Trend and cycles in UK stock prices*

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Abstract

In this paper, the present-value model is used to decompose the forecastable component of stock prices into the share of the cycle due to forecastable dividend growth on the one hand, and that due to time-varying discount rates on the other. The decomposition is then applied to the analysis of the forecastable component of UK stock prices over the past 85 years. Although dividends are found to be significantly cyclical, it is showed that they play a minor role in the forecastability of stock-price movements, virtually all of which being explained by volatile expectations about future returns.

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1 Introduction

That stock prices contain a sizeable forecastable component, and thus depart substantially from the so-called 'random-walk' model of stock-price behaviour, is now widely accepted amongst finance specialists. Early studies, essentially based on returns' auto-regressions and variance-ratio tests, seemed at best limited to establishing the forecastability of stock returns over several years' horizons (eg Poterba and Summers (1988), Fama and French (1988a), Culter *et al.* (1990)). However, multivariate estimates have unambiguously revealed that yearly returns are strongly forecastable on the basis of the 'great' financial ratios, such as price/dividend ratios, price/earning ratios, or dividend payout ratios (see, amongst others, Campbell and Shiller (1988), Fama and French (1988b), Cochrane (1994)).

The close connection between the forecastable component of a time-series and its 'transitory', or 'cyclical', component was highlighted by the seminal contribution of Beveridge and Nelson (1981). More specifically, they identify the trend of a series with the value it is expected to reach in the infinite future, corrected for the cumulated increase in the series due to the drift. If we denote $\{X_t\}$ as the time series under study and γ as the average growth rate of the series, its stochastic trend is:

$$\tau_t^x \equiv \lim_{n \to \infty} E_t\left(x_{t+n} - n\gamma\right)$$

With this definition, the dynamics of the trend is entirely unforcastable (i.e. it follows a random walk). In contrast, the Beveridge-Nelson cycle, $c_t^x = x_t - \tau_t^x$, encompasses all that is forecastable in the behaviour of the series, at all horizons:

$$E_t\left(c_{t+i}^x - c_t^x\right) + \gamma i = E_t\left(x_{t+j} - x_t\right), \, i = 0, ...\infty$$

Although Beveridge and Nelson (1981) have confined the application of their trend-extraction technique to univariate time-series, the methodology has been generalised to multivariate forecastability by Stock and Watson (1988). In the present paper, I shall follow these authors in identifying the cycle in stock prices with their forecastable component, estimated from a VAR.

The observation that stock prices are significantly forecastable, that is, contain a Beveridge and Nelson-type cycle, raises several questions. At the theoretical level, one may wonder whether this component merely reflects rational market valuations under changing preferences or investment opportunities, or if they are indicative of the fact that market psychology, bubbles or other 'animal-spirits' play an important rôle in the determination of stock-price movements (see, amongst others, Summers (1986), Poterba and Summers (1988) on this view). At a more empirical level, one may ask oneself what the sources of the forecastability of stock price movements are; that is, whether such forecastability primarily reflects changing dividend prospects, or whether volatile expectations in future returns are the cause.

In some cases, the Beveridge-Nelson trend/cycle decomposition gives an unambiguous answer to the latter question. For example, if dividends approximately follow a random walk, then they are entirely unpredictable, so the Beveridge-Nelson cycle in stock prices unambiguously picks the contribution of *time-varying discount rates* to the forecastability of prices and returns. Conversely, if constant expected returns are assumed, then the cycle in prices directly reflects the extent to which *dividend growth* is forecastable. The issue becomes, however, more complicated when both dividends and discount rates are suspected to be cyclical. In such a situation, the cycle in stock prices comes from the forecastable component of both variables, so the Beveridge-Nelson cycle becomes uninformative about the *sources* of the forecastability of stock prices (i.e. beyond offering an accurate measurement of its *size*).

In order to overcome this difficulty, the present paper suggests a simple generalisation of the Beveridge-Nelson trend/cycle decomposition of stock prices for the case where *both* dividend growth and discount rates are persistently time-varying, which allows one to decompose the contribution of each of these factors into the forecastability of price movements.

