Non-fundamental speculation and index replacements: a market microstructure approach*

Paolo Colla
London School of Economics
p.colla@lse.ac.uk

Abstract

We develop a dynamic trading game in which fundamental insiders coexist with non-fundamental speculators. Non-fundamental speculators possess superior information about the future noise trades and are able to make sharper inference about the fundamental value with respect to the market maker. We show that non-fundamental speculators decrease market depth as well as the insider's ex-ante gains. We study inclusions in the S&P500 after October 1989 as an example in which non-fundamental speculation may arise due to the preannouncement practice in index replacements. Evidence on the trading activity and the bid-ask spread pattern is consistent with our theoretical analysis.

1 Introduction

The effect on stock prices induced by changes in the composition of broad market indexes has been addressed by many researchers. Most of the empirical work conducted up to know focus on the S&P500[1]. There is a general agreement that inclusion in the index results in raising stock prices due to the demand shift represented by index funds' trading activity[2]. Passive portfolio managers mimic a certain benchmark, and the portfolio rebalancing is usually carried out by means of some tracking error procedure. Index replacements therefore represent a clear rebalancing opportunity: when a stock is added to the S&P500, passive funds should buy it replicating the weight it has in the S&P500. Furthermore since

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[1] Investigations on indices other than the S&P500 can be found in Bos [3] and Jain [10] who report evidence for supplementary S&P Indices; Beneish and Gardner [2] focus on the DJIA; Deininger et al. [5] study the German DAX and MDAX.

[2] Schleifer [15] does not find any significant price impact over a sample consisting of 144 additions during 1966-76, and relates this evidence to the small value of the S&P500 owned by index funds (less than 0.5% in 1975). Similarly prices for S&P500 inclusions are not significantly affected in Harris and Gurel [8] over the period 1973-77.

the tracking error minimization is usually implemented on a daily basis, funds pegged to the S&P500 submit their orders during the effective day of inclusion. While the timing of the announcement of index replacements does not seem to affect passive managers' behavior, it has relevant consequences on other market participants. Until October 1989, S&P announced the inclusion of a new stock in the S&P500 after the close, the change becoming effective by the following open. Since index funds would step into the market at the open after this public announcement, there would not be profitable speculation unless the announcement would be anticipated by some traders. After October 1989 S&P switched to preannouncing the change in the S&P500 composition usually five days before the inclusion. This new practice makes attractive buying the stock cheap ahead of passive funds and selling it at possibly higher prices after the funds' demand is satisfied . Beneish and Whaley [1] refer to this strategy as the "S&P game". Evidence on price jumps at the inclusion under the pre-1989 announcement policy is documented by several studies[3]. According to Jain [10], the price increase documented for the S&P500 as well as for other S&P indexes supports the information hypothesis: inclusion in the index conveys information to the market signalling higher management quality and reduced security riskiness rather than shifting the index funds' demand. The information hypothesis is nevertheless in sharp contrast with the claim by the authorities monitoring the indexes replacements, that address the change in listing as a non-informational event[4]. Beneish and Gardner [2] provide evidence against the information hypothesis. The authors find the price and trading volume of newly included firms in the DJIA not affected and they point at the scarcity of funds pegged to the DJIA as the main reason for this. Beneish and Whaley [1] focus on changes in the S&P500 over the period 1986-1994. Close-to-close data display an average 3.67% price increase following the announcement for the subsample 1986-89. The authors relate this variation to the growth in the index funds industry. During 1989-1994 the postannouncement abnormal return is even larger (5.90%), providing evidence for the presence of risk arbitrageurs playing the "S&P game". Prices tend to increase until the effective day as well as from the open to the close on the effective day. The latter finding supports last day buying pressure by index funds[5]. The trading activity dynamics exhibit abnormal average volume

[3] Researchers disagree on the magnitude of this positive price reaction as well as on its transitory/permanent nature. Over roughly the same sample, Shleifer [15] reports a 2.79% increase in stock prices for index additions; according to Harris and Gurel [8] there is a 3.13% positive price change; Jain [10] documents a 3.07% excess return; Dhillon and Johnson [6] find a 2.26% price increase. The price effect analysis in the long-run appear to be even more controversial among these works. Shleifer [15] documents positive price effects even one month after replacement and concludes that the long-run demand curves for stocks are not perfectly elastic. Similarly, prices in Dhillon and Johnson [6] steady decline but do not revert. On the other hand Harris and Gurel [8] report a complete reversal in three weeks thus relying to the price-pressure hypothesis.

[4] The S&P's Security Price Index Record states that 'Judgements as to the investment appeal of the stocks do not enter into the selection process'. Beneish and Gardner [2] quote a similar disclaimer by the editors of the Wall Street Journal ruling the DJIA replacements.

[5] Similarly, the temporary upwards shift in the average trade size shows that many index funds wait until the effective day to rebalance. This evidence is consistent with the daily

following the announcement, even though the preannouncement practice after October 1989 appears to be effective in ease order imbalances in the day after the announcement. Results on the bid-ask spread are less supportive than the ones reported by other trading activity proxies: significant reductions in the spread are documented for some days after inclusion without any sort of pattern.

On the theoretical side little work has been done on index replacements and more generally on the value of anticipating uninformed trades such as passive funds' demand. Building up on Kyle [11] some extensions have been proposed addressing this issue. Rochet and Vila [14] develop a static game in which the insider is aware of both the final liquidation value and the noise traders' demand while submitting his (limit) order. With respect to the static Kyle [11] equilibrium they show that the informed trades less aggressively on his price signal and in the opposite direction of his volume signal, offsetting half of the uninformed trades. Prices as well as the insider unconditional expected profits are unaltered, while the aggregate order flow and the market liquidity decrease. Yu [17] extends Rochet and Vila [14] in a dynamic setting. At every batch auction the insider's information set -in addition to the final liquidation value- comprises a noisy signal of the uninformed trades. Comparing the expected profits arising from this model and the sequential auction equilibrium in Kyle [11], it is shown that the value of knowing (current) noise trades -as well as market liquidity- depends on the signal's precision. Our analysis is closely related to the two-period trading model in Madrigal [13] where a speculator profits from the superior information on the past uninformed trades he is endowed with. The author shows that knowledge of the market trading process allows the (non-fundamental) speculator to make superior estimates of the final liquidation value with respect to the market maker.

In this paper we explicitly model the preannouncement practice in S&P500 replacements after October 1989 considering a market in which insiders coexist with traders who possess superior information with respect to *future* uninformed trades, i.e. passive funds' entry. The model is presented in Section 2. The equilibrium with and without non-fundamental speculation is analyzed. Section 3 presents the empirical evidence on S&P500 inclusions after October 1989, focusing on trading volume and market depth. Finally Section 4 concludes.

2 Non-fundamental speculation and the 'S&P game'

2.1 Model setup

We develop a two period sequential trading game along the lines of Kyle [11]. Trading takes place in two dates $t = 1, 2$ and the market operates as a batch auction. There are two traded assets: a riskfree asset whose payoff is normalized

tracking error being the performance evaluation criterion.

to zero, and a stock with final liquidation value $\tilde{v} \sim N(p_0, \Sigma_0)$ realized after date 2 trading. The trading dates capture the timing in index replacements as follows. Before trading takes place at time 1 the authorities announce the change in the index composition. Further to the stock(s) added to/removed from the index, it is announced that the change is effective after the second trading round. There are four types of agents in the market: an insider, a non-fundamental speculator[6], a market-maker and noise traders. The insider, the speculator and the noise traders submit their demands -denoted respectively by $\tilde{x}_t, \tilde{y}_t$ and $\tilde{u}_t$- at both dates; thus the time t aggregate order flow is given by $\{\tilde{\omega}_t \equiv \tilde{x}_t + \tilde{y}_t + \tilde{u}_t\}_{t=1,2}$. The quantity traded by noise traders reflect liquidity reasons and we assume that $\tilde{u}_1 \sim N(0, \sigma_0)$, $\tilde{u}_2 \sim N(\bar{u}_0, \sigma_0)$ with $(\tilde{v}, \tilde{u}_1, \tilde{u}_2)$ mutually independent. The joint distribution of $(\tilde{v}, \tilde{u}_1, \tilde{u}_2)$ is common knowledge among market participants before the trading game starts. The noise trading specification departs slightly from the usual assumptions in that we consider a shift in expected liquidity trades between the two trading dates. In our setup we regard passive funds entering the market as responsible for this shift in the average noise trading. Pegged funds aim at tracking the market index and their performance is assessed via tracking error procedures. Every time the index list changes, passive managers should rebalance their portfolios in order to replicate the new weights. Since passive trades are driven by no on other information than the index composition, we consider them as noise trades. In our setup the index replacement is effective after date 2, implying that pegged funds should trade the included/excluded stocks at the second trading date. Thus the shift in the average noise trading activity at the second round reflects passive funds stepping into the market. The magnitude of this shift will depend on the weight pegged funds have relative to other liquidity traders. For an included (resp. excluded) stock we therefore expect $\bar{u}_0$ to be positive (resp. negative). At both dates the trading process is modeled via a two-stage game: in the first stage the insider, the speculator and the liquidity traders submit their orders to the market maker; in the second stage the market maker determines the price at which the market is cleared. The price in period t is assumed to satisfy the efficiency condition[7]:

$$p_t = E\left(\tilde{v} \middle| \Phi_t^M\right) \tag{1}$$

where Φ_t^M denotes the market maker's information set, i.e. $\Phi_t^M = \{\tilde{\omega}_s = \omega_s, s \leq t\}$. After each trading round the price becomes common knowledge among all market participants. In describing the information structure of our trading game, let Φ_t^I (resp. Φ_t^S) denote the insider's (resp. speculator's) information set at date

[6] In this strategic trading setup uncertainty among market participants is captured by two random variables: the final liquidation value and liquidity trades. We therefore distinguish the information related to these random variables as fundamental and non-fundamental respectively. Similarly we distinguish informed traders between insiders and speculators: the former receive both fundamental and non-fundamental information while the latter are endowed with non-fundamental information only.

