Inventory Effects and Trading Horizons. *

Giovanni Cespa †

June, 2000

Abstract

I study the effects of the heterogeneity of traders' horizon in the context of a 2-period NREE model where all traders are risk averse. Owing to inventory effects, myopic trading behavior generates multiplicity of equilibria. In particular, two distinct patterns arise. Along the first equilibrium, short term traders anticipate higher second period price reaction to information arrival and, owing to risk aversion, scale back their trading intensity. This, in turn, reduces both risk sharing and information impounding into prices enforcing a high returns' volatility-low price informativeness equilibrium. In the second one, the opposite happens and a low volatility-high price informativeness equilibrium arises.

Keywords: Financial Economics, Information and Market Efficiency.

*Support from the EC through the TMR grants n. ERBFMBI-CT97-2528 and n. ERBFMRX-CT96-0054, the Université des Sciences Sociales de Toulouse (GREMAQ) and Ente per gli Studi Monetari e Finanziari "Luigi Einaudi," is gratefully acknowledged. This is a revised version of the first chapter of my Ph.D dissertation. I thank my supervisor, Xavier Vives, Bruno Biais, Sandro Brusco, Jordi Caballé, Marco Pagano, Jozséf Sáckovics, participants to the TMR Meeting on Financial Market Efficiency, Corporate Finance and Regulation (Barcelona, 5/2000) and seminar audience at the Università degli Studi di Venezia (Dipartimento di Matematica Applicata) for helpful comments and suggestions. The usual disclaimer applies.

†Departament d'Economia i Empresa, Universitat Pompeu Fabra, Ramon Trias Fargas, 21–22, Barcelona. E-mail: giovanni.cespa@econ.upf.es

1 Introduction

This paper analyzes the effect of myopic behavior on stock market patterns. Both empirical and theoretical considerations motivate the analysis. On the one hand, the increasingly important role that institutional investors are playing in modern economies, [1] coupled with the alleged short termistic behavior that these traders display, [2] calls for an understanding of the effects that their trading activity has on the performance of stock markets. On the other hand, intuitive reasoning suggests that in a realistic market where price movements are due both to information arrival and supply shocks, the risk borne by an agent holding a positive inventory of the traded asset should have a different effect on his behavior depending on his time preferences. Indeed, myopic traders, faced with the need of liquidating their position in the short run, have fewer opportunities to smooth their inventory holdings' decisions. As a consequence, their behavior is strongly influenced by the anticipation of price reaction to future order flows. On the contrary, thanks to their longer horizon, long termists, can attain a better inventory allocation. This difference, in turn, should make market patterns dependent on the composition of the market.

I analyze these issues in a 2-period noisy, rational expectations equilibrium model of stock-market trading where 2 classes of traders interact: a sector of short-term, informed traders of measure $\mu > 0$, and a sector of long-term, informed traders of measure $1 - \mu$. [3]

Owing to risk aversion, *(a)* informed agents, besides speculating on their private information, absorb liquidity shocks and *(b)* prices are influenced by both information arrival and liquidity supply. [4] The effect of point *(b)* above may be either to *increase* or to *decrease* second period market depth with respect to a semi strong efficient market. Indeed, adverse selection effects are mitigated because of speculators' market making activity, while owing to risk aversion, prices react to noise traders' demand.

Depending on the trading horizon of informed speculators, the resulting effect on trading activity changes. Long-termists, when in period 1 choose the size of their

[1] See e.g. Gompers and Metrick (1998), Sias (1996) and Friedman (1996). Gompers and Metrick (1998), report that "[...] *large* institutional investors (...) nearly doubled their share of the common-stock market from 1980 to 1996. By December 1996, these large institutions held discretionary control over more than half of the U.S. equity market."

[2] For instance, Kahn and Winton (1998) argue that "...traditionally [Institutional Investors] were *stock pickers* who tried to beat the market through trading; if a firm whose stock they held seemed headed for trouble, these investors headed for the door (*the Wall Street rule*)." Also, Wermers (1998) "Many newsmedia commentators (...) tend to believe that institutional investors focus excessively on short-term trading strategies..." Finally, Tirole (2000) "...institutions shy away from sitting on boards and mostly act as short-term players."

[3] A number of authors have analyzed dynamic rational expectations equilibrium models, see e.g. Brown and Jennings (1989), Grundy and Nichols (1989), Vives (1995) and He and Wang (1995).

[4] As in Admati and Pfleiderer (1991) and Subrahmanyam (1991).

position, anticipate the volatility of the asset liquidation value using both the private and public information available. Short-termists, on the contrary, cannot hold the asset until the liquidation date and are, therefore, interested in forecasting second period price reaction to the incoming order flow which, in turn, depends on their first period behavior. As a consequence, two possible equilibria arise. If short term traders anticipate that second period prices overreact to public information, they know that the weight of second period public news will be *high* in affecting price formation. This increases the risk of their speculative position in the first period and makes them scale back their trades. Thus, the aggregate risk bearing capacity of the market in the second period decreases and this makes the market thinner and second period prices overreactive to information arrival. The opposite happens if they anticipate second period price *underreaction*. Therefore, myopic behavior induces multiplicity of equilibria.

The consequences for market performance depend on which of the two equilibria arises. Along the high trading intensity equilibrium, given that second period prices *underreact* to information arrival, short term traders speculate more intensively on their private signal and accomodate unexpected liquidity shocks. This has two effects: on the one hand it increases price informativeness in both periods; on the other hand it improves risk sharing, reducing returns' volatility. However, a higher trading intensity also affects the severity of adverse selection effects. Numerical simulations show that when the quality of prior information is sufficiently poor, an increase in the size of the short term trading sector, leads to a deeper market.

Summarizing, inventory effects coupled with myopic behavior give rise to two different equilibrium patterns. Along the *high* trading intensity equilibrium, short term traders *overreact* to private information, inducing a *low* volatility, *high* price informativeness equilibrium. On the contrary, along the *low* trading intensity equilibrium, prices are less informative and returns more volatile. Thus, in the presence of myopic behavior, periods of high volatility are a signal of poor price informativeness.