I start by noting that, under constant discount rates, the cycle in stock prices is entirely explained by the corresponding cycle in dividends. This allows a definition of the 'dividend cycle' as the cycle in stock prices that would prevail if discount rates were constant, that is, if the forecastability of stock-price movements were entirely explained by fluctuations in dividend growth. Similarly, the 'discount-rate cycle' is defined as the cycle in stock prices that would prevail if dividend growth was entirely unpredictable, that is, if dividends followed a pure random walk. These definitions yield a simple additive decomposition of the level of stock prices into the price (stochastic) trend, the 'dividend cycle' and the 'discount rate cycle', the relative size of which providing a direct measure of the sources of the forecastability of stock-price movements. This decomposition is then applied to a broad index of UK stock-market data, where it is found that, although dividend growth is significantly forecastable, its fluctuations have a negligible impact on the cycle in stock prices, virtually all of which being explained by volatile expectations about future returns.

Other researchers have studied the historical volatility of UK stock prices in some detail. Bulkley and Tonks (1989), applying Shiller's (1981) original excess volatility test, reject the joint hypothesis of market efficiency/constant discount-factor, which suggests that discount rates are indeed time-varying. Cuthbertson *et al.* (1997) apply the VAR methodology of Campbell and Shiller (1987, 1988) to test several asset-pricing models (constant discount rate, constant safe rate, constant risk premium, CAPM), and again find that discount rates are more variable than any of these models predict. Rather than focusing on the volatility of prices, returns or the dividend/price ratio, the present contribution aims to re-assess the evidence by asking *how much* do forecastable dividend growth and time-varying discount factors contribute to the forecastability of stock-price movements, and to measure the impact of their innovations on the dynamic of the variables under study. Therefore, rather than assuming and then testing a specific asset-pricing model, we shall basically think of the discount rate as a *residual*, whose measure and effect on prices can be infered from the data.

The paper is organised as follows. Section 2 uses the log-linear present value model to analytically decompose the level of stock-prices into the price trend and the two cyclical components. Section 3 applies this methodology to the long-run behaviour of the UK stock market, using a VAR to compute expectations of future dividend/price ratios and dividend growth. Section 4 completes the analysis by evaluating the dynamic effect of 'permanent' and 'transitory' innovations to stock prices, stock returns, and dividend/price ratios. Section 5 concludes.

2 Theoretical framework

2.1 The log-linear present value formula

The present section describes how to use the present value model, in its log-linear formulation, in order to first extract, and then decompose, the forecastable component of stock prices. Defining Q_t as the real price of a stock at date t, D_t the income received from holding the stock during period t, and $R_{t+1} = (Q_{t+1} + D_{t+1})/Q_t$ as the ex post return from holding stock from date t to date $t+1$, Campbell and Shiller (1988) show that the log-linear counterpart of ex post returns is given by (see appendix A1 for a sketch of the derivation):

$$r_{t+1} - r^* = (\lambda_t - \lambda^*) - \rho(\lambda_{t+1} - \lambda^*) + (\Delta d_{t+1} - g),$$

where $r_t = \log(R_{t+1})$ denotes log-returns, $\lambda_t = d_t - q_t$ the log-dividend/price ratio, $\rho = (1+g)/R^* < 1$, and where $\lambda^* = E(\lambda_t)$ and $R^* = E(R_t)$ are the mean of the log dividend/price ratio and stock returns, respectively. Solving the latter equation for $\lambda_t - \lambda^*$, iterating it forwards under the no-bubble condition $\lim_{j\to\infty} \rho^j \lambda_{t+j} = 0$, and taking time-$t$ expectations on both sides, yields the following *log-linear present value model* for the dividend/price ratio:

$$d_t - q_t = \lambda^* + \sum_{j=0}^{\infty} \rho^j E_t(r_{t+1+j} - r^*) - \sum_{j=0}^{\infty} \rho^j E_t(\Delta d_{t+1+j} - g) \qquad (1)$$

In other words, the dividend/price ratio at date t is an increasing function of current, $E_t(r_{t+1})$, and expected future discount rates, $E_t(r_{t+1+j})$ $(j = 1, ...\infty)$, and a decreasing function of current dividends, d_t, and expected future dividend growth, $E_t(\Delta d_{t+1+j})$ $(j = 1, ...\infty)$. In the derivation it has been assumed that the dividend/price ratio was stationary, and hence both the discount rate and dividend *growth* must be stationary. Since many stock price and dividend series appear to have a unit-root (section 3 confirms that such is the case for UK data), we interpret the present-value model as featuring *cointegrated* prices and dividends.

