice that both p^*_{t-1} and p_{t-1} cancel out. Then, by collecting terms we can rewrite eq. (32) as follows:

$$\begin{aligned}
i_t &= i^*_t + \mu_s q^e_{t+1} - [\phi + \mu_s(\rho^*_\pi - \phi)]\pi^*_t + \mu_s \pi^e_{t+1} + \phi(1-\mu_s)\pi_t \qquad (33) \\
&\quad +(\phi - \mu_s\phi - \mu_s)q_t - \phi(1-\mu_s)q_{t-1} + \varphi_t
\end{aligned}$$

Of course, if the weight of the forward looking component in the exchange rate formation $\mu_s = 1$, then working with expressions (5) and (33) it is possible to get the 'textbook' expression of the UIP, i.e.

$$i_t = i_t^* + \Delta s_{t+1}^e + \varphi_t$$

By contrast, if $\mu_s = 0$, then the UIP in the model collapses to

$$i_t - \phi\pi_t = i_t^* - \phi\pi_t^* + \Delta q_t + \varphi_t$$

which is the backward-looking expression used by Debelle and Wilkinson (2002) under the restriction $\phi = 1$.

Then, it is possible to express q_{t+1}^eas:

$$\begin{aligned} \mu_s q_{t+1}^e &= i_t - i_t^* + [\phi + \mu_s(\rho_\pi^* - \phi)]\pi_t^* - \mu_s \pi_{t+1}^e \\ &\quad -\phi(1-\mu_s)\pi_t - (\phi - \mu_s\phi - \mu_s)q_t + \phi(1-\mu_s)q_{t-1} - \varphi_t \end{aligned} \tag{34}$$

which is useful for our purposes.

All the information coming from equations (22)-(34) may be compacted as follows:

$$A_0 \begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix} = A_0 X_{t+1} = A_1 X_t + B_1 i_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2 x1} \end{bmatrix} \tag{35}$$

The $(nx1)$ column vector X_t in eq. (35) is featured by $n = n_1 + n_2 = 12$, $n_1 = 9$, $n_2 = 3$. In particular, x_{1t} is the vector collecting the predetermined variables, i.e. $x_{1t} = \begin{bmatrix} \pi_t & y_t & \varphi_t & q_{t-1} & \pi_{t-1}^M & i_t^* & \pi_t^* & y_t^* & i_{t-1} \end{bmatrix}'$.[31] Instead, x_{2t} collects all the jump variables of this state-space representation, and it is defined as $x_{2t} = \begin{bmatrix} \pi_{t+1}^e & y_{t+1}^e & q_t \end{bmatrix}'$. Let's define the vector e_i as the $1x(n1+n2)$ vector assuming zero values in all the cells but the i^{th} one, and the value 1 in the i^{th} cell. Then, the matrices and vectors forming the state-space representation (35) are the following:

[31] To be precise, i_{t-1} is not a pre-determined variable in this framework. In fact, its presence in the state-space representation is due to the interest rate smoothing argument belonging to the policy-makers' loss function.

$$A_0 = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 + \widetilde{\rho_{i*}}\ f_{\pi *} e_7 + \widetilde{\rho_{i*}}\ f_{y*} e_8 \\ e_7 \\ e_8 \\ e_9 \\ \mu_\pi e_{10} + \alpha_q e_{12} \\ \mu_y e_{11} \\ \mu_s e_{12} \end{bmatrix}$$

$$A_1 = \begin{bmatrix} e_{10} \\ e_{11} \\ \rho_\varphi e_3 \\ e_{12} \\ \theta e_1 - \theta e_4 + (1-\theta) e_5 + \theta e_{12} \\ \rho_{i*} e_6 \\ \rho_{\pi *} e_7 \\ \rho_{y*} e_8 \\ 0_{1x12} \\ \widetilde{\mu_\pi}\ e_1 - \alpha_y e_2 + e_{10} \\ \widetilde{\mu_y}\ e_2 - \beta_y e_8 - \beta_r e_{10} + e_{11} - \beta_q e_{12} \\ [\phi\ \widetilde{\mu_s}\ e_1 - e_3 - \phi\ \widetilde{\mu_s}\ e_4 - e_6 \\ +[\phi + \mu_s(\rho_{\pi *} - \phi)] e_7 - \mu_s e_{10} + (\phi\ \widetilde{\mu_s} + \mu_s) e_{12}] \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0_{1x8} & 1 & 0 & \beta_r & 1 \end{bmatrix}'$$