[7] As in Kyle [11] the zero-profit condition arises from price competion à la Bertrand in the market making sector. In equilibrium one can consider a single market maker that operates according to a zero expected profits condition.

t. The insider possesses superior information regarding both the stock's fundamental value and other non-fundamental aspects of the asset markets, while the speculator is endowed with privileged non-fundamental information only. In particular for the insider we set $\Phi_1^I = \{\tilde{v} = v, \tilde{u}_1 = u_1\}$ and $\Phi_2^I = \Phi_1^I \cup \{p_1, \tilde{u}_2 = u_2\}$, while for the speculator $\Phi_1^S = \{\tilde{u}_2 = u_2\}$ and $\Phi_2^S = \Phi_1^S \cup \{p_1\}$. Our information structure departs from the existing literature in the following aspects. As in Kyle [11] and Foster and Viswanathan [7] the insider is endowed with (long-lived) information on the final payoff $\tilde{v}$; in our game, before trading at time t, the insider is also aware of time t noise trades. This makes our setup closer to (a two-period extension of) Rochet and Vila [14] or, equivalently, to the trading game in Yu [17] with a non-distorted signal on the final payoff. Our structure shares with Madrigal [13] the focus on the role of non-fundamental speculation (not considered in [11],[7],[14],[17]), with two main differences: 1) the speculator is endowed with superior knowledge about the *future* liquidity trades and 2) the speculator acts as a monopolist on the non-fundamental information about $\tilde{u}_2$ in the *first* trading round. Thus in our model the speculator trades at both dates, while in Madrigal [13] he enters in period 2 only. As mentioned above, we regard noise trades at time 2 as reflecting passive funds' entry. Therefore our information structure captures the idea that both the insider and the speculator know the demand from pegged funds at time 2, but the speculator gathers this non-fundamental information one period ahead of other market participants. Thus the speculator has long-lived information on $\tilde{u}_2$.

As a consequence of these assumptions, our trading game inherits several interesting features. When trading at date 1 both the insider and the speculator impound their information into orders x_1 and y_1. Time 1 noise trades keep the aggregate order flow away from fully revealing both the insider's information on $\tilde{v}$ and the speculator's information about $\tilde{u}_2$. After observing the aggregate order flow ω_1, the market maker forms an estimate $\bar{u}_1$ of $\tilde{u}_2$:

$$\bar{u}_1 = E\left(\tilde{u}_2 \middle| \Phi_1^M\right) \tag{2}$$

Note that our trading game allows the market maker to update his beliefs on both the final liquidation value and the second period noise trades, whereas the existing literature concentrates on the former type of inference[8].

Furthermore the information on $\tilde{u}_2$ -together with the price observation p_1- allows the speculator to form a superior estimate of $\tilde{v}$ relative to the market maker. After the first trading round, the speculator disentangles[9] the insider's and the noise traders' demand $\tilde{x}_1+\tilde{u}_1$ out of the aggregate order flow ω_1 and thus

[8] In Yu [17] the insider receives at each date t (a signal of) time t noise trades. Nonetheless the independence through time of liquidity trades prevents the market maker from extracting any signal on time $t+1$ noise trading based on the current order flow.

[9] Similarly the insider would infer the speculator's information on $\tilde{u}_2$ after observing p_1. Our model displays a hierarchical information structure during the second trading round, i.e. $\Phi_2^S \subset \Phi_2^I$. Borrowing the terminology in Foster and Viswanathan [7] the insider is the 'better informed trader' and the speculator is the 'lesser informed trader'. Similarly Madrigal [13] imposes a hierarchical information structure. In his model the insider and the speculator share the non-fundamental information -knowledge of past noise trades-, but the insider is also aware of the final liquidation value. Note that in our trading game the first period information sets

extracts a signal s of $\tilde{v}$ that is more precise than the market maker's expectation p_1:

$$s = E\left(\tilde{v} \middle| \Phi_2^S\right) \tag{3}$$

Therefore the speculator can profit on the difference $(s - p_1)$ because the liquidity traders in period 2 will keep the order flow from revealing the speculator's information. The insider reacts to the speculator's presence incorporating an estimate of s when trading in the first round. Madrigal [13] focuses on a similar signal extraction problem and the incentives for the insider to manipulate the first period price.

2.2 Equilibrium construction and description

A BNE for our trading game is defined by $\{x_t, y_t, p_t\}_{t=1,2}$ such that the following conditions hold:

1. *insider's profit maximization*: the insider chooses x_1 to maximize total profits:

$$E\left[\left(\tilde{v} - p_1(\omega_1)\right)x_1 + \left(\tilde{v} - p_2(\omega_1, \omega_2)\right)x_2 \middle| \Phi_1^I\right] \tag{4}$$

   given that x_2 maximizes second period profits:

$$E\left[\left(\tilde{v} - p_2(\omega_1, \omega_2)\right)x_2 \middle| \Phi_2^I\right] \tag{5}$$

2. *speculator's profit maximization*: the speculator chooses y_1 to maximize total profits:

$$E\left[\left(\tilde{v} - p_1(\omega_1)\right)y_1 + \left(\tilde{v} - p_2(\omega_1, \omega_2)\right)y_2 \middle| \Phi_1^S\right] \tag{6}$$

   given that y_2 maximizes second period profits:

$$E\left[\left(\tilde{v} - p_2(\omega_1, \omega_2)\right)y_2 \middle| \Phi_2^S\right] \tag{7}$$

3. *market efficiency*: the market maker sets prices according to equation (1):

$$p_1 = E\left(\tilde{v} \middle| \Phi_1^M\right) \tag{8}$$

$$p_2 = E\left(\tilde{v} \middle| \Phi_2^M\right) \tag{9}$$

In characterizing the equilibrium we focus on linear equilibria in which the component functions of the trading strategies and prices are linear. For the speculator this amounts to look for a pair of linear functions $y_1(\cdot)$ and $y_2(\cdot)$ such that $y_1 = y_1(u_2)$ and $y_2 = y_2(u_2, s)$ where s is defined by (3). Recall that within our informational structure the insider knows -prior to trading at time 2- the signal s that the speculator extracts from p_1. As a consequence, when trading at

are non-nested, even though this assumption can be easily modified including u_2 into Φ_1^I. In this case the speculator would lose his informational advantage (with respect to the insider) during the first trading round.

date 1 the insider keeps into account the effect his order has on the speculator's estimate of the final liquidation value $E\left(\tilde{s}|\,\Phi_1^I\right) = E\left(E\left(\tilde{v}|\,\Phi_2^S\right)\middle|\,\Phi_1^I\right)$. The insider's trading strategies are therefore given by a pair of linear functions $x_1(\cdot)$ and $x_2(\cdot)$ such that $x_1 = x_1\left(v, u_1, E\left(\tilde{s}|\,\Phi_1^I\right)\right)$ and $x_2 = x_2(v, u_2, s)$. At $t = 1$ the insider trades on the speculator's (expected) mispricing, i.e. the difference between $E\left(\tilde{s}|\,\Phi_1^I\right)$ and the true liquidation value, as well as the market maker's mispricing, i.e. the difference between the realization v and the first period price. This amounts to conjecture the following form for x_1:

$$x_1 = \alpha(v - p_0) + \beta u_1 + \gamma\left(E\left(\tilde{s}|\,\Phi_1^I\right) - v\right)$$

Note that the insider's date 1 trade depends on the (estimate) of the speculator's conjecture of the final liquidation value, which depends itself on the insider's first period trade. Thus one needs to solve for $E\left(\tilde{s}|\,\Phi_1^I\right)$ and then verify the consistency between the resulting expression for x_1 and the speculator's belief s. In the appendix (see Lemma 1) we show that in equilibrium $E\left(\tilde{s}|\,\Phi_1^I\right) = v + \chi(v - p_0) + \psi u_1$ and $s = p_0 + \phi(x_1 + u_1)$ are mutually consistent (expressions for coefficients χ, ψ and ϕ are given in the appendix). As a consequence the insider's first period trade can be expressed as:

$$x_1 = a_1(v - p_0) + c_1 u_1$$

The unique linear BNE is given in Proposition 1.

Proposition 1 *There exists a unique linear BNE in which trading strategies and prices are of the form:*

$$x_1 = a_1(v - p_0) + c_1 u_1 \tag{10}$$

$$x_2 = a_2(v - p_1) - \frac{1 + B_2}{2}(u_2 - \bar{u}_1) - \frac{C_2}{2}(s - p_1) \tag{11}$$

$$y_1 = B_1(u_2 - \bar{u}_0) \tag{12}$$

$$y_2 = B_2(u_2 - \bar{u}_1) + C_2(s - p_1) \tag{13}$$

$$p_1 = p_0 + \lambda_1(x_1 + y_1 + u_1) \tag{14}$$

$$p_2 = p_1 + \lambda_2(x_2 + y_2 + u_2 - \bar{u}_1) \tag{15}$$

where s is the speculator's belief on $\tilde{v}$ conditional on Φ_2^S and $\bar{u}_1$ is the market maker's belief on $\tilde{u}_2$ conditional on Φ_1^M. The coefficients $a_1, B_1, c_1, a_2, B_2, C_2, \lambda_1, \lambda_2$ and the conditional variances

$$\Sigma_1 = var\left(\tilde{v}|\,\Phi_1^M\right) \tag{16}$$

$$\Sigma_2 = var\left(\tilde{v}|\,\Phi_2^M\right) \tag{17}$$

$$\sigma_1 = var\left(\tilde{u}_2|\,\Phi_1^M\right) \tag{18}$$

are defined in the appendix.

The equilibrium strategies in Proposition 1 have the following interpretation. Before trading takes place at date 1 the market maker's forecast of the random variables $(\tilde{v}, \tilde{u}_1)$ coincides with their unconditional means $(p_0, 0)$. Thus at time 1 the insider trades on the market maker's misperception of the fundamental value $(v - p_0)$ as well as on the speculator's (expected) estimate error $\left(E\left(\tilde{s} \middle| \Phi_1^I\right) - v\right)$ and the current noise trades u_1. Prior to the first trading round, the market maker's belief on $\tilde{u}_2$ is given by $\bar{u}_0$. Therefore the speculator trades on the informational advantage over the market maker $(u_2 - \bar{u}_0)$. After the first trading round, the market maker updates his beliefs about the liquidation value and the time 2 noise trades to $(p_1, \bar{u}_1)$. Thus at time 2 the insider trades on the market maker's misperceptions of 1) the final liquidation value $(v - p_1)$, 2) the current noise trading $(u_2 - \bar{u}_1)$ and 3) the speculator's forecast error $(s - v)$. Similarly the speculator trades on his informational advantage (with respect to the market maker only) captured by the terms $(u_2 - \bar{u}_1)$ and $(s - p_1)$. The variances $(16, 17)$ capture how much of the (insider's) fundamental information is conveyed by prices. Similarly, the variance (18) measures how much of the (speculator's) non-fundamental information is conveyed by p_1[10].