2.2 Trend and cycle in stock prices

The trend in stock prices, in its relation to the trend in dividends, must first be constructed. The Beveridge-Nelson trend in prices is given by:

$$\tau_t^q = \lim_{n\to\infty} E_t\left(q_{t+n} - ng\right) = q_t + \sum_{j=1}^{\infty} E_t\left(\Delta q_{t+j} - g\right) \qquad (2)$$

The VAR that is estimated in the next section does not include stock returns as a dependent variable, so it is not possible to directly use eq. (2) to compute the price trend. However, denoting $\tau_t^d = \lim_{n\to\infty} E_t\left(d_{t+n} - ng\right)$ as the Beveridge-Nelson trend in dividends, one can see that:

$$\tau_t^d - \tau_t^q = \lim_{n\to\infty} E_t\left(\lambda_{t+n}\right) = \lambda^*$$

Hence, the price trend is a simple translation of the trend in dividends (i.e. their expected long-run level), from which it can be computed:

$$\tau_t^q = -\lambda^* + \tau_t^d = -\lambda^* + d_t + \sum_{j=1}^{\infty} E_t\left(\Delta d_{t+j} - g\right) \qquad (3)$$

As emphasised in the introduction, the trend in prices encompasses the unpredictable component of stock price movements; that is, the price trend follows a pure random walk with drift ($E_t\left(\Delta\tau_{t+1}^q\right) = g$). The price cycle is obtained by removing the price trend from the current price: $c_t^q = q_t - \tau_t^q$.

Since the Beveridge-Nelson cycle extracts the forecastable component of the series, its size provides a measure of the degree by which prices depart from the so-called *random-walk model* (which, literally, says prices contain no forecastable component). If discount rates were constant, then transitory stock-price fluctuations would be entirely accounted for by the forecastability of dividend growth, whose impact on stock prices would be accurately measured by the Beveridge-Nelson cycle in prices. This configuration corresponds to Samuelson's (1965) *martingale model*, which features constant discount rates, together with unrestricted dividend growth. Alternatively, if dividends approximately followed a pure random-walk, then the Beveridge-Nelson cycle would measure the extent to which *time variations in discount rates* render prices and returns forecastable. In other words, if only one component of the right hand-side of the present-value model (eq. (1)) is cyclical, then it entirely accounts for the forecastable component in stock prices, and hence is adequately picked by the Beveridge-Nelson cycle. As emphasised in the introduction, there is no reason to assume that such is the case as, in general, both discount rates and dividend growth are forecastable.

2.3 The 'dividend cycle' and the 'discount rate' cycle

The cycle in stock prices can now be decomposed into two forecastable components, measuring the contributions of dividend growth and time-varying discount rateson the overall cycle. Let us first define the '*dividend cycle'*, denoted c_t^d, as the Beveridge-Nelson cycle in prices that would obtain if discount rates were constant, that is, if $E_t(r_{t+1} - r^*)$ were equal to zero at all time[1]. When such is the case, the present value model reduces itself to:

$$q_t = -\lambda^* + d_t + \sum_{j=0}^{\infty} \rho^j E_t (\Delta d_{t+1+j} - g) \tag{4}$$

The dividend cycle is then obtained by removing the trend (eq. (3)) from the current price, as is given by eq (4). One finds:

$$c_t^d = -\sum_{j=1}^{\infty} \left(1 - \rho^j\right) E_t (\Delta d_{t+1+j} - g) \tag{5}$$

Similarily, one can define the *discount rate cycle* as the cycle in stock prices that would prevail if dividend growth was unpredictable, that is, if $E_t(\Delta d_{t+1} - g) = 0$ for all t. When such is the case, the present-value model reduces to:

$$q_t = -\lambda^* + d_t - \sum_{j=0}^{\infty} \rho^j E_t (r_{t+1+j} - r^*) \tag{6}$$