A number of authors have analyzed myopic behavior in financial markets. Vives (1995), in a $N \geq 2$-period model, shows that controlling for patterns of information arrival, price informativeness depends on the trading horizons of speculators. However, in his model prices are set by a risk neutral market making sector and this, eliminating inventory effects, makes the *low trading intensity* equilibrium in the short term trading model, unique. [5] As a consequence the highly informative-low volatility equilibrium disappears. Holden and Subrahmanyam (1996), determine the conditions under which traders choose to acquire short-term information, deriving endogenously myopic behavior. Dow and Rahi (1999) in a model with *hedgers*, concentrate on the welfare effects of a tax on speculation. They show that for some parameter configurations a tax, by reducing the informativeness of prices, can enhance speculators'

[5] At least in the 2-period case.

profits and allow hedgers larger insurance opportunities, eventually leading to a welfare improvement.

Others, have investigated the *feedback* effect that volatility has on short term traders' investment decisions. Pagano (1989), in a OLG model, shows that anticipated high volatility levels, make traders unwilling to enter the market, reducing risk sharing and leading to thin markets. Dennert (1991), shows that *high* volatility equilibria can be self-fulfilling in the steady state of a OLG market with differential information.

The paper is organized as follows: in the next section I outline the model assumptions, define notation and show existence and uniqueness of the equilibrium in the case in which only long term traders are in the market. In the third section I introduce a positive measure of myopic traders and show existence and multiplicity of the equilibrium in this market. I then study the effects on market performance of an increase in the size of the short term trading sector. A final appendix collects most of the proofs.

2 The Model

Trading happens over 2 periods, and there are two types of agents: a continuum of informed speculators (when long termists, maximizing the expected utility of their final wealth $W_{i,2} = \sum_{t=1}^{2} \pi_{i,t}$ when short-termists, maximizing the expected utility of next period profits) and noise traders. The asset payoff is normally distributed $v \sim N(\bar{v}, \tau_v^{-1})$. Informed speculators have CARA utility function with risk-tolerance parameter γ and receive a noisy signal of the asset liquidation value $s_{i,t} = v + \epsilon_{i,t}$ in each period, where $\epsilon_{i,t}$ and $\epsilon_{j,k}$ are independent for all i, j, t, k and $\epsilon_{i,t} \sim N(0, \tau_{\epsilon_t}^{-1})$, $\forall i$. I will make the assumption that the strong law of large numbers holds, i.e. $\int_0^1 s_{i,t} di = v$, almost surely.

In period 1 informed agents have the private signal $s_{i,1}$ available, while in period 2 they have the vector $s_i^2 = (s_{i,1}, s_{i,2})$. It follows from normal theory that the statistic $\tilde{s}_{i,2} = (\sum_{t=1}^{2} \tau_{\epsilon_t})^{-1} (\sum_{t=1}^{2} \tau_{\epsilon_t} s_{i,t})$ is sufficient for the sequence s_i^2 in the estimation of v. An informed agent i in period n submits a limit order $X_i(\tilde{s}_{i,n}, p^{n-1}, \cdot)$ indicating the position desired at every price p_n, contingent on the information available. Noise traders' demand is assumed to be normally distributed $u_t \sim N(0, \tau_u^{-1})$, u_t and u_{t+1} are independent. [6] Finally, u_t, v and $\epsilon_{i,t}$ are independent for all i, t. The subscript l (s) will be used to indicate the parameters of a market populated of only informed long term traders (only informed short term traders). I restrict attention to linear equilibria where a centralized mechanism aggregates orders and sets the equilibrium price that clears the market for the asset.

[6] The random variables $\{u_1, u_2\}$ can equivalently be interpreted as the increments in the stock supply in the two trading periods as in He and Wang (1995).

2.1 The Benchmark

In this section, I derive the unique linear equilibrium of the market where only long term traders act. The result obtained in proposition 1, can be seen either as two-period extension of Admati (1985) or as a generalization of the two-period case of Vives (1995) model to a situation where market makers are risk averse.

Proposition 1 *In the market with long term, informed speculators there exists a unique linear equilibrium where prices are given by* $p_o = \bar{v}$, $p_3 = v$, *and for* $n = 1, 2$, $p_n = \lambda_n z_n + (1 - \lambda_n \Delta a_n) p_{n-1}$ *and strategies are given by*

$$x_{i,n} = a_n(\tilde{s}_{i,n} - p_n) + \gamma \tau_n (E[v|z^n] - p_n), \tag{1}$$

where $a_n = \gamma(\sum_{t=1}^{n} \tau_{\epsilon_t})$, $\Delta a_n = a_n - a_{n-1}$, $z_n = \Delta a_n v + u_n$, $z^n = \{z_t\}_{t=1}^{n}$, $\tau_n = (Var[v|z^n])^{-1}$, $\tau_{in} = (Var[v|z^n, \tilde{s}_{in}])^{-1}$, $\lambda_n = \beta_{v|z_n;z^n}(1 + (1/(a_n + \gamma\tau_n))(\beta^{-1}_{v|z_n;z^n} - a_n))$ *and* $\beta_{v|z_n;z^n} = \Delta a_n \tau_u / \tau_n$ *(i.e. the OLS regression coefficient of* v *over* z_n *in the regression of* v *over* z^n*).*

Proof. See appendix. QED

Informed speculators in period n, trade on their private information according to the (cumulative) precision of the signals they have received thus far, [7] and, owing to risk aversion, absorb liquidity shocks. This, in turn, makes the price react both to information arrival and liquidity supply (as in Subrahmanyam (1991) and Admati and Pfleiderer (1991)). To see this, consider that if in the second period no *new* information arrives, $\lambda_2 = (1/\gamma\tau_{i2}) \neq 0$ differently from what happens in a semi-strong efficient market.