The equilibrium values for the coefficients are reported in Table 1 (Panel A). When trading at date 1 the speculator places a positive weight $(B_1 > 0)$ on the market maker's initial forecast error $(u_2 - \bar{u}_0)$ and then reverses his strategy at date 2 $(B_2 < 0)$. Consider the case in which the demand from passive funds at date 2 is large relative to its expected value, i.e. $(u_2 - \bar{u}_0) > 0$. Ceteris paribus, unexpectedly high liquidity trades at time 2 would determine an increase in the p_2 due to the linear pricing rule (15) and $\lambda_2 > 0$. The speculator would offset one third of the (current) noise trades[11] at date 2. However the speculator knows the realization u_2 one period ahead of the market maker, as to say that he forecasts the increase in p_2 induced by large u_2. As a reaction the speculator trades positively on $(u_2 - \bar{u}_0)$ at the first date and profits from the reversal between the two trading rounds. Note the difference in the trading intensity on u_2 between the two dates. In particular the trading aggressiveness on u_2 increases through time, i.e. $|B_2| > |B_1|$. This arises from the fact that the information about $\tilde{u}_2$ impounded by the speculator's trades at date 1 allows the market maker[12] to make a sharper inference about the second period liquidity trades via the posterior $\bar{u}_1$. During the second trading round the insider acts

[10] The latter variance is crucial in our setup due to the dynamic correlation in the (informed) order flows via u_2, while in the existing literature on uninformed trades it does not play any role. Since in these models the current liquidity trading (as in Rochet and Vila [14]) or a signal of it (as in Yu [17]) belongs to the insider's information set at time t, the variance of noise trading at time $t+1$ conditional on the order flow is equal to its unconditional variance. On the other hand in our two-period framework the speculator's time 1 trade conveys information on the noise trading at time 2. The aggregate order flow is therefore used by the market maker to reduce his uncertainty about the future liquidity trades, i.e. $\sigma_1 < \sigma_0$.

[11] This finding is consistent with Rochet and Vila [14] keeping into account that in our trading game the speculator *and* the insider trade on the same information u_2, thus offsetting 2/3 of the date 2 noise trades. In Rochet and Vila [14] there is no speculator in the market and the insider therefore offsets 50% of the liquidity trades.

[12] In our setup the speculator has long-lived non-fundamental information, as in Kyle [11] the insider has long-lived information about the final liquidation value $\tilde{v}$. Thus it is not surprising that the speculator's behaviour with respect to u_2 closely resembles the insider's aggressiveness on v in Kyle [11].

on the market maker's misperception about u_2 in a similar fashion via the parameter $-\frac{1+B_2}{2}=-1/3$. Further the insider's trades are positively related to the market maker's mispricings $(v-p_0)$ and $(v-p_1)$ via the parameters $a_1>0$ and $a_2>0$, consistently with the existing literature on fundamental speculation. However note that the insider's trading aggressiveness decreases through time as a result of the externality imposed by the speculator, similarly to Madrigal [13]. Further the insider partially offsets the first period noise trading, consistently with Rochet and Vila [14] and Yu [17]. We anticipate that the speculator causes the insider to trade more aggressively on the current noise trade in period 1. Eventually the insider trades at date 2 in the opposite direction of the signal extracted by the speculator as it happens in Madrigal [13].

2.3 The effects of non-fundamental speculation

In order to investigate more closely the insider's and the market maker's reaction to the presence of the speculator we remove the speculator from our setup. This amounts to preclude any preannouncement in index replacements. In this case the insider acts as a monopolist with respect to both the fundamental and non-fundamental information in the market[13]. First period prices would no longer include useful information about the future liquidity trading $\tilde{u}_2$ and the market maker -observing the aggregate order flow $\omega_1=x_1+u_1$- extracts a signal about $\tilde{v}$ only. A linear BNE for this modified trading game is given by conditions $(4),(5)$ and pricing rules $(8),(9)$. The unique linear equilibrium for this trading game is characterized in Proposition 2 (we use the superscript N to denote parameter values in the no speculator case).

Proposition 2 *In the absence of the speculator there exists a unique linear BNE in which trading strategies and prices are of the form:*

$$x_1=a_1^N(v-p_0)+c_1^N u_1 \tag{19}$$

$$x_2=a_2^N(v-p_1)-\frac{1}{2}(u_2-\bar{u}_0) \tag{20}$$

$$p_1=p_0+\lambda_1^N(x_1+u_1) \tag{21}$$

$$p_2=p_1+\lambda_2^N(x_2+u_2-\bar{u}_0) \tag{22}$$

The coefficients $a_1^N, c_1^N, a_2^N, \lambda_1^N, \lambda_2^N$ *and the conditional variances*

$$\Sigma_1^N=var\left(\tilde{v}|\,\Phi_1^M\right) \tag{23}$$

$$\Sigma_2^N=var\left(\tilde{v}|\,\Phi_2^M\right) \tag{24}$$

are defined in the appendix.

[13] The insider's information sets are the same as before, i.e. $\Phi_1^I=\{\tilde{v}=v,\tilde{u}_1=u_1\}$ and $\Phi_2^I=\Phi_1^I\cup\{p_1,\tilde{u}_2=u_2\}$. However in this case he cannot infer u_2 from the first period aggregate order flow as in Proposition 1.

The effect of the speculator on the market equilibrium can be evaluated comparing the parameter values for Proposition 1 and 2 reported in Table 1 (Panel A and B respectively). The presence of the speculator makes the insider trade more aggressively on the current liquidity trades at date 1. This arises from the externality imposed by the speculator on the insider via the signal s extracted from p_1. The insider has an incentive to tilt his trades at date 1 and manipulate p_1 in order to avoid the speculator's inference. In other words, when index replacements are not announced beforehand the insider does not manipulate prices. At the second trading date the insider's intensity on u_2 decreases in the presence of the speculator due to the competition on the current noise trading (note however that the insider and the speculator together trade more aggressively than the insider alone). Further, the insider's intensity on v increases at time 2 in the absence of the speculator. Market depth (measured by $1/\lambda_t, t = 1, 2$) as well as second period market efficiency (measured by Σ_2/Σ_1) is lowered by the speculator's entry. When both the speculator and the insider are aware of the second period liquidity trading as in Proposition 1, market liquidity is lower at both trading dates with respect to the equilibrium values in Proposition 2. This is due to the adverse selection costs faced by the market maker in the trading game in Proposition 1 that are higher than the ones embedded in the information structure underlying Proposition 2. Further the speculator's entry reduces the insider's ex-ante gains.

Panel A: with speculator		**Panel B**: no speculator	
first period variables			
a_1	$0.47536(\sigma_0/\Sigma_0)^{1/2}$	a_1^N	$0.47751(\sigma_0/\Sigma_0)^{1/2}$
c_1	-0.42707	c_1^N	-0.35173
B_1	0.08991		
λ_1	$0.84538(\Sigma_0/\sigma_0)^{1/2}$	λ_1^N	$0.73659(\Sigma_0/\sigma_0)^{1/2}$
μ	0.15990		
ϕ	$0.85771(\Sigma_0/\sigma_0)^{1/2}$		
second period variables			
a_2	$0.41422(\sigma_0/\Sigma_0)^{1/2}$	a_2^N	$0.62100(\sigma_0/\Sigma_0)^{1/2}$
B_2	-1/3	B_2^N	-0.5
C_2	$0.27615(\sigma_0/\Sigma_0)^{1/2}$		
λ_2	$1.20708(\Sigma_0/\sigma_0)^{1/2}$	λ_2^N	$0.80515(\Sigma_0/\sigma_0)^{1/2}$
residual variances			
Σ_1/Σ_0	0.59814	Σ_1^N/Σ_0^N	0.64827
Σ_2/Σ_1	0.54950	Σ_2^N/Σ_1^N	0.5
σ_1/σ_0	0.98562		
ex-ante profits			
insider	$0.76363(\Sigma_0\sigma_0)^{1/2}$	insider	$0.77087(\Sigma_0\sigma_0)^{1/2}$
speculator	$0.14279(\Sigma_0\sigma_0)^{1/2}$		

Table 1: Equilibrium values and the effect of non-fundamental speculation. Panel A considers the trading game with the speculator described in Proposition 1, while Panel B provides the values for the equilibrium without the speculator in Proposition 2 (the superscript N denotes variables for the model with no speculator).

3 Empirical study

3.1 Data set description

Between October 1989 and December 1999 there have been 248 replacements in the S&P500[14]. As in the previous literature, we concentrate on market additions due to the fact that stocks deleted from the S&P500 often do not trade after the list change. From the total sample we removed some stocks. Since we regard passive funds' rebalancing as the main source of uninformed trades, we drop companies added and deleted from the index due to name changes [15]. Furthermore we exclude companies for which we are not certain about the announcement date and/or the effective date. Eventually we require stock data availability for a period ranging from 120 days before the announcement to 60 days after the announcement. The final data set comprises 147 stocks. For each company we collected daily data from CRSP on (1) bid price, (2) ask price, (3) volume (number of shares traded) and (4) outstanding shares. For notational convenience let AD denote the announcement day (i.e. the day in which after the close the announcement is made) and CD as the effective day (i.e. the day in which after the close the change in list is effective). As previously noted, after October 1989 the change is effective at least one day after AD. Figure 1 shows the frequency distribution of the number of trading days between AD and CD for the inclusions occurred under the preannouncement practice. The support ranges from one to sixteen trading days and the mode (resp. mean) is five (resp. 3.81), documenting the common practice from S&P to preannounce changes five days beforehand [16].

[14] We are grateful to Nicholas Barberis and Jeffrey Wurgler for sharing their dataset.

[15] For instance on June 18, 1999 RJR Nabisco Holdings was dropped and replaced by Nabisco Group Holdings. This change does not generate demand from pegged funds. As described above, past studies are mainly concerned with the information conveyed by (announcements of) changes in listing, resulting in the exclusion from the sample of the announcements contaminated by firm specific releases such as earnings and dividends announcements. Since our setup accomplishes for information regarding both the asset's value and the uninformed trades, we do not need to carry out this refinement.

[16] This evidence was first documented by Beneish and Whaley [1] for announcements between October 1989 and June 1994.