If dividends follow a pure random walk, the price trend is simply given by $-\lambda^* + d_t$ (see eq. (3)). Therefore, the 'discount rate cycle' is given by:

$$c_t^r = -\sum_{j=0}^{\infty} \rho^j E_t (r_{t+1+j} - r^*) \tag{7}$$

By solving the present value formula (eq. (1)) for q_t, it is easy to check that the decomposition obtained is additive, so that:

$$q_t = \tau_t^q + c_t^d + c_t^r \tag{8}$$

From equations (2) and (5) the price trend and dividend cycle can be computed on the basis of expectations of future dividend growth, which in turn can be inferred from a VAR (see next section and appendix A2). The discount rate cycle cannot be directly observed, but is recovered using eq. (8).

[1]Note that under this definition the 'dividend cycle' is *not* the forecastable component of dividends, but rather the contribution of the forecastability of dividends to that of stock prices. The same applies for the 'discount-rate cycle' below.

3 An application to the UK stock market, 1919-2001

This section applies the cycle decomposition described above to UK stock prices and dividends. The database used includes nominal stock prices and related dividends from the Barclays value-weighted equity index, over the period 1919-2002. Dividends and prices are recorded in December, so that prices incorporate information about dividends paid out during the entire year. Nominal values are divided by the Barclays cost of living index, to obtain real prices and dividends. All the data is available in Barclays Capital (2003), where further information about its construction is provided.

Panel A of Table 1 gathers some summary statistics about the variables under study, and Panel B displays the parameters' estimates that enter the present-value model. Two observations can already be made from these tables. Firstly, price growth and the dividend/price ratio appear substantially more volatile than dividend growth. Secondly, the contemporaneous correlation between price growth and dividend growth is very small, suggesting that the latter has virtually no influence on the level of stock prices *in the short run*.

To verify that the theoretical framework presented in section 2 is congruent to the data used, Table 2 reports unit-root and cointegration tests for the variables involved in the present-value model. ADF tests do not reject the null hypothesis that the levels of dividends (d_t) and stock prices (q_t) contain a unit-root, but does reject the null at the 1 percent level for their differences, indicating that both variables are $I(1)$. The null hypothesis of non-stationarity of the log dividend/price ratio (δ_t) is almost rejected at the 1 percent level. We interpret these tests as evidence that the data is does not blatantly contradict the assumption that d_t and q_t are $CO(1,1)$ where the cointegrating vector is $(1,-1)$.

Expectations of future dividend growth and future dividend/price ratios, which are necessary to compute the trend and the two cycle components, are obtained using a bivariate VAR, similar to that estimated by Campbell and Shiller (1988) using US stock-market data, and by Cuthbertson *et al.* (1997) using UK data. The coeficient estimates, as well as corrected standard errors and usual specification tests, are shown in Table 3. The VAR includes two lags and is applied to de-meaned series.

Two significant features of the behaviour of the variables are worth noting, as they will play an important rôle in the subsequent analysis. The first equation of the VAR shows that departures of the dividend/price ratio from its long-run equilibrium value are highly persistent, suggesting the presence of a significant cycle in either (or both) of the two right-hand variables of the present value model (eq. (1)). The second equation reveals that dividend growth is indeed cyclical and hence predictable, especially on the basis of information on *past* dividend growth. (the hypothesis that dividends follow a random walk is statistically rejected). Therefore, the cycle decomposition that we sketched out in the previous section applies in a non-trivial way, since one cannot trace trend deviations of stock prices to changing discount factors only.

Appendix A2 shows how the VAR coefficients can be used to compute the price trend, the 'dividend cycle' and the 'discount rate cycle'. These are represented on figure 1. The first panel displays the price series against its trend (as computed from equation (3)), indicating the size of the forecastable component of stock prices. The second panel shows the corresponding 'dividend cycle' and 'discount rate' cycle, as measured by equation (5) and (7) in section 2. The variances and autocorrelation functions of each cycle are summarised in table 3.