Market making performed by risk-averse, informed investors, can enhance or reduce price reaction to information arrival compared to a semi-strong efficient market. Indeed, on the one hand by being risk-averse, market makers bear the risk incorporated in the order flow due to noise trades; on the other hand, by being informed, they speculate on private information, increasing the risk bearing capacity of the market. To see this, rewrite market depth as follows:

$$\lambda_n = \frac{1}{a_n + \gamma\tau_n} + \frac{\gamma\tau_n}{a_n + \gamma\tau_n}\beta_{v|z_n;z^n}.$$

The term $\beta_{v|z_n;z^n}$ accounts for the *adverse selection* effect, while $(a_n + \gamma\tau_n)^{-1}$ is the reciprocal of traders' total trading intensity and accounts for the *inventory* effect. Clearly, $\gamma\tau_n/(a_n + \gamma\tau_n) < 1$, therefore, owing to risk sharing, the *adverse selection*

[7] This generalizes the demand function found in Vives (1995) to a market with risk averse market makers.

effect is weaker in this market; however, $(a_n + \gamma\tau_n)^{-1} > 0$, thus *inventory risk* shifts upwards the reaction function of a risk neutral market maker. [8]

In the first period, the positive effect on depth due to the reduction in adverse selection is *never* strong enough to overcome the negative effect due to inventory risk and $\lambda_1 > \beta_{v|z_1}$. The same result is obtained by Subrahmanyam (1991) in a static, strategic context with uninformed, risk averse market makers. In a dynamic market, the result of the comparison depends on the sign of the intensity of informed net trades. To see this, consider the following argument. An increase in the aggregate position (signalled by $z_2 > 0$) can be taken as a signal that speculators have received good news about the asset pay off. However, if market makers know that speculators trading intensity has reduced over time, the same signal could most likely come from a demand pressure of liquidity traders. Therefore, depending on the sign of speculators' net trading intensity, the statistical inference the market makes about $z_2 > 0$ can lead either to an upward or to a downward revision of the asset pay off expectation. Suppose now $\Delta a_2 > 0$ and $z_2 > 0$. A risk neutral market maker revises upwards his estimate of the asset pay off and increases p_2. If market makers are risk averse, price reaction to adverse selection effects is mitigated. [9] However, existence of noise trading garbling the public signal *adds* to the price reaction due to adverse selection and may make the price more reactive to the (informational content of the) order flow. Therefore, if the improving effect on market depth due to the reduction in adverse selection costs overcomes the worsening effect due to inventory risk, the market is deeper. [10] Suppose now $\Delta a_2 < 0$ and $z_2 > 0$. Then a risk neutral market maker turns good news into a price reduction. If market makers are risk averse, price reaction to adverse selection is mitigated while again the existence of noise trading garbling the public signal shifts upwards the reaction function of the market maker. If the adjustment due to inventory risk is smaller than that due to adverse selection, the market is deeper.

Given that, depending on traders behavior, second period price can overreact (underreact) to information arrival, one expects this effect to feed-back into the conditional volatility of returns.

Proposition 2 *In every equilibrium of the market with long term, informed speculators*

1. *price precision in period n is given by $\tau_n = \tau_v + \tau_u \sum_{t=1}^{n} (\Delta a_t)^2$;*

[8] Price reaction noise trading can be given the following intuition. Suppose $\Delta a_2 = 0$, and $z_2 > 0$. Then market makers face a positive demand coming from noise traders. The risk they run, in this case, is that of selling the asset for a price $p_2 < v$, therefore they accomodate the trade by increasing p_2. Suppose $z_2 < 0$, then market makers face a positive supply and the risk they run is that of buying the asset at $p_2 > v$, therefore to accomodate the trade, they set a lower price.

[9] By the increased risk bearing capacity of the market due to informed market making.

[10] Analytically the condition is $(a_2 + \gamma\tau_2)^{-1} < \beta_{v|z_2}(1 - (\gamma\tau_2/(a_2 + \gamma\tau_2)))$.

2. *the conditional volatility of returns is given by* $\mathrm{Var}[p_2|p_1] = \alpha^{-2}(\tau_1^{-1} - \tau_2^{-1})$, *where* $\alpha = \beta_{v|z_2}/\lambda_2$;

Proof. The first part of the proof comes from the properties of the normal distribution. Next, consider that in every linear equilibrium p^n is informationally equivalent to z^n. Hence, $\mathrm{Var}[p_2 - p_1|p_1] = \mathrm{Var}[p_2 - p_1|z_1] = \lambda_2^2 \mathrm{Var}[z_2|z_1]$ and $\mathrm{Var}[v|p^n] = \mathrm{Var}[v|z^n]$. For part 1, given that at equilibrium p_1 is o.e. to z_1, $\mathrm{Var}[p_2|p_1] = \mathrm{Var}[p_2|z_1] = \lambda_2^2 \tau_2 \tau_1^{-1} \tau_u^{-1}$. Multiplying numerator and denominator of the above expression by $(\Delta a_2)^2 \tau_u \tau_2$ and collecting parameters, I obtain the result in the proposition. QED

The conditional volatility of returns is the result of the composite effect of *(a)* the reduction in the conditional variance of the asset liquidation value due to the arrival of *news* (the term $(\tau_1^{-1} - \tau_2^{-1})$) and *(b)* the cumulative effect that market makers' risk aversion and private information, induce on second period price reaction to the incoming order flow (the term α^{-2}). Point *(a)* above refers to the standard explanation of returns' volatility in a semi-strong efficient market where the only sources of price movements are due to information arrival; point *(b)* is peculiar to the present market structure where inventory effects also play a role in affecting returns' volatility. Therefore, whenever owing to risk sharing the reduction in *adverse selection* compensates the effect of *inventory risk*, the market is *deeper* in the secod period and the conditional volatility of returns is lower than in a semi-strong efficient market. On the contrary, whenever the reverse happens, a *thinner* market induces higher volatility of returns. The negative relation between return volatility and market depth is well documented both at an empirical and theoretical level (Pagano 1989).

Price informativeness is not affected by market makers' risk aversion. Long term, informed speculators' trading intensity does not depend on the risk preferences of market makers.