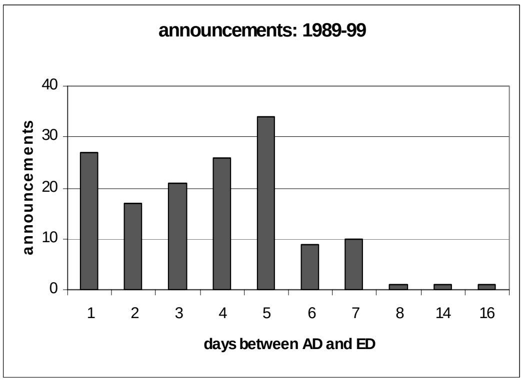

Figure 1: Announcement frequencies. Frequency distribution of the number of trading days between the announcement and the effective day over the period October 1989-December 1999 for S&P500 (147 inclusions)

3.2 Trading volume

The appeal to investors of passive techniques is widely documented by the growth in the net asset value experienced by the major funds pegged to the S&P500 in the last two decades (see Beneish and Whaley [1] and Harris and Gurel [8] among others). The widespread use of indexing funds can be assessed by looking at the trading volume pattern around the announcement day (AD) and the effective day (CD). Let V_{it} denote the trading volume for stock i on day t as measured by the ratio between the number of shares traded and the number of outstanding shares in a given day t. Moreover define the abnormal trading volume[17] on day t as the ratio between V_{it} and the average trading volume in the 40 days preceding the announcement day $\bar{V} \equiv \left(\sum_{t=AD-40}^{AD-1} V_t\right)/40$. Eventually let $MAVR_t$ be the cross-section average for the abnormal trading volume over a sample of size N_t:

$$AVR_{it} = V_{it}/\bar{V} \quad ; \quad MAVR_t = \frac{1}{N_t}\sum_{i=1}^{N_t} AVR_{it} \tag{25}$$

After October 1989 there is at least one trading day between AD and CD. Assuming that the indexed managers' performance is assessed by some daily tracking error measurement, pegged funds' rebalancing should occur at CD.

[17] Using the daily turnover as a measure of the daily trading volume accounts for splits experienced by the stock. Other authors use market adjusted trading turnover, i.e. $V_{it}/\bar{V}_i$ divided by the market turnover of the total NYSE. Market adjustment results in stronger tests by taking into account market variation. Harris and Gurel [8] report that the qualitative results are not affected by the way one measures trading volume. Along the same lines, our results are qualitatively the same when using raw trading volume V_{it}. Lynch and Mendenhall [12] and Cusick [4] use a logarithmic transformation of the market adjusted trading volume.

The presence of risk arbitrageurs (i.e. the speculator in our model in Section 2) can be detected by the abnormal trading volume in the interval from AD+1 through CD-1 for the stocks with more than one trading day between AD and CD. Abnormal trading volume on AD may provide evidence that leakage of information regarding index inclusion has occurred.

Results from the inclusions in the S&P500 are summarized in Table 2 and Figures 2 and 3, taking into account the number of trading days between AD and CD. Under the assumption that the AVR_{it}'s are (cross-sectionally) independently and identically normally distributed, the resulting statistic for $MAVR_t$ follows a Student-t distribution with $N_t - 1$ degrees of freedom. Further, we perform a binomial test for the null that the magnitude of each $MAVR_t$ has probability 0.5 of being different from one[18]. For each day Table 2 reports the sample size[19], the mean abnormal volume ratio ($MAVR_t$), the cross-sectional t-ratio ($t\,(MAVR)$) and the percentage of companies for which AVR_{it} is greater than one. Figure 2 (resp. Figure 3) plots the $MAVR_t$ and its 95% confidence interval around AD (resp. CD).

On the day after the announcement trading volume is more than 4 times larger than the daily mean volume over the 8 weeks base period (with a 11.496 t-ratio) . Mean abnormal volume appears to be persistent in that $MAVR_t$ is greater than one during each day from AD+2 to AD+10, even though its magnitude is far from the increase experienced during AD+1 and $MAVR_t$ is not significantly different from one after AD+5. This evidence is consistent with the presence of non-fundamental speculators stepping into the market after the announcement, and diluting their order flow over the days preceding the effective change. During AD the estimated mean abnormal volume is 25% above the level in the 40 days preceding the announcement. Further, the t-statistic rejects $MAVR_{AD} = 1$ at 5% significance level. This finding is consistent with Beneish and Whaley [1] suggesting that leakage of information regarding the index replacement has occurred. Results from the ten days preceding AD are less clear. An estimate for the mean abnormal volume significantly greater than unity is documented for AD-2 only. Comparing the percentage of individual firms whose abnormal volume ratio is different from one is also illustrative. More than 90% of the cross-section have individual AVR_{iAD+1} greater than one, this percentage being statistically different from 1/2 at 5% significance level. More than half of the stocks in our sample experience AVR_i greater than one in in the days from AD+2 to AD+5. For all the days between AD-10 and AD the fraction of companies with statistically significant values for AVR_{it}

[18] When computing the $t-$statistic we opt for the cross-sectional dispersion of $MAVR_t$ to estimate its variance. See Lynch and Mendenhall [12] for an alternative method of computing standard errors. Details on the binomial test are in Hollander and Wolfe [9].

[19] Since the number of trading days between AD and CD varies across firms (see Figure 1), the column labeled N in each panel in table 2 reports the number of companies included in the sample. For each of the ten days after AD in Panel A, only those firms for which CD has not yet occurred are included. This is why the sample size in the second column decreases over the days after AD in Panel A. Similarly for the ten days preceding CD, only firms for which AD has not yet occurred are included, such that the sample size increases over the ten days before CD in Panel B.

different from one ranges from 31.29% (AD-10) to 40.14% (AD-5).

Trading volume on CD is roughly 17 times higher than the base period (with $t\left(MAVR_{CD}\right) = 11.893$), suggesting that passive managers actually wait until the effective day to rebalance their portfolios. Trading activity for the six days before CD is at least twice the average volume during the 8 weeks preceding AD and can be attributed to risk-arbitrageurs' activity. The increase in volume tends to be permanent, in that $MAVR_t$ is significantly different from one in all the days from CD+1 to CD+10. The percentage of companies with AVR_{it} different from one around CD show that these findings do not appear to be driven by outliers.

	panel A -- event day: AD				panel B -- event day: CD			
day	*N*	MAVR	*t*(MAVR)	% AVR>1	*N*	MAVR	*t*(MAVR)	% AVR>1
-10	147	0.963	-0.592	**31.29**	2	0.834	-1.66	0
-9	147	1.043	0.688	**36.73**	2	1.415	1.099	100
-8	147	0.987	-0.243	**38.10**	2	1.344	0.496	50
-7	147	0.941	-1.53	**39.46**	3	3.029	1.23	66.67
-6	147	1.108	1.298	**36.73**	13	**2.069**	3.402	**84.62**
-5	147	1.047	0.853	**40.14**	22	**2.204**	2.365	63.64
-4	147	1.096	1.624	45.58	56	**3.037**	5.638	**76.79**
-3	147	1.081	1.394	43.54	82	**2.842**	7.078	**79.27**
-2	147	**1.179**	2.106	44.90	103	**2.588**	8.271	**85.44**
-1	147	0.999	-0.026	42.86	120	**3.396**	8.95	**88.33**
0	147	**1.261**	2.954	50.34	147	**16.996**	11.893	**97.28**
1	120	**4.437**	11.496	**94.17**	147	**4.017**	7.897	**93.88**
2	103	**2.452**	8.044	**86.41**	147	**2.437**	8.035	**82.99**
3	82	**2.276**	6.611	**75.61**	147	**2.14**	7.582	**73.47**
4	56	**2.193**	5.613	**67.86**	147	**1.908**	8.238	**72.79**
5	22	**1.792**	2.827	**72.73**	147	**1.62**	6.211	**64.63**
6	13	1.555	2.10	61.54	147	**1.628**	7.055	**70.07**
7	3	2.464	1.21	66.67	147	**1.479**	5.904	**63.27**
8	2	1.268	0.348	50	147	**1.507**	5.792	**61.22**
9	2	1.433	0.695	50	147	**1.442**	5.407	**62.59**
10	2	1.123	0.296	50	147	**1.514**	4.281	**58.50**

Table 2: Abnormal trading volume around the announcement and the effective days. Abnormal trading volume is defined for each stock in (25). In the sample AD precedes CD by at least one day. Since the number of trading days between AD and CD varies across firms, the columns labeled N reports the number of companies included in the cross-section for each day. Boldface numbers in columns labeled $MAVR$ denote mean trading volume significantly different from one (5% significance level). Boldface numbers in columns labeled $\% \ AVR > 1$ denote percentage significantly different from 0.5 (5% significance level).

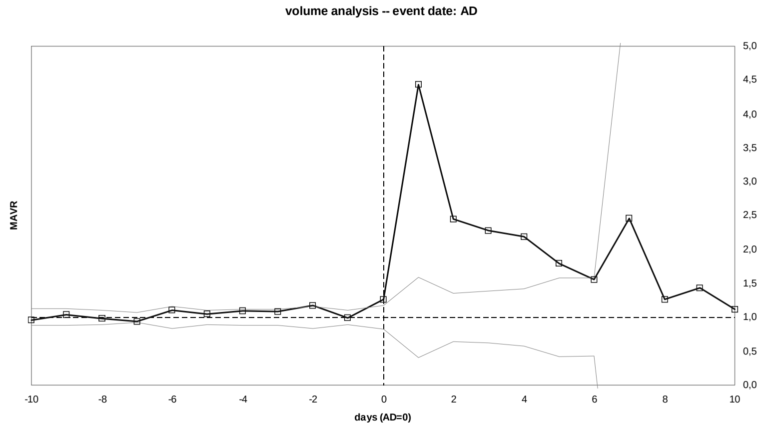

Figure 2: Abnormal trading volume around the announcement day. Mean abnormal volume is defined in (25). The $MAVR$'s are displayed (bold solid line) for each trading day in the window (AD-10,AD+10) together with the 95% confidence interval for the null hypothesis $MAVR = 1$.

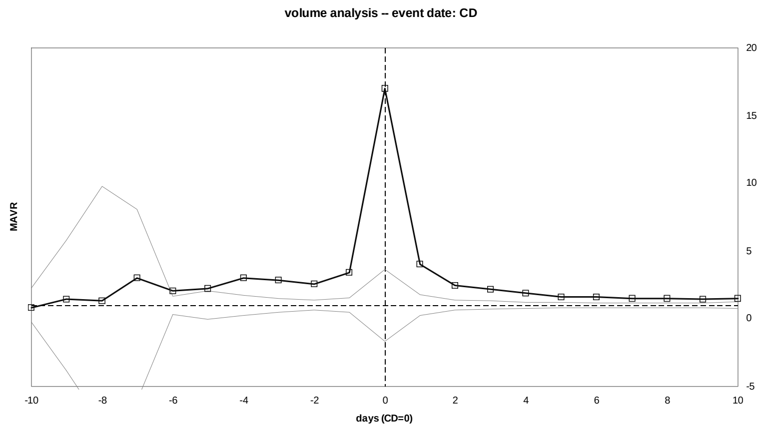

Figure 3: Abnormal trading volume around the effective day. Mean abnormal volume is defined in (25). The $MAVR$'s are displayed (bold solid line) for each trading day in the window (CD-10,CD+10) together with the 95% confidence interval for the null hypothesis $MAVR = 1$.