The results are rather unambiguous. The dividend cycle is almost flat; it has a very small variance, about 70 times smaller than that of the overall cycle. Therefore, dividends' fluctuations have virtually no impact on the transitory components of stock prices, when compared to the forecastability of stock-price movements stemming from time-varying discount rates. Moreover, this occurs in spite of the fact that dividends *are* significantly forecastable, but this forecastability is completely overwhelmed by that coming from the discount rates. As a result, prices behave 'as if' dividend followed a pure random walk.

4 Impulse-response analysis

To complete the analysis of the behaviour of UK stock markets, impulse-response functions are drawn and the forecast-error variances of price/dividend ratios and stock returns are decomposed. To avoid problems related to the ordering of variables and to possible contemporaneous feedback (likely to arise in annual data) between variables in the identified VAR, shocks are orthogonalised using a permanent/transitory decomposition[2] (see Blanchard and Quah (1989)). More specifically, defining $X_t = \left[\begin{array}{cc} \lambda_t - \lambda^* & \Delta d_t - g \end{array} \right]'$ as the vector under examination, we can write its dynamics as:

$$X_t = C(L)\,\eta_t \ ,$$

where $C(L)$ is a matrix polynomial and where $\eta_t = \left(\eta_t^1, \eta_t^2\right)$ is a vector of normalised and orthogonalised shocks such that $\text{var}(\eta_t) = I$ and $C_{21}(1) = 0$. The latter restriction imposes that η_t^1, the 'transitory shock', does not move the long-run level of dividends, whereas only η_t^2, the 'permanent shock', may move the stochastic trend common to dividends and prices. Since, under the assumption that dividend/price ratios are stationary, movements in the common trend are entirely driven by changes in dividends, (see eq. (1)), a rather natural interpretation of the orthogonalisation adopted here is to say that η_t^2 represents a 'dividend innovation', and that, by contrast, η_t^1 is a 'discount-rate innovation' with temporary effects only (see Cochrane (1994) for an alternative justification of this labelling).

[2] It should be noted that, with our sample, results are not substantially altered if one uses a more conventional Cholesky decomposition where dividends are ordered first. Thus, we regard our decomposition as reasonably robust within the context of our sample.

Appendix A3 shows how this orthogonalisation allows one to compute the dynamic effect of transitory and permanent shocks on the dividend/price ratio, stock prices and stock returns. Each of the impulse-response functions, depicted in figure 2, gives an assessment of the size and persistence of transitory innovations to stock prices relative to that of permanent innovations. They confirm the predominent rôle of transitory 'discount-rate' shocks in the fluctuations of stock prices. For instance, the contemporaneous responses of prices and returns to a transitory shock are about three times as large as the corresponding responses to a permanent shock. The contrast is even more striking if one looks at the responses of the dividend/price ratio, which is left almost unchanged after a permanent shock, despite the auto-correlation pattern of dividend growth. Transitory shocks are also quite persistent, with a half-life of about two years. Finally, a transitory shock is associated with high returns, followed by about five years of negative returns due to trend-reversion.

The share of forecast error variance explained by transitory and permanent shocks at various horizons is calculated in table 5. As could be expected from the observation of impulse-response functions, transitory shocks are largely predominent in generating forecast errors of future dividend/price ratios and future returns. Note that this estimate of the influence of transitory shocks over the variance of stock returns is substantially higher than that obtained by Cochrane (1994) on US data. By contrast, transitory shocks are found to be about half as persistent in UK data as they are in US data (which feature a half-life for transitory shocks of about 5 years, see Cochrane (1994)).

5 Conclusion

Whereas there is little ambiguity about the fact that dividends govern the long-run value of stock market indexes, much less is known about the forces leading stock-price fluctuations in the short- and medium-run. This paper has addressed this issue by suggesting a method for measuring the relative rôles of dividends and discount rates in generating forecastability in stock price movements. Applying this method to the UK stock market, we concluded that mean-reverting, but persistent, deviations of the discount rate from its long-run average *are* the dominant force behind stock-price fluctuations in the medium-run, *even though dividends do not follow a random walk and display a clear cyclical pattern*. Whether these observed large swings in stock prices and discount rates can be reconciled with the joint hypothesis of market efficiency together with the specification of a 'reasonable' asset-pricing model remains an open issue, which we have avoided by defining the discount rate as a mere residual (rather than the outcome of an explicit behavioural model). However, an accurate measurement of the contribution of time-varying discount rates to the volatility and forecastability of stock price movements might be useful in helping one to choose amongst the competing families of asset-pricing models (i.e. equilibrium asset-pricing versus fad-based models) available in the literature. More generally, the decomposition used in this paper can usefully be applied to the analysis of the sources of stock market fluctuations when *both* dividends and discount rates are cyclical.