$$\varepsilon_{t+1} = \begin{bmatrix} u_{t+1} & v_{t+1} & \psi_{t+1} & 0_{1x2} & \xi^*_{t+1} & u^*_{t+1} & v^*_{t+1} & 0 \end{bmatrix}'$$

where $\widetilde{\rho_{i*}} \equiv -(1 - \rho_{i*})$ and $\widetilde{\mu_x} \equiv -(1 - \mu_x)$.

From the system (35) it is immediate to obtain the standard law of motion of the optimal stochastic regulator problem, as follows:

$$X_{t+1} = AX_t + Bi_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2 x1} \end{bmatrix} \tag{36}$$

where

$$A \equiv {A_0}^{-1} A_1 \tag{37}$$

$$B \equiv {A_0}^{-1} B_1 \tag{38}$$

Eq. (36) represents the constraint faced by the central banker when solving his optimal control problem. In doing that, the central banker minimizes the following intertemporal loss function:

$$J_0 = E_t \sum_{\tau=1}^{\infty} \delta^{\tau} L_{t+\tau} \tag{39}$$

In our set up, the contemporaneous penalty function is given by

$$L_t = \lambda_{\Delta s} {s_t}^2 + \lambda_{\pi CPI} (\pi_t^{CPI})^2 + \lambda_y y_t^2 + \lambda_{\Delta i} \Delta i_t^2 \tag{40}$$

where CPI inflation is defined according to equation (10), that we rewrite here below:

$$\pi_t^{CPI} = (1-\chi)\pi_t + \chi[(1-\theta)\pi_{t-1}^M + \theta(\pi_t + \Delta q_t)] \tag{41}$$

We exploit this expression (given the vector x_{1t} of pre-determined variables) in order to re-express the CPI inflation target in matrix form. In fact, also the instantaneous loss (40) may be rewritten in a compact manner, as follows:

$$Y_t = C_x X_t + C_i i_t \tag{42}$$

$$L_t = Y_t' K Y_t \tag{43}$$

where

$$C_x = \begin{bmatrix} e_1 - e_4 - e_7 + e_{12} \\ (1-\chi+\chi\theta)e_1 - \chi\theta e_4 + \chi(1-\theta)e_5 + \chi\theta e_{12} \\ e_2 \\ -e_9 \end{bmatrix}$$

$$C_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad K = diag\left\{\lambda_{\Delta s}, \lambda_\pi^{CPI}, \lambda_y, \lambda_{\Delta i}\right\}$$

with *diag* being a diagonal matrix whose elements are indicated in curly brackets.

Now the problem is ready to be solved with standard techniques. In particular, notice that the instantaneous loss function (43), given (42), may be decomposed as follows:

$$\begin{aligned} L_t &= \begin{bmatrix} X_t' & i_t' \end{bmatrix} \begin{bmatrix} C_x' \\ C_i' \end{bmatrix} K \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix} \\ &= X_t' C_x' K C_x X_t + X_t' C_x' K C_i i_t + i_t' C_i' K C_x X_t + i_t' C_i' K C_i i_t \\ &= X_t' Q X_t + X_t' U i_t + i_t' U' X_t + i_t' R i_t \end{aligned}$$

where

$$Q \equiv C_x' K C_x$$

$$U \equiv C_x' K C_i$$

$$R \equiv C_i' K C_i$$

Hence the CB's optimal control problem is given by the conventional Bellman equation

$$J_t = E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} (X_\tau' Q X_\tau + X_\tau' U i_\tau + i_\tau' U' X_\tau + i_\tau' R i_\tau) \qquad (44)$$

subject to the law of motion of the economy (36). Söderlind (1999) shows that the solution of the optimal control problem *under discretion* is given by[32]

$$i_t = fx_{1t} \tag{45}$$

where f is a $(1xn_1)$ vector whose elements are convolutions of the preferences of the regulator (i.e. the λ_s in the loss function (43), stored in the K matrix) and the semi-structural parameters of the recursive system (36).