Finally, when $N = 1$, the model collapses to the Admati (1985) case.

3 The Market with Myopic Traders

In this section, I study the effect of introducing a positive measure μ of myopic traders on the equilibrium derived in proposition 1. Short termists are endowed with the same information of long term traders but maximize the expected utility of short run profits. Short termism has been typically associated with institutional investors' behavior (Friedman (1996), Jacobs (1991), Tirole (2000)), thus the characteristics of the short term trading sector should be taken as a proxy of the effect that institutions have on market patterns.

The following result applies

Proposition 3 *Linear equilibria of the market where a positive measure* μ *of short termists and a measure* $1-\mu$ *of long termists trade exist and are characterized by the following couple of prices and strategies:*

1. *prices:* $p_o = \bar{v}$, $p_1 = \lambda_1 z_1 + c_1$, $p_2 = \lambda_2 z_2 + (1-\lambda_2 \Delta a_2) p_{1l}$ *and* $p_3 = v$*;*

2. *second period strategies are as in proposition 1, short and long term traders' first period strategies are given respectively by*

$$x_{is,1} = a_{1s}(s_{i,1} - p_1) + \gamma\rho\tau_1(E[v|z_1] - p_1) + \left(\frac{\gamma\rho\tau_{i1}(1-\lambda_2\Delta a_2)}{\lambda_2\Delta a_2}\right)(p_{1l} - p_1), \quad (2)$$

$$x_{il,1} = a_{1l}(s_{i,1} - p_1) + \gamma\tau_1(E[v|z_1] - p_1) + \left(\frac{\gamma\tau_u(1-\lambda_2\Delta a_2)^2}{\lambda_2^2}\right)(p_{1l} - p_1), \quad (3)$$

where $p_{1l} = \lambda_{1l} z_1 + (1-\lambda_{1l} a_1)\bar{v}$, $\lambda_{1l} = (1+\gamma\tau_u a_1)/(a_1+\gamma\tau_1)$, $\lambda_2 = (1+\gamma\tau_u\Delta a_2)/(a_2+\gamma\tau_2)$, $a_1 = \mu a_{1s} + (1-\mu)a_{1l}$, $a_{1l} = \gamma\tau_{\epsilon_1}$,

$$a_{1s} = \gamma\alpha(\tau_{\epsilon_1}^{-1} + \tau_2^{-1})^{-1}, \quad (4)$$

$\rho = a_{1s}/a_{1l}$*, and explicit expressions for* λ_1 *and* c_1 *are given in the appendix.*

Proof. See the appendix. QED

As in proposition 1 in the first period, long term traders trade on their private information the more intensively, the higher is the precision of their private signal and the more risk tolerant they are. Short termists, react positively to γ and τ_{ϵ_1} and take into account the informativeness of second period price. The reason is as in Vives (1995): given that they have to liquidate their position in the second period, they try to predict p_2. However, their signal is about v, therefore the closer is p_2 to v (the higher is τ_2) the more informative is their private signal about p_2 and the more intensively they trade. Furthermore, to the extent that second period prices can overreact (underreact) to the arrival of public information, they scale down (up) their trades.

Corollary 1 *In every equilibrium of the market with myopic traders*

1. $a_{1l} > 0$ *and* $a_{1s} > 0$*;*

2. $\alpha > 0$.

Proof. For part 1, given that $a_{1l} = \gamma\tau_{\epsilon_1}$, the result follows. Next, for a_{1s}, consider the following argument. Suppose $\mu = 1$ (the same holds for any μ) and assume that $\Delta a_2 < -1/\gamma\tau_u$, in this case, $\Delta a_2 < 0$ and $\alpha > 0$, hence $a_{1s} > 0$. Next, suppose $-1/\gamma\tau_u < \Delta a_2 < 0$, then $\alpha < 0$, hence $a_{1s} < 0$. However, in this case, $\Delta a_2 = a_2 - a_1 > 0$, contradicting the assumption.

For part 2, the result follows from $a_{1s} > 0$. QED

Therefore, in equilibrium both long and short term traders put a positive weight on their first period private information and *inventory effects* cannot offset *adverse selection* effects.

To understand the effect of short term horizons on the market, it is useful to to consider first the two extreme cases where $\mu = 0$ and $\mu = 1$.

3.1 The Case $\mu = 0$

If $\mu = 0$, then $p_{1l} = p_1$. Therefore, if there is only one short term trader in the market, it is easy to see that (2) reduces to

$$\begin{aligned} x_{is,1} &= a_{1s}(s_{i,1} - p_1) + \gamma\rho\tau_1(E[v|z_1] - p_1) \\ &= \rho x_{il,1}, \end{aligned}$$

where it can be checked that $\rho < 1$. In other words, a short term trader suffers from the unpredictability of second period price and, given that he has to liquidate his position in $t = 2$, his position is *scaled back* w.r.t. that of a long termist. Given that the short term trading sector has measure zero, the other market characteristics are not affected by the presence of a short termist, hence the results given in section 2.1 apply.

3.2 The Case $\mu = 1$

Suppose now that $\mu = 1$. In this case, equilibrium prices are affected by the time horizon of traders in the market. In particular, consider the problem of a short termist. When choosing his position in the first period, he anticipates the fact that he will unload it in the next period. This makes first period trading intensity depend on second period market depth. However, price reaction to second period information (i.e. λ_2), in turn depends on traders' first period behavior. Therefore, second period depth and first period trading intensity are determined simultaneously in equilibrium and the following result applies.

Proposition 4 *In the market with only short term traders, when $\tau_{\epsilon_2} = 0$, there exist two equilibria a_{11s}, a_{12s}, where:*

1. *prices are given by* $p_3 = v$, $p_o = \bar{v}$, $p_2 = \lambda_2 z_2 + (1 - \lambda_2 \Delta a_2) p_{1l}$ *and*

$$p_1 = (1 - \lambda_2 \Delta a_2) p_{1l} + \lambda_2 \Delta a_2 \left(\frac{z_1 + \gamma \rho \tau_1 E[v|z_1]}{\gamma \rho \tau_{i1}} \right); \qquad (5)$$

2. *strategies are as in proposition 3;*

3. *first period trading intensities are such that*

$$a_{12s} < a_{1l} < a_{11s},$$

and $\alpha(a_{12s}) < 1 < \alpha(a_{11s})$.