Appendix

A1. The log-linear present value formula

This appendix recalls how one can derive the log-linear present value model of Campbell and Shiller (1988). Let us first decompose the equation for ex post returns as follows:

$$R_{t+1} = (Q_{t+1}/Q_t) + (D_{t+1}/D_t) \times (D_t/Q_t) \tag{9}$$

Assuming that the dividend/price ratio is stationary, one can linearise (9) around the means of the variable under consideration and obtain:

$$R_{t+1} - R^* = (Q_{t+1}/Q_t - 1 - g) + (1+g)(\Lambda_t - \Lambda^*) + \Lambda^* (D_{t+1}/D_t - 1 - g) \ ,$$

where $R^* \equiv E(R_t)$, $\Lambda_t \equiv D_t/Q_t$ and $\Lambda^* \equiv E(\Lambda_t) = (R^* - 1 - g)/(1+g)$. Rewriting the latter equation in terms of proportional deviations from trend, and then using the fact that, for $X_t - X^*$ 'small', one has $(X_t - X^*)/X^* \simeq \log X_t - \log X^*$, one obtains:

$$\begin{aligned} r_{t+1} - r^* &= \rho(\Delta q_{t+1} - g) + (1-\rho)(\lambda_t - \lambda^*) + (1-\rho)(\Delta d_{t+1} - g) \\ &= (\lambda_t - \lambda^*) - \rho(\lambda_{t+1} - \lambda^*) + (\Delta d_{t+1} - g) \end{aligned}$$

A2. Calculation of the price trend, the discount-rate cycle and the dividend cycle

The VAR that is estimated can be written as $X_t = \sum_{i=1}^{2} A_i X_{t-i} + \epsilon_t$, where $X_t = \left[\begin{array}{cc} \lambda_t - E(\lambda_t) & \Delta d_t - E(\Delta d_t) \end{array} \right]'$. Let us define the matrix Π and vectors Y_t, ε_t and J as follows:

$$\Pi \equiv \begin{bmatrix} A_1 & A_2 \\ I & 0 \end{bmatrix}, \ Y_t \equiv \begin{bmatrix} X_t \\ X_{t-1} \end{bmatrix}, \ \varepsilon_t = \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix}, \ J \equiv \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

The evolution of X_t is then described by the equation:

$$Y_{t+k} = \Pi Y_{t+k-1} + \varepsilon_{t+k} = \Pi^k Y_t + \sum_{i=0}^{k-1} \Pi^i \varepsilon_{t+k-i}$$

From eq. (3) in section 2, the price trend at date t is given by:

$$\tau_t^q = -\lambda^* + d_t + J \sum_{k=1}^{\infty} E_t(Y_{t+k}) = -\lambda^* + d_t + J\Pi(I - \Pi)^{-1} Y_t$$

Let us denote $\left[\pi_{ij}^k\right] \equiv \Pi^k$, which reads as "$\pi_{ij}^k$ is the element of Π^k that lies on its i-th row and its j-th column", and scalars $\varphi_j = \sum_{k=1}^{\infty}\left(1-\rho^k\right)\pi_{2j}^{k+1}$. From eq. (3) in section 2, the 'dividend cycle' can then be computed as:

$$c_t^d = -\sum_{k=1}^{\infty}\left(1-\rho^k\right) J E_t\left(Y_{t+1+k}\right) = -\left[\begin{array}{cccc} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{array}\right] Y_t$$