Under the assumption of rational expectations, the equilibrium law of motion of the predetermined variables is given by

$$x_{1t+1} = Mx_{1t} + \varepsilon_{t+1} \tag{46}$$

while the jump variables are defined as

$$x_{2t} = Nx_{1t} \tag{47}$$

Details on how to compute the matrices M and N are provided by Söderlind (1999). Importantly enough, when the discount factor $\delta \to 1$, the intertemporal loss function (44) approaches the unconditional mean of the period loss function, i.e.

$$E(L_t) = \lambda_{\Delta s} Var(\Delta s_t) + \lambda_{\pi^{CPI}} Var(\pi_t^{CPI}) + \lambda_y Var(y_t) + \lambda_{\Delta i} irsmooth \tag{48}$$

Impulse Response Functions

The impulse response functions presented in this paper were produced by shocking the pre-determined variables at the 'very first period and then nothing' (standard approach). To produce the responses of the system to a domestic monetary policy shock, the value of the short term nominal interest rate i_t in steady-state was perturbed of one unit at the very first period.

[32] In fact, when forward-looking variables are part of the dynamics of the economy, a distinction between the solution under discretion and the one under commitment has to be drawn. The optimal policy in the commitment case is more complicated, and depends also on the shadow prices of the forward-looking variables. The software for solving the optimal stochastic regulator problem both under discretion and under commitment may be found in Söderlind's web-page, i.e http://home.tiscalinet.ch/paulsoderlind/.

7.2 Appendix ii): The formalization of the 'controlled' exchange rate regime in a linear-quadratic world

We thought of at least three alternative manners for mimicking a fixed exchange rate regime, i.e. a nominal exchange rate targeting, an interest rate peg, and a change in the policy instrument. The first option may be identified with the following loss function:

$$Var(s_t - \overline{s}) + irsmooth \tag{49}$$

However, given the unit-root nature of the nominal exchange rate in our set up, we know that its in-population volatility is unbounded. Then, this choice would be incorrect.

As a second option, a simple interest rate peg of the form

$$i_t = i_t^* + \varphi_t \tag{50}$$

might be implemented. But this would not allow us to pin down any unique equilibrium value for the nominal exchange rate, then leading to exchange rate and real economy indeterminacy as shown by Benigno, Benigno, and Ghironi (2003). In a nutshell, Benigno *et al*'s paper suggests the following. Let's focus for simplicity on a UIP condition such as $i_t = i_t^* + s_{t+1}^e - s_t$. Then, the imposition of the fixed rule $i_t = i_t^*$ would imply the condition $s_{t+1}^e = s_t$ to be respected in equilibrium. Does this guarantee the possibility for the Central Bank to pin down a desired nominal exchange rate s^*? No, for two reasons. i) $s_{t+1}^e = s_t = s^*$ must hold just in *expected* terms. Then we might observe *ex-post* jumps in the nominal exchange rate, like $s_{t+1} = s^{*'} \neq s^*$, jumps that would be perfectly consistent with the *ex-ante* imposition of the rule $i_t = i_t^*$. Therefore, this rule is not of help for stabilizing the exchange rate level. ii) $s_{t+1}^e = s_t = \overline{s}$ holds true for infinite number of fixed exchange rate $\overline{s} = s^* + c$, with c assuming any value. Hence, the rule proposed above would also fail to pin down a *unique* equilibrium value for s_t. Notice that, via the link between nominal and real exchange rate, the instability and indeterminacy problems would affect the real side of the economy as well. The solution of these problems is provided by a rule like $i_t = (1 + i_t^*)\phi(\frac{S_t}{S^*}) - 1$, with $\phi()$ being a function whose features are stated in

Benigno, Benigno, and Ghironi (2003). The non-linearity of this rule implies some difficulties in implementing it in our linear-quadratic world.

Third, we could modify the set-up of our problem in order to enable the Central Banker to control *directly* the nominal exchange rate s_t. This would provide us with a stationary process s_t, i.e. $s_t = \overline{s}\ \forall t$.[33] But most of the examples provided in Table 1 in the text refer to less-than-perfectly fixed exchange rate regime when dealing with 'controlled' flexibility, than this choice might bring us to a too 'extreme' scenario with respect to what it happened in those countries.