Proof. See the appendix. QED

When the arrival of information is concentrated in the first period, short term horizons induce multiplicity of equilibria. The intuition is as follows. In choosing their first period speculative position, traders anticipate that second period prices can either *overreact* or *underreact* to information arrival. Suppose they anticipate $\alpha < 1$. Then, second period prices overreact to news and their position in the first period becomes riskier. As a consequence, they underreact to their first period signal. This attenuates the *weakening effect* that risk sharing has on the adverse selection component of λ_2, and increases the effect of *inventory risk* leading to an equilibrium with $\alpha < 1$. Suppose, on the contrary, they anticipate that second period prices underreact to information arrival. Then, they overreact to their first period signal enforcing an equilibrium with $\alpha > 1$. [11]

Remark 1 If we compare (5) with the price that would arise in a market with long term traders, then we can write

$$p_{1l} - p_1 = \frac{\lambda_2 \Delta a_2}{(a_{1s} + \gamma \tau_1)(a_{1s} + \gamma \rho \tau_1)} (\rho - 1) \tau_v (z_1 - E[z_1]),$$

and, indicating with $x^*_{i1,l}$ the position of a long term trader in a market with $\mu = 1$ and $p_1 = p_{1l}$,

$$x_{i1,s} = \rho x^*_{i1,l} + (\rho - 1) \frac{\gamma \tau_v}{a_2 + \gamma \tau_2} (z_1 - E[z_1]).$$

Thus, $\forall z_1 > E[z_1]$, $p_{1l} > p_1$ if and only if $\rho > 1$, i.e. price *underreacts* to unexpected order flow realizations, if and only if traders *overreact* to their private signals. Furthermore, if (and only if) $\rho > 1$, short term speculators scale up their trades w.r.t. a long termist and accomodate unexpectedly high order flows. The

[11] Numerical simulations were ran for the case $\tau_{\epsilon_2} \neq 0$ and confirmed the multiplicity result.

opposite happens if $\rho < 1$. The intuition is straightforward. Given that $\rho > 1 \Leftrightarrow \alpha > 1$, anticipating second period price underreaction to the incoming order flow reduces the risk of holding a larger inventory and this boosts market making activity in the first period.

The final effect on λ_{1s} depends on the relative importance that adverse selection problems have over risk sharing. The idea is that while an increase in trading intensity (the case $\rho > 1$) on the one hand improves risk sharing and thus boosts market making activity, on the other hand, by leading to stronger speculation, may also make adverse selection more severe. Denote with z_{1l} the informational content of the order flow in the market with long term traders, then the following result applies

Proposition 5 *With concentrated arrival of information, in the market with short term traders,*

1. *If*

$$a_{1s} > \frac{1 - a_{1l}\beta_{v|z_{1l}}}{\beta_{v|z_{1l}}},$$

then $\rho > 1$ if and only if $\lambda_{1s} < \lambda_{1l}(a_{1l})$;

2. $\lambda_2(a_{11s}) < 0 < \lambda_2(a_{12s})$;

3. $|\lambda_2(a_{11s})| < \lambda_2(a_{12s})$.

Proof. See the appendix. QED

Note that if $\rho > 1$, then $\lambda_2 < 0$, i.e. the asset becomes a Giffen good. This type of anomalies, typical in a multi-asset framework as Admati (1985), has a different explanation here. For, suppose $\rho > 1$ and $z_2 < 0$, then, given $\Delta a_2 < 0$, market makers in the second period attribute a negative order flow to a selling pressure coming from noise traders and update upwards their estimate of the asset pay off.

If $\rho > 1$, short term traders overreact to private information and this, on the one hand improves risk sharing, on the other hand may worsen adverse selection problems. When the prior information held by the market is not very good with respect to the informativeness of the order flow, adverse selection effects are weaker than inventory risk. In this case, proposition 5, shows that a market populated with short term traders, is *deeper* along the *high* trading intensity equilibrium than a market with long termists. For example, if $\tau_v = 10$, $\tau_u = \gamma = 1$, $\tau_{\epsilon_1} = .1$ and $\tau_{\epsilon_2} = 0$, then $a_{1s} = 1.191$, $(1 - a_{1l}\beta_{v|z_{1l}})/\beta_{v|z_{1l}} = 100$ and comparing, $\lambda_{1s} = 0.173$ while $\lambda_{1l}(a_{1l}) = 0.108$.

I conclude this section, by studying the effects of short horizons on returns' volatility and price informativeness. Indicate with $\tau_n(a_{1k})$ the period n price precision calculated in the market with k-type $(=l, s)$ traders.

Proposition 6 *With concentrated arrival of information, in the market with short term traders*

1. $\tau_n(a_{12s}) < \tau_n(a_{1l}) < \tau_n(a_{11s})$, *for* $n = 1, 2$;
2. $Var[p_2|p_1; a_{11s}] < Var[p_2|p_1; a_{12s}]$.

Proof. See the appendix. QED

Thus in both periods, short term horizons deliver more informative prices in the high trading intensity equilibrium w.r.t. long horizons. While for $n = 1$, the reason is obvious, it is interesting to contrast the result for $n = 2$ with a two period version of Vives (1995) which is a particular case of this model when $\alpha = 1$. [12] If $\tau_{\epsilon_2} = 0$, long term traders concentrate all their trading activity in the first period while short termists spread it across both trading dates. Given that *(a)* price precision is a quadratic function of net trading intensities, *(b)* total trading intensity is the same independently of speculators' horizons and *(c)* short term traders always trade less intensively than long termists, the precision of the final price is always higher with long term traders. Indeed, in this case, spreading trades across time reduces price informativeness. In the present model, this is what happen when $\rho < 1$. However, owing to inventory effects, the *high* trading intensity equilibrium can realize. In this case $\rho > 1$, hence the degree of inequality in the distribution of net trades becomes higher when short term traders are in the market and second period price informativeness is higher.