A3. Shock orthogonalisation

First, let us write the VMA representation of the VAR as $X_t = B(L)\epsilon_t$, where $B(L) \equiv \left(I - A_1 L - A_2 L^2\right)^{-1}$ and $\operatorname{cov}(\epsilon) = \Sigma$. It can identically be rewritten as follows:

$$X_t = C(L)\eta_t \ , \tag{10}$$

where $\operatorname{cov}(\eta) \equiv \Omega$, $C(L) \equiv B(L)C(0)$ and $\eta_t \equiv C(0)^{-1}\epsilon_t$. By construction one has $\operatorname{var}\left(C(0)\eta_t\right) = \operatorname{var}(\epsilon_t)$, that is, $C(0)\Omega C(0)' = \Sigma$. The latter equation generates three equations for seven unknowns. The normalisation $\Omega = I$ adds three equations to the system. The identifying assumption that η_t^2 has non long run effect on dividends, which is written as $C_{21}(1) = 0$, allows to computes all components of $C(0)$. The rest of $C(L)$ if computed using $C(L) = B(L)C(0)$ (cf. Blanchard and Quah (1989)).

The effect of η_t on *returns* Δq_t is computed by differencing the first equation in (10), and then 'adding' to it the second equation. One finds:

$$\Delta q_t - g = \left[\begin{array}{cc} C_{21}(L) - (1-L)C_{11}(L) & C_{22}(L) - (1-L)C_{12}(L) \end{array}\right]\eta_t$$

Finally, note that if we define X_t' as $\left[\begin{array}{cc} \Delta q_t - E(\Delta q_t) & \Delta d_t - E(\Delta d_t) \end{array}\right]'$, we can write its dynamics as $X_t' = C^*(L)\eta_t$, where:

$$C^*(L) = \left[\begin{array}{cc} C_{21}(L) - (1-L)C_{11}(L) & C_{22}(L) - (1-L)C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{array}\right]$$

Due to the assumption that d_t and q_t are cointegrated, the matrix of long-run multipliers $C^*(1)$ has rank 1. Under our identifying restriction, it is indeed given by $C^*(1) = \left[\begin{array}{cc} \mathbf{0} & \mathbf{C}_{22}(1) \end{array}\right]$, which says that a η_t^1 shock has no long-run effect on either dividends or prices, whereas the long-run responses of prices and dividends to an η_t^2 shock are identical. Thus, the long-run restriction used in this paper is formally analogous to the permanent/transitory decomposition used in the VECM litterature (e.g. King *et al.* (1991)).

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Table 1. Statistics and Parameters (1919-2002)

A. Summary statistics

Variable	Mean	St. Dev.	Correlation matrix		
	$\overline{x}$	σ_x	Δd_t	Δq_t	$d_t - q_t$
Δd_t	0,013	0,127	1,000	-0,019	0,264
Δq_t	0,014	0,218		1,000	-0,391
$d_t - q_t$	-3,039	0,274			1,000

B. Parameter Values

Div. growth	Div. Yield	Weights	Disc. rate
$g = E(\Delta d_t)$	$\Lambda^* = E(\Lambda_t)$	$\rho = \frac{1}{1+\Lambda^*}$	$r^* = \frac{1+g}{\rho} - 1$
1,25%	4,79%	0,954	6,10%

Panel A reports the mean, standard deviation and contemporaneous correlations of dividend growth (Δd_t), returns (Δq_t), and the log dividend/price ratio ($\Lambda = D/Q$). Panel B shows the parameters used in the present value formula, that is, the average dividend growth (g), the average dividend yield (Λ^*), the weights (ρ), and the average discount rate (r^*). ρ and Λ^* are recovered using the fact that $r^* = 6.10\%$ over the whole sample.

Table 2. Unit-root tests (1919-2002)

ADF

d_t	q_t	Δd_t	Δq_t	λ_t
-1.54	-2.15	-6.19**	-6.37**	-3.39*

* 5% significance. (-2.90), ** 1% significance (-3.51)

Table 3. Estimates from a bivariate VAR (1919-2002)

Dependent variables	λ_{t-1}	Δd_{t-1}	λ_{t-2}	Δd_{t-2}	R^2	F-prob	Q^*
	Explanatory variables						
λ_t	0.712	0.141	-0.086	0.071	0.461	0.000	1.164
	(0.200)	(0.317)	(0.210)	(0.161)			
Δd_t	-0.108	0.422	0.099	-0.168	0.316	0.000	3.979
	(0.036)	(0.060)	(0.035)	(0.051)			

Heteroscedasticity-consistent standard errors are shown below each coefficient. F-prob is the probability value of an F-test for the joint significance of the explanatory variable. $Q^*$is the Ljung-Box statistics for up to 4th-order residuals' autocorrelation.