Another possibility of setting up a 'controlled' exchange rate regime is the following:

$$Var(i_t - i_t^* - \varphi_t) + irsmooth \tag{51}$$

which is intended to resemble a situation in which the domestic Central Banker focuses his attention on external shocks gives up the monetary policy instrument for maintaining the level of the nominal exchange rate s_t at its predetermined value. The rationale for this comes from our reading of the UIP condition (34). If domestic monetary policy makers must operate in order to keep the nominal exchange rate $s_t = \overline{s}$, then they should basically operate in a manner such that the UIP holds under this constraint. In other words, they would have to target the foreign interest rate i_t^*, while taking the risk-premium φ_t also into account.

By means of eq. (3), we may rewrite (51) as:

$$Var\left\{\mu_s s_{t+1}^e + (1-\mu_s)[(1+\phi)s_t - \phi s_{t-1}] - s_t\right\} + irsmooth \tag{52}$$

Then, under this exchange rate regime the Central Bank aims at minimizing a function $g(s_{t+1}^e, s_t, s_{t-1})$. Notice that the solution of the optimal control problem at hand delivers a policy rule relating i_t to all the predetermined variables of the system. This is so because the regulator aims at computing a unique stable solution under rational expectations. This means

[33] This equilibrium outcome would hold true in case of *absence* of any interest rate smoothing motive.

that while targeting (51), the Central Banker also cares of determining equilibrium values of the other variables present in the economy. Of course, this implies a less-than-perfect nominal exchange rate peg.

In fact, we employed this last option to double-check the results obtained with the loss function (15). The check confirmed us that our qualitative results are robust to this variation in the CB's loss.

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Country	*Pre-Shift classification*	*Post-Shift classification*
Argentina	March 1986-April 1991: freely falling/floating	April 1991-Dec. 2001: currency Board/Peg to US $
Australia	Dec.1971-Nov. 1982: peg/band around US $	Nov. 1982-Dec. 2001: managed/freely floating
New Zealand	July 1973-March 1985: Peg to US $ (up to '73) Band around Australian dollar	March 1985-Dec. 2001: managed floating
Norway	July 1987-Dec. 1992: moving band around DM	Dec. 1992-Dec. 2001 managed floating
Sweden	March 1973-Nov.1992: crawling around DM	Nov. 1992-Dec. 2001: managed floating
Switzerland	Jan.1973-Sept. 1981: managed floating	Sept. 1981-Dec. 2001: moving band around DM
UK	Oct.1990-Sept. 1992: Band around ECU/DM	Sept. 1992-Dec. 2001: managed floating

Table 1: **HISTORICAL EXCHANGE RATE REGIMES SHIFTS**. Source: Reinhart and Rogoff (2002). The labels for the exchange rate regimes are those referring to Reinhart and Rogoff (2002)'s coarse grid (Table 4 in their paper), and go (from 'fixed' to 'floating') approximately like this: 'peg', 'crawling peg', 'moving band', 'managed floating', 'freely floating'. Our cut-off for shifting from 'controlled' to 'floating' exchange rate regime is located between 'moving band' and 'managed floating'.