The second result shows that along the *low* (*high*) trading intensity equilibrium, the conditional volatility is *high* (*low*). The intuition is that if short term traders anticipate higher second period price reaction to the incoming order flow, they scale back their trading intensity, reducing the total risk bearing capacity of the market in the second period. This reduces second period market depth and leads to a *high* volatility equilibrium. Therefore, according to proposition 6, in the presence of myopic traders, a market with high conditional volatility of returns, delivers less informative prices. [13]

Remark 2 The result on volatility, is reminiscent of the one obtained by Dennert (1991). Indeed, Dennert shows that the *steady state* solution of a OLG model of a stock market with asymmetric information, is characterized by the coexistence of an equilibrium with high price volatility and one with low price volatility. [14] In

[12] Allowing for a more general model that includes a class of risk averse market makers with risk aversion parameter $1/\gamma^U$, it can be shown that this is the case when $\gamma^U \to \infty$, see Cespa (1999).

[13] Numerical simulations show that the same result obtains when $\tau_{\epsilon_2} = \tau_{\epsilon_1} \in \{.1, .4, .7, 1\}$ and $\tau_v, \tau_u, \gamma \in \{.1, .4, .7, 1\}$.

[14] According to Dennert (1991), an economy is in a *steady state*, if prices are identically distributed i.e. $p_t \sim p \sim N(E[p], \mathrm{Var}[p])$.

the present context, this effect obtains independently of the assumption over the distribution of prices.

3.3 The Case $\mu < 1$

Building on the insight gained by studying the market with only short term traders, in this section I focus on the *general* model with short and long termists. The aim is to study the effect on market performance of an increase in the size of the short term trading sector. When the arrival of information is concentrated in the first period, a straightforward generalization of the results obtained in propositions 4, 5 and 6 gives the following corollary

Corollary 2 *In the market where a sector of short term traders and one of long termists (respectively of measure μ and $1-\mu$) interact, when the arrival of information is concentrated in the first period (i.e. when $\tau_{\epsilon_2} = 0$), there exist two equilibria a_{11s}, a_{12s} such that*

1. $a_{12s} < a_{1l} < a_{11s}$;
2. $\lambda_2(a_{12s}) > 0$ *and* $\lambda_2(a_{11s}) < 0$;
3. $\forall \mu \in (0,1)$, $Var[p_2|p_1; a_{11s}] < Var[p_2|p_1; a_{12s}]$ *and* $\tau_n(a_{11}) > \tau_n(a_{12})$, $n = 1, 2$, *where* $a_{1k} = \mu a_{1ks} + (1-\mu)a_{1l}$, $k = 1, 2$.

Proof. See the appendix. QED

All the results in the above corollary mirror what has been shown for the case $\mu = 1$ and the intuitions given for that case apply to this case too. [15]

Consider now the effect that short term behavior has on long term traders' positions. It is easy to show that

Proposition 7 *In every equilibrium of the market with myopic traders,*

1. $$p_{1l} - p_1 = \frac{\lambda_2^2 \Delta a_2 (1 - \lambda_{1l} a_1)}{D} \mu(\rho - 1)(z_1 - E[z_1]);$$

2. *long and short term traders strategies can be written as follows:*

$$x_{i1,l} = a_{1l}(s_{i1} - p_1) + \gamma\tau_1(E[v|z_1] - p_1) + K_1\mu(\rho - 1)(z_1 - E[z_1]), \quad (6)$$

$$x_{i1,s} = a_{1s}(s_{i1} - p_1) + \gamma\rho\tau_1(E[v|z_1] - p_1) + K_2\mu(\rho - 1)(z_1 - E[z_1]), \quad (7)$$

[15] Numerical simulations were ran for the case $\tau_{\epsilon_2} \neq 0$ and confirmed the multiplicity result.

where D, $K_1 > 0$ and $K_2 > 0$ are constants defined in the appendix.

Proof. See the appendix. QED

As happened for the market where $\mu = 1$, if $\rho > 1$, short term traders accomodate unexpected order flow realizations by increasing their market making activity. Furthermore, what can be seen in (6), is that by affecting second period market depth, short term trading induces an *externality* on long term traders' market making activity. Indeed, if the *high* trading intensity equilibrium realizes, then second period prices *underreact* to z_2, therefore the risk of holding a larger position *decreases* both for short *and* long termists, and this makes both classes of traders increase their market making activity.

The next proposition characterizes the effects of an increase in the size of the short-term trading sector on price informativeness.

Proposition 8 *In every equilibrium of the market with short and long term traders, when the arrival of information is concentrated in the first period $\partial\tau_2(a_{11s})/\partial\mu > 0$ and*

$$\frac{\partial\tau_2(a_{12s})}{\partial\mu}\begin{cases} < 0 & \text{for } 0 < \mu < 1/2 + \gamma a_{1l}\tau_u/4 \\ > 0 & \text{otherwise;} \end{cases}$$

Proof. See the appendix. QED

Therefore, along the *high* trading intensity equilibrium, an increase of the measure of short term traders, induces more informative second period equilibrium prices. This follows directly from 6. Moreover, also along the low trading intensity equilibrium an increase in μ may lead to more informative second period prices, the reason is that the inequality in the intertemporal distribution of net trades is high for μ close to zero (remember that in this case $\Delta a_2 \sim 0$) and decreases as μ increases; for $\mu = 1/2 + \gamma a_{1l}\tau_u/4$ it reaches its minimum ($\Delta a_2 = a_{1l}$ and $a_1 = \gamma a_{1l}^2/(2 + \gamma a_{1l}\tau_u)$) and then increases again.

4 Numerical Simulations

In this section, I collect the results of numerical simulations showing the effects that an increase in μ has on first period market depth, volatility and volume.