3.3 Market depth

While several authors focused on trading volume around inclusions in the S&P500, market depth has received little attention[20]. In what follows we employ the bid-ask spread as a proxy for market liquidity. A measure for abnormal depth can be constructed along the same lines used for the trading volume analysis. Let S_{it} denote the bid-ask spread for stock i on day t and $\bar{S} \equiv \left(\sum_{t=AD-40}^{AD-1} S_t \right) /40$ the average bid-ask spread over the base period. The abnormal spread ratio ASR_{it} and the cross-section counterpart $MASR_t$ are defined as follows:

$$ASR_{it} = S_{it}/\bar{S} \quad , \quad MASR_t = \frac{1}{N} \sum_{i=1}^{N} AS_{it} \tag{26}$$

For the announcement and the inclusion not to affect market depth one should observe $MASR_t$ close to one around both AD and CD . On the other hand, a situation in which $MASR_t$ is less (resp. greater) than one detects a reduction (raise) in the average spread during day t relative to the base period, i.e. market depth increases (decreases) on day t.

Table 3 and Figures 4, 5 report the mean abnormal spread around AD and CD (see Section 3.2 for details on the reported statistics). $MASR_t$ is significantly greater than one during AD and the following three days (the average increase in volume ranges from 11% for AD to 40% for AD+1), while the percentage of companies for which $ASR_{it} > 1$ is statistically greater than 50% on AD+1 only. After AD+3 no clear pattern emerges from our data. Before AD the market is more liquid with respect to the previous 40 days for AD-8 and AD-7. On the other hand market liquidity decreases around the inclusion. There is an average 20% increase in the spread during the 5 days preceding the effective day; further $MASR_{CD}$ is roughly twice the base period. The abnormal spread ratio is statistically different from one (5% significance level) in four cases out of six over the window (CD-5,CD). Notice that the reduction in market liquidity during CD is not driven by outliers (approximately 85% stocks of the cross-section experience abnormal spreads). This finding is in contrast with Beneish and Whaley[21] [1] and at a first sight might be counterintuitive. Recall from Section 3.2 that on CD the included stocks experience an important increase in trading volume related to the demand from passive funds (see Table 2-Panel B and Figure 3). Beneish and Whaley[1] argue that the specialist might temporarily charge a lower spread, given that the increase in trading volume would cover the operation costs. However the trading game we develop in Section 2.2 is consistent with an increase in the spread during the day of inclusion: liquidity should decrease as a consequence of the higher adverse selection costs faced by the market maker in the presence of non-fundamental

[20] Among the cited bunch of literature, Beneish and Whaley [1] study the bid-ask spread for inclusions: as mentioned in the Introduction they find reductions in the bid-ask spread after the effective date.

[21] The authors report a spread decrease during CD and the following days, even though the spread is significantly below normal only for CD+1. Note that the data set in Beneish and Whaley [1] comprises 30 index inclusions from October 1989 through June 1994.

speculation. Therefore the documented evidence on $MASR_{CD}$ clearly supports our theoretical analysis[22].

	panel A -- event day: AD				panel B -- event day: CD			
day	*N*	MASR	*t*(MASR)	% ASR>1	*N*	MASR	*t*(MASR)	% ASR>1
-10	147	0.972	-0.676	**39.46**	2	0.576	-1.752	0
-9	147	0.951	-1.352	**40.82**	2	1.011	1.00	50
-8	147	**0.934**	-2.178	**38.78**	2	1.499	0.598	50
-7	147	**0.926**	-2.304	**34.69**	3	2.596	1.664	100
-6	147	1.034	0.69	**39.46**	13	1.204	1.551	69.23
-5	147	0.996	-0.10	42.18	22	1.184	1.299	54.55
-4	147	1.023	0.595	42.18	56	**1.296**	2.861	55.36
-3	147	1.063	1.406	43.54	82	1.14	1.915	52.44
-2	147	1.085	1.706	45.58	103	**1.190**	2.968	57.28
-1	147	1.026	0.546	**38.10**	120	**1.274**	4.667	58.33
0	147	**1.111**	2.24	46.94	147	**1.936**	9.227	**84.35**
1	120	**1.401**	6.396	**68.33**	147	1.082	1.815	45.58
2	103	**1.123**	2.05	55.34	147	1.053	1.072	**39.46**
3	82	**1.183**	2.673	53.66	147	1.114	1.96	**41.50**
4	56	1.199	1.90	48.21	147	1.05	1.042	42.86
5	22	1.058	0.468	40.91	147	1.031	0.665	**40.14**
6	13	1.294	1.294	53.85	147	1.042	0.961	44.22
7	3	2.056	0.925	66.67	147	1.037	0.771	42.86
8	2	0.999	-0.003	50	147	1.038	0.889	42.18
9	2	1.345	1.069	100	147	1.055	1.205	46.26
10	2	0.807	-1.00	0	147	1.098	1.815	45.58

Table 3: Abnormal bid-ask spread around the announcement and the effective days. Abnormal bid-ask spread is defined for each stock in (26). In the sample AD precedes CD by at least one day. Since the number of trading days between AD and CD varies across firms, the columns labeled N reports the number of companies included in the cross-section for each day. Boldface numbers in columns labeled $MASR$ denote mean bid-ask spread significantly different from one (5% significance level). Boldface numbers in columns labeled $\% \ ASR > 1$ denote percentage significantly different from 0.5 (5% significance level).

[22] The model in Section 2 predicts a decrease in market liquidity between the trading rounds. In an applied setup we interpret days as rounds. This way the first date coincides with the day following the announcement (AD+1), while the second date is the effective day (CD). In our dataset however there are only 17 companies fitting this timing, while for the remaining 103 stocks there are at least two days between AD and CD. For the first batch we define $\Delta MAVR_{CD} = MASR_{CD} - MASR_{AD+1}$ and proceed to test the null $\Delta MAVR_{CD} > 0$ against the alternative $\Delta MAVR_{CD} > 0$. For the second batch we compute for each stock the average abnormal spread over the window (AD+1,CD-1) and we use it in place of $MASR_{AD+1}$ in the above testing procedure. The following evidence is consistent with the theoretical model developed in Section $2.1 - 2.2$.

N	$\Delta MAVR_{CD}$	$t(\Delta MAVR_{CD})$	$\% \ \Delta MAVR_{CD} > 1$
17	**0.560**	1.904	52.94
103	**0.595**	5.824	**75.73**

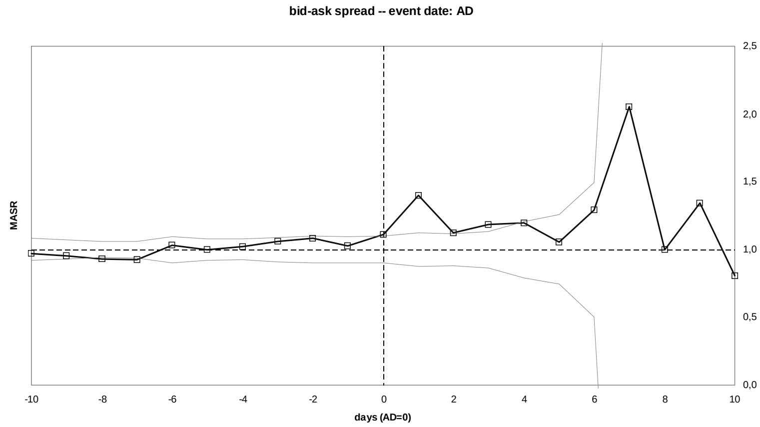

Figure 4: Abnormal bid-ask spread around the announcement day. Mean bid-ask spread is defined in (26). The $MASR$'s are displayed (bold solid line) for each trading day in the window (AD-10,AD+10) together with the 95% confidence interval for the null hypothesis $MASR = 1$.

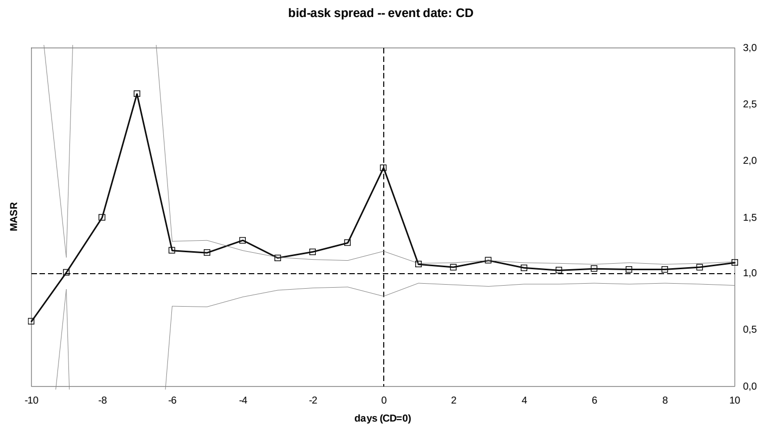

Figure 5: Abnormal bid-ask spread around the effective day. Mean bid-ask spread is defined in (26). The $MASR$'s are displayed (bold solid line) for each trading day in the window (CD-10,CD+10) together with the 95% confidence interval for the null hypothesis $MASR = 1$.

4 Conclusion

In the last two decades passive funds have gained an increasing consideration among investors as a relatively cheap tool to achieve portfolio diversification. Passive funds aim at mimicking a benchmark index. Portfolio rebalancing, as well as performance evaluation, is carried out by means of tracking error procedures. Index replacements appear as a clear rebalancing opportunity for passive managers. Starting from October 1989 changes in the S&P500 composition are preannounced by Standard and Poor's five days beforehand. Passive funds are not affected by the announcement timing and the portfolio rebalancing occurs during the effective day. On the other hand this preannouncement policy induces non-fundamental speculators to enter the market. Non-fundamental speculators do not possess any information on the asset's fundamental value, rather they buy the included stock ahead of passive funds and sell it a few days later at a possibly higher price. We develop a dynamic model that explicitly keep into account this preannouncement practice. We show that strategies based on non-fundamental information generate ex-ante gains, and determine a drop in market liquidity, as a direct consequence of the increased adverse selection costs faced by the specialist. Examining S&P500 inclusions from October 1989 to December 1999 we find evidence supporting our theoretical analysis.