Table 4. Properties of the cycles

	St. deviation	Autocorrelations				
	$\sigma(\%)$	r_1	r_2	r_3	r_4	r_5
c_t	0.274	0.707	0.472	0.291	0.188	0.068
c_t^r	0.271	0.703	0.461	0.267	0.156	0.044
c_t^d	0.003	0.670	0.357	0.150	0.107	0.002

c_t denotes the overall cycle, c_t^r the discount-rate cycle, and $c_t^d$the dividend cycle.

Table 5. Variance Decomposition

h	$\text{var}(\lambda_t - E_{t-h}(\lambda_t))$		$\text{var}(\Delta q_t - E_{t-h}(\Delta q_t))$	
	η_t^1	η_t^2	η_t^1	η_t^2
1	0.996	0.004	0.920	0,080.
2	0.992	0.008	0.913	0.087
4	0.983	0.017	0.922	0.078
∞	0.982	0.018	0.922	0.078

η_t^1 is the transitory shock, and η_t^2 is the permanent shock. The table reports the share of each shock in the forecast error variance of the dividend/price ratio and the stock return at various horizons.

Figure 1: Trend and cycles in UK stock prices

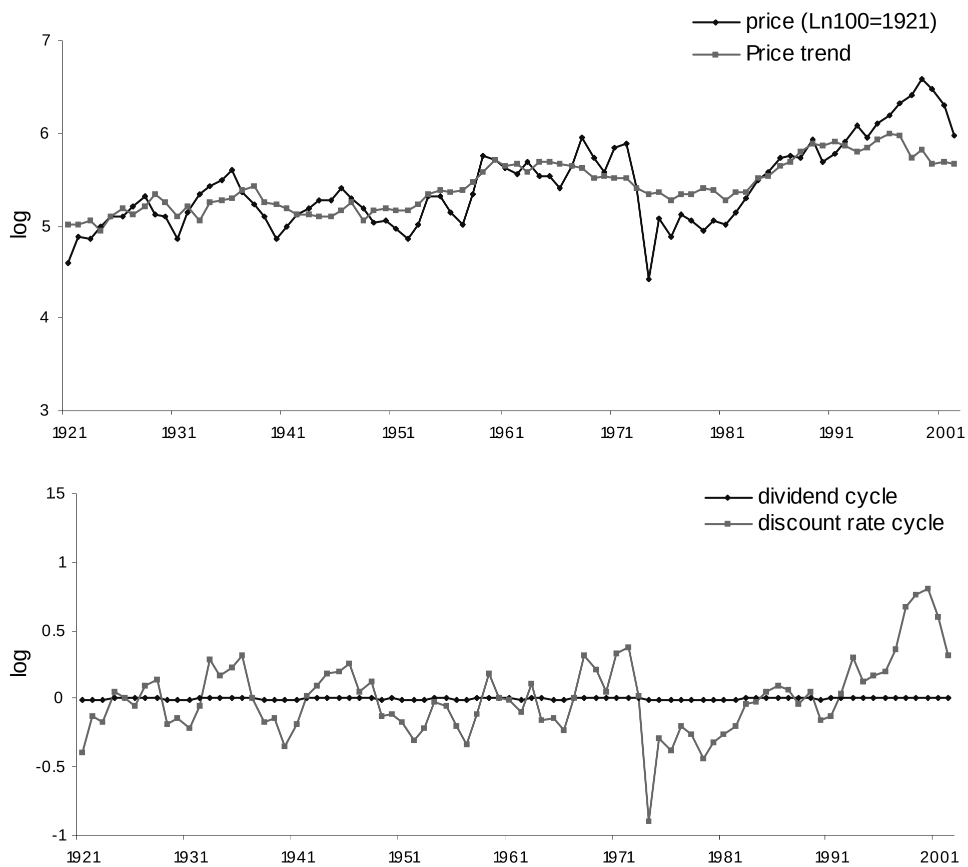

Figure 2: Impulse-response functions