Starting with depth, consider figure 1. In the left panel, I plot λ_1 (l_1) as a function of μ [16] Thicker (thinner) lines indicate the equilibrium values of these variables associated with the *high* (*low*) trading intensity equilibrium. As one can see, along

[16] The parameter values are $\tau_v = \tau_{\epsilon_1} = \tau_u = \gamma = 1$.

the low trading intensity equilibrium, an increase in the size of the short term trading sector leads to an less liquid market while the reverse happens along the high trading intensity equilibrium. The intuition is that for the chosen parameter values, the positive effect on risk sharing overcomes the negative effect on adverse selection due to an increase in the speculative activity of short term traders. As a consequence, and along the high trading intensity equilibrium, as the presence of short termists increases, the market becomes deeper. In other words, long termists have a negative effect on market depth. The opposite happens along the *low* trading intensity equilibrium. In this case as the market becomes increasingly concentrated in the hands of short termists, first period market depth decreases. In the right panel, I set $\tau_v = 10$ and $\tau_{\epsilon_1} = .1$, leaving the value of the other parameters unchanged. In this case, the quality of prior information that traders have about the asset has improved. As a consequence, inventory effects are less important than adverse selection. Thus, an increase in the size of the short term trading sector leads to a thinner market both along the *high* and the *low* trading intensity equilibrium. Notice that in both simulations, the volatility of returns increases for both parameter configurations. The reason for this result is as follows. Returns' volatility is the product of the reduction in the asset conditional variance due to information arrival and the ratio $(1/\alpha)^2$. Along the *high* trading intensity equilibrium $\alpha > 1$, thus prices underreact to information arrival. However, $\tau_1^{-1} - \tau_2^{-1}$ is high. The second effect overcomes the first and volatility increases with μ. Along the *low* trading intensity equilibrium, prices overreact to information arrival while the reduction in the asset conditional variance is low. In this case, the first effect dominates.

Next, consider volume. [17] In figure, 2 I plot short and long term traders' volume as a function of μ along the high and low trading intensity equilibrium. [18] There are two effects to notice. First of all: volume is higher along the high trading intensity equilibrium for both classes of traders. The intuition is that lower second period volatility boosts market making activity in the first period and leads to higher volume both for short and long term traders. Next, notice that as μ increases, volume decreases for both traders' type along the high trading intensity equilibrium while along the low trading intensity equilibrium it increases for long term traders and decreases for short termists. The idea is as follows. In the high trading intensity equilibrium, price informativeness increases with the size of the short term trading sector, hence as μ grows, less speculative opportunities are left to traders in the market, and this reduces their activity. In the low trading intensity equilibrium, higher returns' volatility makes

[17] Volume is defined as the expected value of the absolute change in short and long term traders' positions across periods 1 and 2, i.e. $E[\mu|x_{js,2} - x_{js,1}| + (1-\mu)|x_{il,2} - x_{il,1}|]$, see He and Wang (1995).

[18] Parameter values are as follows: $\tau_u = .1, \tau_v = 1, \tau_{\epsilon_1} = 1, \tau_{\epsilon_2} = 0$ and $\gamma = 1$. I indicate with $STV1$ the volume of short term traders in the high trading intensity equilibrium, with $STV2\ldots$

short term traders unwilling to take large positions, however, long term traders can benefit from a less informative price sequence and speculate more intensively on their information. In figure 3, I plot aggregate volume which, as a result of the effects discussed above, is decreasing along the high volatility equilibrium and hump shaped along the low trading intensity equilibrium.

5 Conclusions

In this paper, I have analyzed the effects of myopic behavior on stock market patterns in the context of a dynamic rational expectations equilibrium model. Owing to inventory effects, short term horizons induce equilibrium multiplicity. In particular two different outcomes are possible: in the first one, short term traders anticipate second period price overreaction to public information, scale back their trades and enforce a high volatility, low price informativeness equilibrium; in the second the opposite happens and a low volatility, high price informativeness equilibrium arises. In a market with heterogeneous horizons the effect on returns' volatility produces an externality on long term traders' inventory decisions. Along the *high* trading intensity equilibrium, an increase in the size of the short term trading sector, promotes a deeper market, while along the *low* trading intensity equilibrium the opposite happens (provided that inventory risk effects are stronger than adverse selection ones). Finally, owing to higher price informativeness, when the size of the short term trading sector grows, volume decreases in the high trading intensity equilibrium and is hump shaped in the low trading intensity one.

A number of issues are left for future research. First of all, the N-period extension should be considered. While analytical results exist for the case of long term traders only (He and Wang (1995)), there is no general analysis for the case presented here. Next, introducing *hedgers*, welfare considerations could be addressed. Dow and Rahi (1999), show that in a static context, a tax on speculation, by reducing the informativeness of the price, can improve both *hedgers* and speculators' welfare. In the present context, the final effect should depend on the structure of the equilibrium set. Effects on investment decisions could also be considered, to the extent that stock market prices accomplish at the same time the role of indicators for firms' decisions and aggregate information dispersed among traders in the economy, equilibria with low price informativeness, should lead to sub-optimal decisions. Finally, by considering a multi-asset framework (Admati (1985), Cespa (1999)), one could characterize how inventory effects interact across different assets.

Figure 1: First period market depth and return volatility with concentrated arrival of information, $l_1 = \lambda_1$, $m = \mu$.