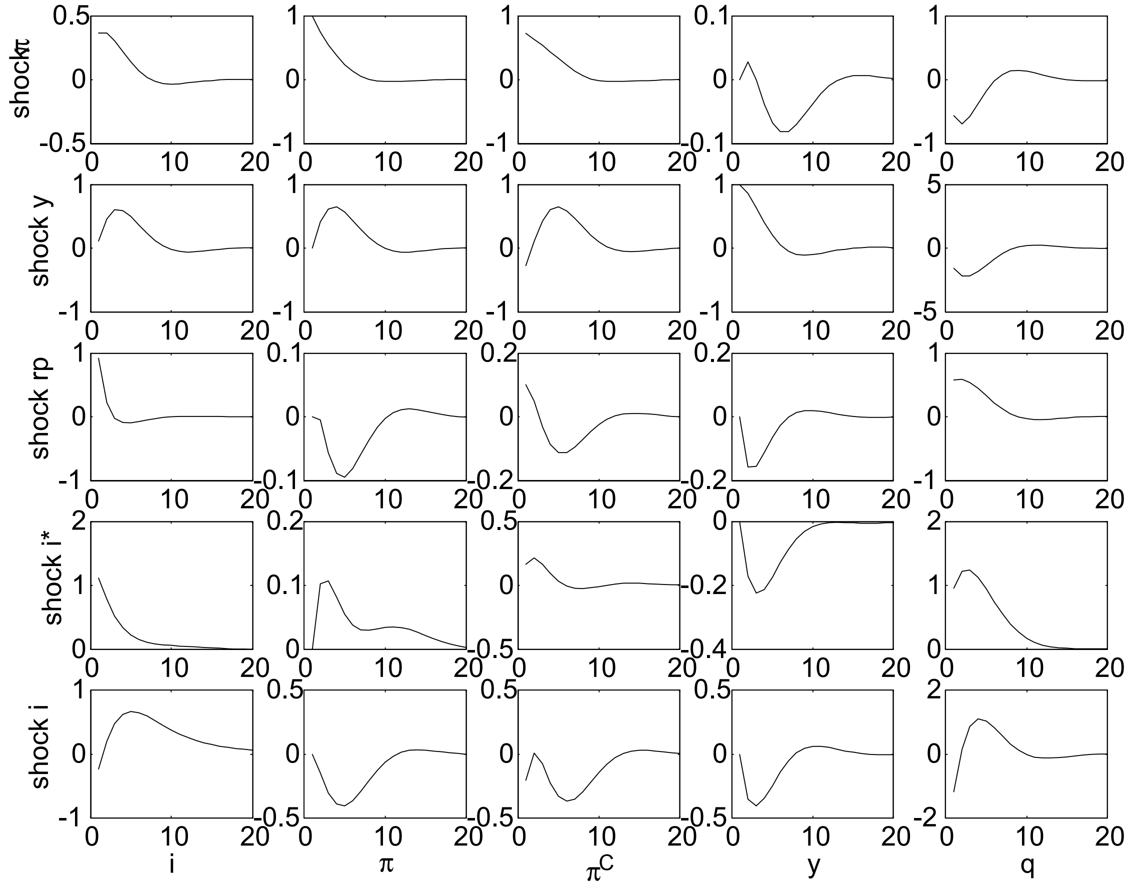

Figure 1: **IMPULSE RESPONSE FUNCTIONS UNDER CONTROLLED EXCHANGE RATE VOLATILITY.**

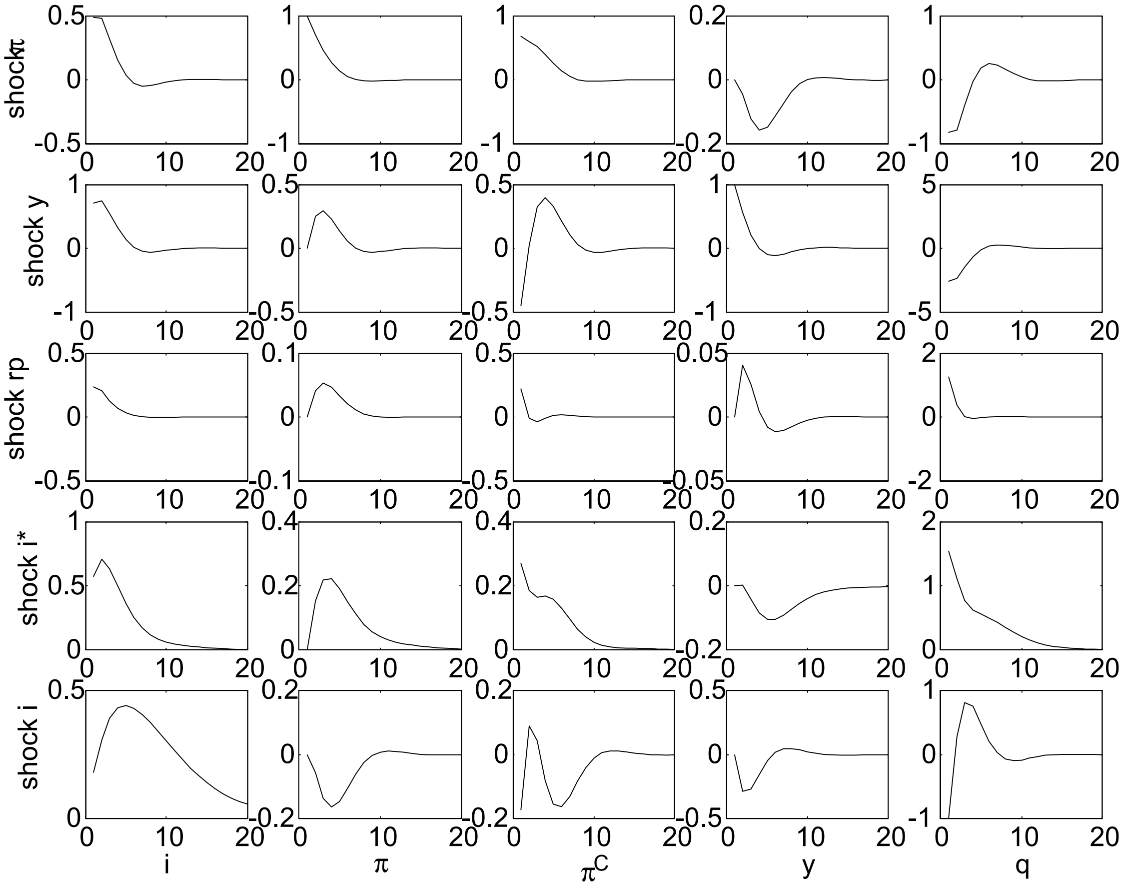

Figure 2: **IMPULSE RESPONSE FUNCTIONS UNDER QUASI-STRICT CPI INFLATION TARGETING.**

Domestic economy							
Phillips curve		IS curve		UIP condition		CPI equation	
μ_π	.5	μ_y	.3	μ_s	.7	χ	.35
α_y	.2	β_r	.15	ϕ	$-.5$	θ	.5
α_q	.04	β_q	.05	ρ_φ	.3		
σ_u^2	1.556	β_y	.12	σ_ψ^2	.844		
		σ_v^2	.656				
Foreign economy						*Central Bank*	
Phillips curve		IS curve		Taylor rule		Key-parameters	
$\rho_{\pi*}$	.8	ρ_{y*}	.8	$f_{\pi*}$	1.5	δ	.99
σ_{u*}^2	.5	σ_{v*}^2	.5	f_{y*}	.5	$\lambda_{\Delta i}$	.2
				ρ_{i*}	.75		
				$\sigma_{\xi*}^2$	.5		

Table 2: **BENCHMARK PARAMETRIZATION**. Sources of the parameters indicated in the text.

Optimal policy rule	π_t	y_t	φ_t	q_{t-1}	π_{t-1}^M	i_t^*	π_t^*	y_t^*	i_{t-1}
$Pre-Shift$	**.36**	**.11**	**.92**	-.13	.00	**1.12**	**-.24**	**-.08**	**.04**
$Post-Shift$	**.49**	**.71**	**.23**	-.01	.03	**.57**	**.18**	**.10**	**.33**