5 Appendix

Proposition 1. There exists a unique linear BNE in which trading strategies and prices are of the form:

$$x_1 = a_1 (v - p_0) + c_1 u_1 \tag{A1}$$

$$x_2 = a_2 (v - p_1) - \frac{1 + B_2}{2} (u_2 - \bar{u}_1) - \frac{C_2}{2} (s - p_1) \tag{A2}$$

$$y_1 = B_1 (u_2 - \bar{u}_0) \tag{A3}$$

$$y_2 = B_2 (u_2 - \bar{u}_1) + C_2 (s - p_1) \tag{A4}$$

$$p_1 = p_0 + \lambda_1 (x_1 + y_1 + u_1) \tag{A5}$$

$$p_2 = p_1 + \lambda_2 (x_2 + y_2 + u_2 - \bar{u}_1) \tag{A6}$$

where s is the speculator's belief on $\tilde{v}$ conditional on Φ_2^S and $\bar{u}_1$ is the market maker's belief on $\tilde{u}_2$ conditional on Φ_1^M:

$$s = p_0 + \phi (x_1 + u_1) \tag{A7}$$

$$\bar{u}_1 = \bar{u}_0 + \mu (x_1 + y_1 + u_1) \tag{A8}$$

The updating coefficients in $(A7)$,$(A8)$ are:

$$\phi = \frac{k_1}{k_1^2 + k_2^2} \sqrt{\frac{\Sigma_0}{\sigma_0}} \tag{A9}$$

$$\mu = \frac{k_3}{k_1^2 + k_2^2 + k_3^2} \tag{A10}$$

Equilibrium parameters are given by:

$$\lambda_1 = \frac{k_1}{k_1^2 + k_2^2 + k_3^2} \sqrt{\frac{\Sigma_0}{\sigma_0}} \tag{A11}$$

$$a_1 = d^{-1} \left(1 - \frac{2\lambda_1 + \phi - 2\lambda_2 \mu}{6\lambda_2} \right) \tag{A12}$$

$$c_1 = d^{-1} \lambda_1 - 1 \tag{A13}$$

$$B_1 = D^{-1} \left[\frac{2 (\lambda_1 - \lambda_2 \mu)}{9\lambda_2} \right] \tag{A14}$$

$$d = 2\lambda_1 - \frac{(2\lambda_1 + \phi - 2\lambda_2 \mu)^2}{18\lambda_2} \tag{A15}$$

$$D = 2\lambda_1 - \frac{2 (\lambda_1 - \lambda_2 \mu)^2}{9\lambda_2} \tag{A16}$$

$$\lambda_2 = \frac{-\Sigma_{us} + \sqrt{\Sigma_{us}^2 + \sigma_1 (9\Sigma_1 - \Sigma_s)}}{2\sigma_1} \tag{A17}$$

$$a_2 = \frac{1}{2\lambda_2} \tag{A18}$$

$$B_2 = -\frac{1}{3} \tag{A19}$$

$$C_2 = \frac{1}{3\lambda_2} \tag{A20}$$

The residual variance-covariance matrix after the first trading date is given by:

$$\Sigma_1 = var\left(\tilde{v} \middle| p_1 \right) = \frac{k_2^2 + k_3^2}{k_1^2 + k_2^2 + k_3^2} \Sigma_0 \tag{A21}$$

$$\sigma_1 = var\left(\tilde{u}_2 \middle| p_1 \right) = \frac{k_1^2 + k_2^2}{k_1^2 + k_2^2 + k_3^2} \sigma_0 \tag{A22}$$

$$\Sigma_s = var\left(\tilde{s} \middle| p_1 \right) = \frac{k_1^2 k_3^2}{(k_1^2 + k_2^2)(k_1^2 + k_2^2 + k_3^2)} \Sigma_0 \tag{A23}$$

$$\Sigma_{us} = cov\left(\tilde{u}_2, \tilde{s} \middle| p_1 \right) = -\frac{k_1 k_3}{k_1^2 + k_2^2 + k_3^2} \sqrt{\Sigma_0 \sigma_0} \tag{A24}$$

The final liquidation value variance after the second trading date is:

$$\Sigma_2 = var\left(\tilde{v} \middle| p_1, p_2 \right) = \frac{1}{2}\Sigma_1 - \frac{\lambda_2}{3}\Sigma_{vu} - \frac{1}{6}\Sigma_s \tag{A25}$$

The constants k_1, k_2, k_3, k_4 are defined by:

$$k_1 = a_1 \sqrt{\frac{\Sigma_0}{\sigma_0}} \tag{A26}$$

$$k_2 = c_1 - 1 \tag{A27}$$

$$k_3 = B_1 \tag{A28}$$

and the inequalities:

$$\lambda_2 > 0 \tag{A29}$$

$$2\lambda_1 - \frac{1}{18\lambda_2}\left(2\lambda_1 + \phi - 2\lambda_2\mu\right)^2 > 0 \tag{A30}$$

$$2\lambda_1 - \frac{2\left(\lambda_1 - \lambda_2\mu\right)^2}{9\lambda_2} > 0 \tag{A31}$$

■

Note that λ_1, the updating coefficients μ, ϕ and the residual variances $\Sigma_1, \sigma_1, \Sigma_s, \Sigma_{vu}$ depend on k_1, k_2, k_3 only. As a consequence λ_2 can be written as a function of k_1, k_2, k_3, and so it is Σ_2. After the variables $\lambda_1, \lambda_2, \mu, \phi$ are defined, so are the inequalities $(A29)$-$(A31)$ and the date 1 trading coefficients a_1, B_1, c_1. The right hand side of equations $(A26)$-$(A28)$ -defining the constants k_1, k_2, k_3- depend only on k_1, k_2, k_3. This way for given values of the exogenous parameters Σ_0, σ_0 all the equilibrium variables $(A1 - A25)$ are obtained.

Proof (Proposition 1). Given the form of the speculator's trade $(A4)$ and the pricing function (15), the insider chooses his second period trade x_2 in order to maximize the objective function (5):

$$E\left[\left(\tilde{v} - p_2\left(\omega_2\right)\right)x_2 \middle| \Phi_2^I\right] = \left[v - p_1 - \lambda_2 x_2 - \left(1 + B_2\right)\lambda_2\left(u_2 - \bar{u}_1\right) - C_2\lambda_2\left(s - p_1\right)\right]x_2 \tag{A32}$$

The optimality conditions for $(A32)$ are:

$$\begin{aligned} x_2 &= \frac{v - p_1}{2\lambda_2} - \frac{1 + B_2}{2}\left(u_2 - \bar{u}_1\right) - \frac{C_2}{2}\left(s - p_1\right) \\ \lambda_2 &> 0 \end{aligned}$$

such that $(A2)$ obtains with $(A18)$. Similarly, given $(A2)$ and (15), the speculator chooses y_2 in order to maximize the objective function (7):

$$E\left[\left(\tilde{v} - p_2\left(\omega_2\right)\right)y_2 \middle| \Phi_2^S\right] = \left(-\frac{1 - B_2}{2}\lambda_2\left(u_2 - \bar{u}_1\right) - \lambda_2 y_2 + \left(1 - \left(a_2 - C_2/2\right)\lambda_2\right)\left(s - p_1\right)\right)y_2 \tag{A33}$$

yielding the following optimality conditions:

$$\begin{aligned} y_2 &= -\frac{1 - B_2}{4}\left(u_2 - \bar{u}_1\right) + \frac{1 - \left(a_2 - C_2/2\right)\lambda_2}{2\lambda_2}\left(s - p_1\right) \\ \lambda_2 &> 0 \end{aligned}$$

In terms of equation $(A4)$ we get $B_2 = -\frac{1 - B_2}{4}, C_2 = \frac{1 - \left(a_2 - C_2/2\right)\lambda_2}{2\lambda_2}$. Solving for these coefficients yields $(A19)$ and $(A20)$ and the time 2 trades $(A2)$ and $(A4)$ become respectively:

$$\begin{aligned} x_2 &= \frac{v - p_1}{2\lambda_2} - \frac{u_2 - \bar{u}_1}{3} - \frac{s - p_1}{6\lambda_2} \\ y_2 &= -\frac{u_2 - \bar{u}_1}{3} + \frac{s - p_1}{3\lambda_2} \end{aligned}$$

Note that time 2 trading intensities depend on λ_2 only. Let π_2^I and π_2^S denote respectively the insider's and speculator's expected profits at date 2. Plugging the above expressions for x_2 and y_2 in the objective functions $(A32)$,$(A33)$ give:

$$\pi_2^I = \frac{(v-p_1)^2}{4\lambda_2} + \frac{\lambda_2 (u_2 - \bar{u}_1)^2}{9} + \frac{(s-p_1)^2}{36\lambda_2} - \frac{(v-p_1)(u_2-\bar{u}_1)}{3} - \frac{(v-p_1)(s-p_1)}{6\lambda_2} + \frac{(s-p_1)(u_2-\bar{u}_1)}{9} \tag{A34}$$

$$\pi_2^S = \frac{\lambda_2 (u_2 - \bar{u}_1)^2}{9} + \frac{(s-p_1)^2}{9\lambda_2} - \frac{2(s-p_1)(u_2-\bar{u}_1)}{9} \tag{A35}$$

Recall that the insider knows the final liquidation value, i.e. $\{\tilde{v} = v\} \subset \Phi_2^I$. Using the decomposition $(s-p_1) = (v-p_1) + (s-v)$ and $(s-p_1)^2 = (v-p_1)^2 + (s-v)^2 + 2(v-p_1)(s-v)$ the insider's expected profits $(A34)$ can be equivalently written as:

$$\pi_2^I = \frac{(v-p_1)^2}{9\lambda_2} + \frac{\lambda_2 (u_2 - \bar{u}_1)^2}{9} + \frac{(s-v)^2}{36\lambda_2} - \frac{2(v-p_1)(u_2-\bar{u}_1)}{9} - \frac{(v-p_1)(s-v)}{9\lambda_2} + \frac{(s-v)(u_2-\bar{u}_1)}{9} \tag{A34'}$$

In the first trading round the insider chooses x_1 to maximize (4):

$$E\left[(\tilde{v} - p_1(\omega_1)) x_1 + \pi_2^I(x_1) \middle| \Phi_1^I \right] \tag{A36}$$

where $\pi_2^I(x_1)$ is given in $(A34')$. Therefore when submitting his order x_1 the insider has to keep into account the impact of his trade on the price $p_1(x_1)$ *and* the speculator's forecast of the final liquidation value $s(x_1)$. Assume that under the insider's conjecture the first period price follows (14) and the speculator updates his beliefs on $\tilde{v}$ according to (3). The expression for x_1 depends on the insider's estimate -conditional on Φ_1^I- of the speculator's forecast $E\left(\tilde{v} \middle| \Phi_2^S\right)$; similarly, the signal s depends on the speculator's conjecture of the form of x_1. The following Lemma gives the expression for the insider's estimate of the speculator's forecast s.