Table 3: **OPTIMAL REACTION FUNCTIONS UNDER ALTERNATIVE REGIMES.** Model parameter as in the benchmark case, see Table 2.

'True' Model			*Phillips curve (21)*			
μ_π	μ_y	μ_s	$\frac{\bar{R}^2}{T=200}$	$\frac{rej.-rate}{T=200}$	$\frac{\bar{R}^2}{T=500}$	$\frac{rej.-rate}{T=500}$
.3	.1	.4	.657	.057	.670	.077
.3	.1	.7	.726	.054	.741	.062
.3	.1	.9	.761	.053	.776	.052
.5	.3	.4	.454	.093	.464	**.216**
.5	.3	.7	.517	.073	.528	.106
.5	.3	.9	.535	.063	.545	.074
.8	.8	.4	.156	**.485**	158	**.946**
.8	.8	.7	.154	**.437**	156	**.944**
.8	.8	.9	.156	**.432**	157	**.933**

Table 4: **ESTIMATED BACKWARD-LOOKING PHILLIPS CURVE IN PRESENCE OF A REGIME SHIFT.** Note: 'rej. rate' indicates the probability of rejecting the Null of stability of the estimated equation at the 5-percent significance level on the basis of the Chow-breakpoint test. The Chow test 5-percent critical values were computed with a Montecarlo experiment under the Null of absence of structural break. VAR lags = 4, interest rate smoothing weight = .2, number of sample draws N = 5,000, simulations run with the parameter values indicated in Table 2.