Lemma 1 *In equilibrium* $E\left(\tilde{s} - \tilde{v} \middle| \Phi_1^I\right) = \chi(v - p_0) + \psi u_1$

Proof. Assuming that the Lemma 1 holds, the insider's trading strategy (10) can be rewritten as $(A1)$

$$x_1 = a_1 (v - p_0) + c_1 u_1$$

where $a_1 \equiv \alpha + \gamma\chi$ and $c_1 \equiv \beta + \gamma\psi$. Under $(A1)$ the speculator updates his belief on $\tilde{v}$ after the first round according to:

$$s = p_0 + \phi(x_1 + u_1) \tag{A37}$$

where the regression coefficient ϕ is:

$$\phi = \frac{cov\left(\tilde{v}, x_1 + u_1 \middle| \Phi_1^S\right)}{var\left(x_1 + u_1 \middle| \Phi_1^S\right)} = \frac{a_1 \Sigma_0}{a_1^2 \Sigma_0 + (1 + c_1)^2 \sigma_0} \tag{A38}$$

Plugging $(A1)$ into the speculator's forecast $(A37)$ and taking expectations conditional on Φ_1^I gives:

$$E\left(\tilde{s} - \tilde{v} \middle| \Phi_1^I\right) = \chi (v - p_0) + \psi u_1$$

where $\chi \equiv a_1 \phi - 1$ and $\psi \equiv (1 + c_1)\phi$. Recall that under equation $(A1)$ the parameters a_1 and c_1 depend on the insider's forecast via parameters χ and ψ such that:

$$a_1 = \frac{\alpha - \gamma}{1 - \gamma\phi} \quad ; \quad c_1 = \frac{\beta + \gamma\phi}{1 - \gamma\phi}$$

For these expressions for a_1, c_1 and the speculator's update $(A37)$ the conjecture in Lemma 1 is verified ■

Note that under the conjectures (14) and $(A37)$ the optimality conditions[23] for the objective function $(A36)$ give:

$$E\left(\tilde{v} - p_1(\omega_1) \middle| \Phi_1^I\right)\left(1 - \frac{2\lambda_1 + \phi - 2\lambda_2\mu}{9\lambda_2}\right) - \lambda_1 x_1 +$$

$$+ E\left(\tilde{u}_2 - \bar{u}_1(\omega_1) \middle| \Phi_1^I\right)\left(\frac{2\lambda_1 + \phi - 2\lambda_2\mu}{9}\right) + E\left(\tilde{s} - \tilde{v} \middle| \Phi_1^I\right)\left(\frac{2\lambda_1 + \phi - 2\lambda_2\mu}{18\lambda_2}\right) = 0$$

$$2\lambda_1 - \frac{1}{18\lambda_2}(2\lambda_1 + \phi - 2\lambda_2\mu)^2 > 0$$

such that the insider's first period trade is of the form $(A1)$:

$$x_1 = d^{-1} e (v - p_0) + \left(d^{-1}\lambda_1 - 1\right) u_1$$

where d is defined in $(A15)$ and $e = 1 - \dfrac{2\lambda_1 + \phi - 2\lambda_2\mu}{6\lambda_2}$.

The speculator chooses his first period trade y_1 maximizing (6):

$$E\left[(\tilde{v} - p_1(\omega_1))\, y_1 + \pi_2^S(y_1) \middle| \Phi_1^S\right]$$

where π_2^S is given by $(A35)$. Plugging the expression for the time 1 price (14) and the beliefs' update $(A37)$ together with the conjecture for x_1 as in $(A1)$ one

[23] Using 14 and A37 the terms in the insider's profits A34' can be written as functions of x_1 according to $(v - p_1) = (v - p_0) - \lambda_1 [x_1 + B_1 (u_2 - \bar{u}_0) + u_1]$, $(s - v) = -(v - p_0) + \phi(x_1 + u_1)$ and $(u_2 - \bar{u}_1) = (u_2 - \bar{u}_0)(1 - B_1\mu) - \mu(x_1 + u_1)$.

gets the optimality conditions[24]:

$$E\left(\tilde{s}-p_1(\omega_1)|\,\Phi_1^S\right)\left(1-\frac{2(\lambda_1-\lambda_2\mu)}{9\lambda_2}\right)-\lambda_1 y_1+$$
$$+E\left(\tilde{u}_2-\bar{u}_1(\omega_1)|\,\Phi_1^S\right)\frac{2(\lambda_1-\lambda_2\mu)}{9}=0$$

$$2\lambda_1-\frac{2(\lambda_1-\lambda_2\mu)^2}{9\lambda_2}>0$$

and the speculator's first period trade is given by $(A3)$:

$$y_1=D^{-1}\left[\frac{2(\lambda_1-\lambda_2\mu)}{9}\right](u_2-\bar{u}_0)$$

where D is defined in $(A16)$.

We now turn to determine the equilibrium prices. Let $\mathbf{z}$ denote the 3×1 random vector containing the final liquidation value, the second period noise trades and the speculator's signal, i.e. $\mathbf{z}\equiv(\tilde{v},\tilde{u}_2,\tilde{s})'$. The unconditional distribution for the random variables $\tilde{v},\tilde{u}_1,\tilde{u}_2$ together with the expressions for the speculator's signal s $(A37)$ and the first period trades $(A1)$,$(A3)$ yield the market maker's prior joint distribution for $(\mathbf{z}',\tilde{\omega}_1)$

$$\begin{bmatrix}\mathbf{z}\\ \tilde{\omega}_1\end{bmatrix}\sim N\left(\begin{bmatrix}E(\mathbf{z})\\ 0\end{bmatrix},\begin{bmatrix}S_{\mathbf{z}} & S_{\mathbf{z}\omega}\\ S'_{\mathbf{z}\omega} & S_\omega\end{bmatrix}\right)$$

where $E(\mathbf{z})=(p_0,\bar{u}_0,p_0)'$, $S_\omega=var(\tilde{\omega}_1)$ and the other components of the variance-covariance matrix are defined as follows:

$$\begin{aligned}S_{\mathbf{z}} &\equiv E\left((\mathbf{z}-E(\mathbf{z}))(\mathbf{z}-E(\mathbf{z}))'\right)=\begin{bmatrix}\Sigma_0 & 0 & S_{vs}\\ 0 & \sigma_0 & 0\\ S_{vs} & 0 & S_s\end{bmatrix}\\ S_{\mathbf{z}\omega} &\equiv E\left((\mathbf{z}-E(\mathbf{z}))\tilde{\omega}_1\right)=\begin{bmatrix}S_{v\omega} & S_{u\omega} & S_{s\omega}\end{bmatrix}'\end{aligned}$$

and:

$$\begin{gathered}S_s=var(\tilde{s})=\phi^2\left(a_1^2\Sigma_0+(1+c_1)^2\sigma_0\right)\\ S_\omega=a_1^2\Sigma_0+\left[B_1^2+(1+c_1)^2\right]\sigma_0\\ S_{vs}=cov(\tilde{v},\tilde{s})=\phi a_1\Sigma_0\\ S_{v\omega}=cov(\tilde{v},\tilde{\omega}_1)=a_1\Sigma_0\\ S_{u\omega}=cov(\tilde{u}_2,\tilde{\omega}_1)=B_1\sigma_0\\ S_{s\omega}=cov(\tilde{s},\tilde{\omega}_1)=\phi\left(a_1^2\Sigma_0+(1+c_1)^2\sigma_0\right)\end{gathered}\qquad\text{(A39)}$$

[24]Note that 14,A37 and A1 allow to write $v-p_1=(1-a_1\lambda_1)(v-p_0)-\lambda_1[y_1+(1+c_1)u_1]$, $s-p_1=(\phi-\lambda_1)[a_1(v-p_0)+(1+c_1)u_1]-\lambda_1 y_1$ and $u_2-\bar{u}_1=-\mu[a_1(v-p_0)+(1+c_1)u_1]-\mu y_1+(u_2-\bar{u}_0)$. Further, since the time 1 speculator's information set does not include the realization of $\tilde{v}$ one has $E\left(v-p_1|\,\Phi_1^S\right)=E\left(s-p_1|\,\Phi_1^S\right)=-\lambda_1 y_1$.

Note that plugging the expression ($A38$) for ϕ in the above unconditional variances yields $S_{vs} = S_s$ and $S_{s\omega} = S_{v\omega}$. After observing the first period aggregate order flow the market maker (posterior) variance-covariance matrix for the vector $\mathbf{z}$ becomes:

$$\Sigma_{\mathbf{z}} \equiv E\left(\left(\mathbf{z} - E\left(\mathbf{z}\right)\right)\left(\mathbf{z} - E\left(\mathbf{z}\right)\right)'\middle|\, \tilde{\omega}_1 = \omega_1\right) = \begin{bmatrix} \Sigma_1 & \Sigma_{vu} & \Sigma_{vs} \\ \Sigma_{vu} & \sigma_1 & \Sigma_{us} \\ \Sigma_{vs} & \Sigma_{us} & \Sigma_s \end{bmatrix}$$

where the projection theorem yields:

$$\begin{array}{ccc} \Sigma_1 = \Sigma_0 - \frac{S_{v\omega}^2}{S_\omega} & ; \quad \sigma_1 = \sigma_0 - \frac{S_{u\omega}^2}{S_\omega} & ; \quad \Sigma_s = S_s - \frac{S_{s\omega}^2}{S_\omega} \\ \Sigma_{vu} = -\frac{S_{v\omega}S_{u\omega}}{S_\omega} & ; \quad \Sigma_{vs} = S_{vs} - \frac{S_{v\omega}S_{s\omega}}{S_\omega} & ; \quad \Sigma_{us} = -\frac{S_{u\omega}S_{s\omega}}{S_\omega} \end{array} \tag{A40}$$

As noted above $S_{vs} = S_s$ and $S_{s\omega} = S_{v\omega}$, implying that $\Sigma_{vs} = \Sigma_s$ and $\Sigma_{vu} = \Sigma_{us}$. After observing the aggregate order flow ω_1 the market maker updates his forecast over the random variables $\tilde{v}, \tilde{u}_2$ according to (14) and ($A8$):

$$p_1 = E\left(\tilde{v}\middle|\, \Phi_1^M\right) = p_0 + \lambda_1\omega_1 \quad ; \quad \bar{u}_1 = E\left(\tilde{s}\middle|\, \Phi_1^M\right) = \bar{u}_0 + \mu\omega_1$$

Using ($A39$) the regression coefficients become:

$$\lambda_1 = \frac{S_{v\omega}}{S_\omega} = \frac{a_1\Sigma_0}{a_1^2\Sigma_0 + \left[B_1^2 + (1 + c_1)^2\right]\sigma_0} \tag{A41}$$

$$\mu = \frac{S_{u\omega}}{S_\omega} = \frac{B_1\sigma_0}{a_1^2\Sigma_0 + \left[B_1^2 + (1 + c_1)^2\right]\sigma_0} \tag{A42}$$

The first period aggregate order flow reduces the variance of the final payoff and the second period noise trades as follows:

$$\Sigma_1 = \Sigma_0\left(1 - a_1\lambda_1\right) \tag{A43}$$

$$\sigma_1 = \sigma_0\left(1 - B_1\mu\right) \tag{A44}$$

In the second trading round the market maker sets the second period price according to (15):

$$p_2 = p_1 + \lambda_2\left(\omega_2 - \bar{u}_1\right)$$

where:

$$\begin{aligned} \lambda_2 &= \frac{cov\left(\tilde{v}, \tilde{\omega}_2\middle|\, \Phi_1^M\right)}{var\left(\tilde{\omega}_2\middle|\, \Phi_1^M\right)} = \\ &= \frac{a_2\Sigma_1 + \frac{1 + B_2}{2}\Sigma_{vu} + \frac{C_2}{2}\Sigma_{vs}}{a_2^2\Sigma_1 + \frac{(1 + B_2)^2}{4}\sigma_1 + \frac{C_2^2}{4}\Sigma_s + a_2\left(1 + B_2\right)\Sigma_{vu} + a_2C_2\Sigma_{vs} + \frac{(1 + B_2)\,C_2}{2}\Sigma_{us}} \end{aligned}$$

Note that using the equilibrium values for the second period trading aggressiveness (see $(A18)$-$(A20)$) the above expression for λ_2 reduces to a quadratic equation in λ_2 and admits the following solutions:

$$\lambda_2 = \frac{-\Sigma_{us} \pm \sqrt{\Sigma_{us}^2 + \sigma_1 (9\Sigma_1 - \Sigma_s)}}{2\sigma_1} \tag{A45}$$

Lemma 2 addresses the existence and uniqueness of solutions for λ_2.

Lemma 2 *In equilibrium:*

$$\lambda_2 = \frac{-\Sigma_{us} + \sqrt{\Sigma_{us}^2 + \sigma_1 (9\Sigma_1 - \Sigma_s)}}{2\sigma_1} \in \mathcal{R}$$

Proof. To prove Lemma 2 we proceed in two steps. First we show that both the solutions for λ_2 in $(A45)$ are real and then we use the second order conditions to pick the positive root and get $(A17)$.

From $(A45)$ a sufficient condition for λ_2 to belong to the real line is $\Sigma_1 - \Sigma_s > 0$. Note that from $(A40)$ the residual variance Σ_1 can be written as:

$$\Sigma_1 = \Sigma_0 \left(1 - \frac{a_1^2 \Sigma_0}{S_\omega}\right) \tag{A46}$$

Now let $\mathbf{S} = var(\tilde{x}_1 + \tilde{u}_1) = a_1^2 \Sigma_0 + (1 + c_1)^2 \sigma_0$ such that from $(A39)$ one has $S_{s\omega}^2 = S_s \mathbf{S}$ and Σ_s in $(A40)$ becomes:

$$\Sigma_s = S_s \left(1 - \frac{\mathbf{S}}{S_\omega}\right) \tag{A47}$$

Since $\mathbf{S} > \alpha^2 \Sigma_0 > 0$ it emerges from $(A46)$,$(A47)$ that a sufficient condition for $\Sigma_1 - \Sigma_s > 0$ to hold is $\Sigma_0 > S_s$. Using the expression $(A38)$ for ϕ, the signal's unconditional variance S_s can be written as:

$$S_s = \Sigma_0 \frac{a_1^2 \Sigma_0}{\mathbf{S}}$$

Since $\alpha^2 \Sigma_0 < a_1^2 \Sigma_0 + (1 + c_1)^2 \sigma_0 = \mathbf{S}$, then $S_s < \Sigma_0$ and we have λ_2 in $(A45)$ lies in the real line. Note that in equilibrium $\lambda_2 > 0$ from the second order conditions for $(A32)$ and $(A33)$. Thus, regardless of the sign of Σ_{us}, the negative root in $(A45)$ can be neglected and $(A17)$ obtains. ■

Eventually the final liquidation residual variance after the second trading round becomes:

$$\Sigma_2 = var(\tilde{v} | p_2, p_1) = \Sigma_1 - \lambda_2 \left(a_2 \Sigma_1 + \frac{1 + B_2}{2} \Sigma_{vu} + \frac{C_2}{2} \Sigma_{vs}\right)$$

Using the equilibrium values for a_2, B_2, C_2 (see $(A18)$-$(A20)$) the above reduces to (17). Therefore an equilibrium for the trading game is described by finding solutions $(a_1, B_1, c_1, \mu, \phi, \lambda_1, \lambda_2)$ for equations $(A12)-(A16)$, $(A42)$, $(A38)$, $(A41)$, $(A17)$

subject to the nonlinear constraints ($A29$)-($A31$). These equations are nonlinear in the unknowns, thus making it difficult to solve by direct substitution. Following Madrigal [13], we proceed in simplifying the above system of equations. First we conjecture that the first period trading intensities a_1, B_1, c_1 in ($A1$) and ($A3$) are of the form:

$$a_1 = k_1 \sqrt{\frac{\sigma_0}{\Sigma_0}} \quad ; \quad 1 + c_1 = k_2 \quad ; \quad B_1 = k_3 \tag{A48}$$

for some k_1, k_2 and k_3. Plugging the conjecture ($A48$) into the expressions for ϕ, μ and λ_1 (respectively ($A38$),($A42$) and ($A41$)) yields ($A9$),($A10$) and ($A11$):

$$\begin{aligned} \phi &= \frac{k_1}{k_1^2 + k_2^2} \sqrt{\frac{\Sigma_0}{\sigma_0}} \\ \mu &= \frac{k_3}{k_1^2 + k_2^2 + k_3^2} \\ \lambda_1 &= \frac{k_1}{k_1^2 + k_2^2 + k_3^2} \sqrt{\frac{\Sigma_0}{\sigma_0}} \end{aligned}$$

Using the ($A9$),($A10$),($A11$) together with the conjecture ($A48$) into ($A43$) and ($A44$) results in ($A21$) and ($A22$):

$$\begin{aligned} \Sigma_1 &= \frac{k_2^2 + k_3^2}{k_1^2 + k_2^2 + k_3^2} \Sigma_0 \\ \sigma_1 &= \frac{k_1^2 + k_2^2}{k_1^2 + k_2^2 + k_3^2} \sigma_0 \end{aligned}$$

Solving for ($A39$) and ($A40$) gives ($A23$) and ($A24$):

$$\begin{aligned} \Sigma_s &= \frac{k_1^2 k_3^2}{(k_1^2 + k_2^2)(k_1^2 + k_2^2 + k_3^2)} \Sigma_0 \\ \Sigma_{us} &= -\frac{k_1 k_3}{k_1^2 + k_2^2 + k_3^2} \sqrt{\Sigma_0 \sigma_0} \end{aligned}$$

Using ($A21$),($A22$),($A23$) and ($A24$) into ($A17$) allows to write λ_2 as a function of the constants k_1, k_2, k_3 and the exogenous parameters Σ_0 and σ_0:

$$\lambda_2 = \left(\frac{k_1 k_3}{2(k_1^2 + k_2^2)} + \frac{3}{2} \left(\frac{k_2^2 + k_3^2}{k_1^2 + k_2^2} \right)^{1/2} \right) \sqrt{\frac{\Sigma_0}{\sigma_0}} \tag{A49}$$

This way we reduced the parameters ϕ, μ, λ_1 and λ_2 as functions of k_1, k_2 and k_3. Therefore the second order conditions ($A29$)-($A31$) also depend on k_1, k_2 and k_3 only. Further from (A12),(A14),(A13) the first period trading intensities a_1, B_1 and c_1 depend on ϕ, μ, λ_1 and λ_2 via the definitions (A15) and (A16) respectively for d and D. Thus the conjecture ($A48$) can be verified using ($A9$),($A10$),($A11$),($A49$),($A29$)-($A31$) giving:

$$k_1 = 0.47536 \quad , \quad k_2 = 0.57293 \quad , \quad k_3 = 0.08991 \tag{A50}$$

The parameters' values at equilibrium are reported in Table 1 using $(A50)$. The value for the ex-ante profits can be found applying the law of iterated expectation in (4) and (6). ■

Proof (Proposition 2).. This trading game is a two-period version of Rochet and Vila [14]. Expressions for parameters at equilibrium can be found in Yu [17] keeping into account that the insider knows the realization $u_{1,}$ and u_2 in our model, while in Yu [17] he receives a noisy signal of liquidity trades. Equilibrium values are:

$$a_1^N = \frac{2\lambda_2^N - \lambda_1^N}{\lambda_1^N \left(4\lambda_2^N - \lambda_1^N\right)} \tag{A51}$$

$$c_1^N = -\frac{2\lambda_2^N - \lambda_1^N}{\left(4\lambda_2^N - \lambda_1^N\right)} \tag{A52}$$

$$\lambda_1^N = \frac{a_1^N \Sigma_0}{\left(a_1^N\right)^2 \Sigma_0 + (1+c_1)^2 \sigma_0} \tag{A53}$$

$$a_2^N = \frac{1}{2\lambda_2^N} \tag{A54}$$

$$\lambda_2^N = \left(1 - a_1^N \lambda_1^N\right)^{1/2} \sqrt{\frac{\Sigma_0}{\sigma_0}} \tag{A55}$$

Further the second order conditions yield:

$$\begin{aligned} \lambda_2^N &> 0 \\ 2\lambda_1^N - \frac{\left(\lambda_1^N\right)^2}{2\lambda_2^N} &> 0 \end{aligned}$$

Proceeding as in the Proof of Proposition 1 we first conjecture that the date 1 insider's intensities are:

$$a_1^N = k_1^N \sqrt{\frac{\sigma_0}{\Sigma_0}} \quad ; \quad 1 + c_1^N = k_2^N \tag{A56}$$

Using the conjecture $(A56)$ into $(A53)$,$(A55)$ give:

$$\begin{aligned} \lambda_1^N &= \frac{k_1^N}{\left(k_1^N\right)^2 + \left(k_2^N\right)^2} \sqrt{\frac{\Sigma_0}{\sigma_0}} \\ \lambda_2^N &= \frac{k_2^N}{\left[\left(k_1^N\right)^2 + \left(k_2^N\right)^2\right]^{1/2}} \sqrt{\frac{\Sigma_0}{\sigma_0}} \end{aligned}$$

The conjecture can be verified substituting the above expressions for λ_1^N and λ_2^N into $(A51)$,$(A52)$, yielding $k_1^N = 0.47751$ and $k_2^N = 0.64827$.

■

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