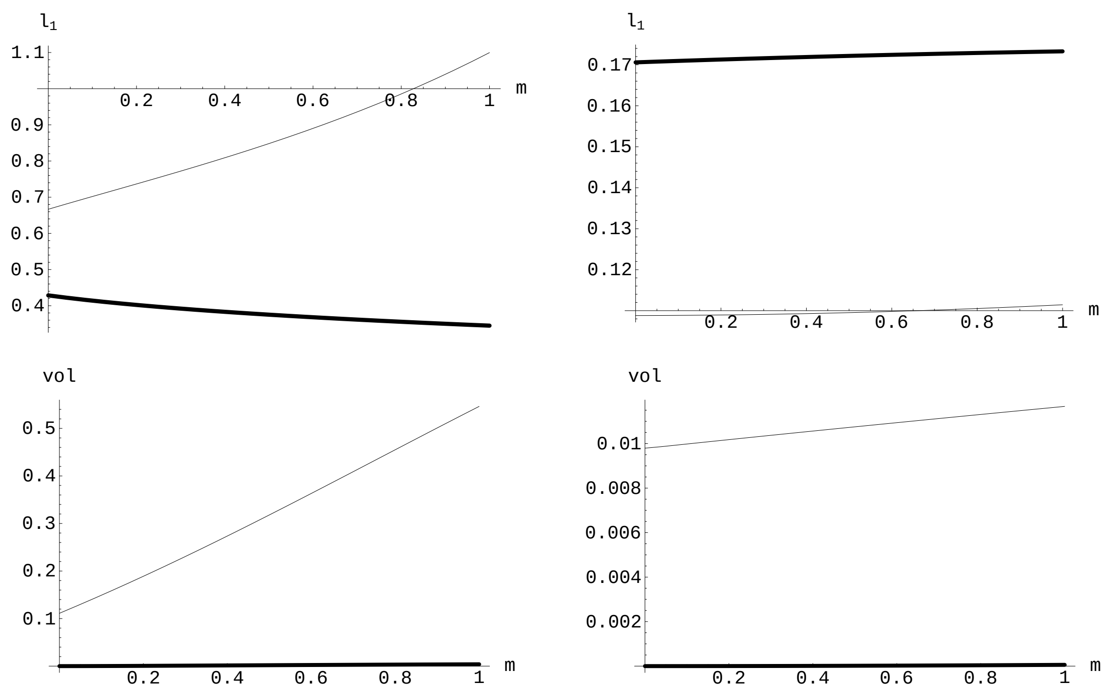


Figure 2: Short and Long term traders volume with concentrated arrival of information, m = μ.

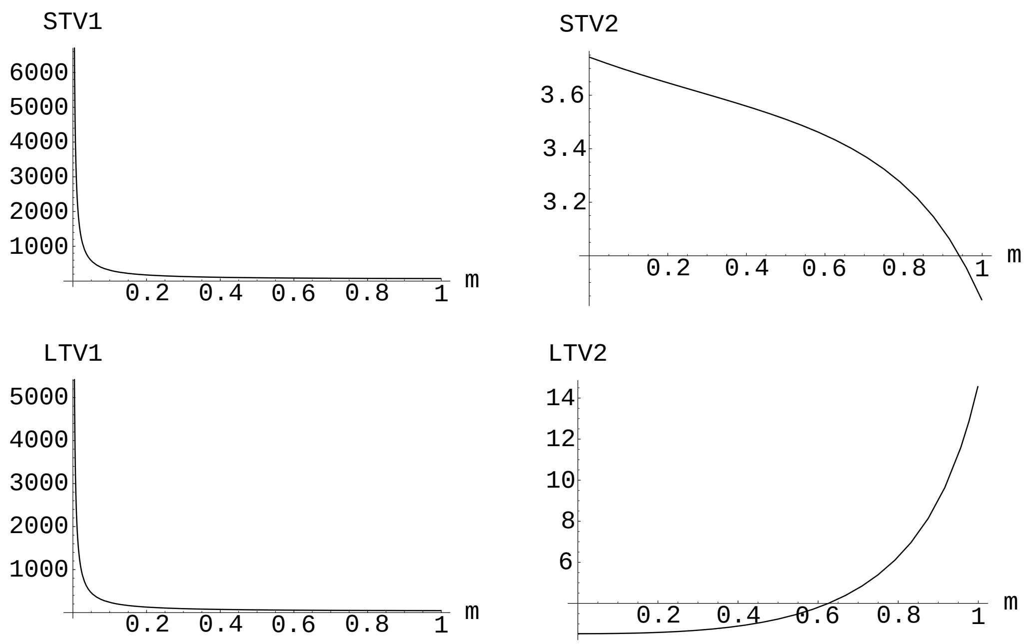
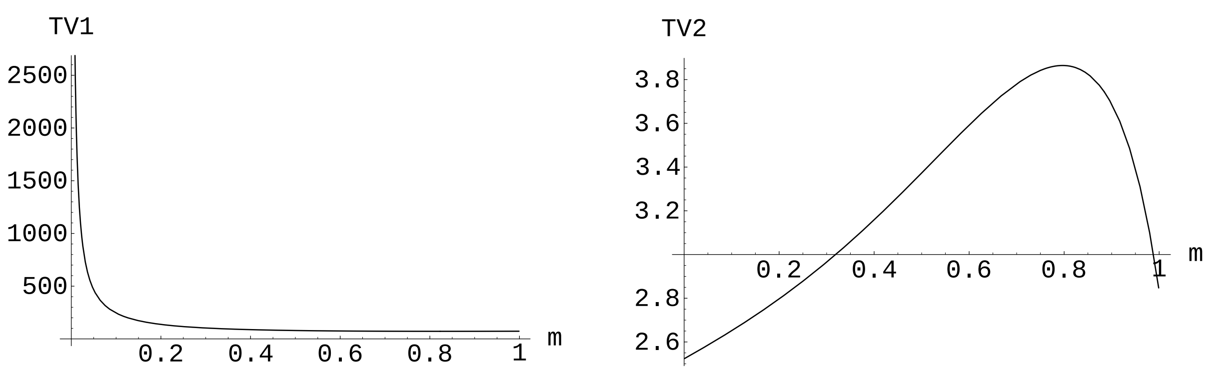


Figure 3: Aggregate trading volume with concentrated arrival of information, m = μ.



6 APPENDIX

First of all, I state a well known result on multivariate normal random variables (see e.g. Danthine and Moresi (1992)).

Lemma 1 *Let $Q(\boldsymbol{w})$ be a quadratic function of the vector $\boldsymbol{w}$: $Q(\boldsymbol{w}) = D + \mathbf{b}'\boldsymbol{w} - \boldsymbol{w}'\mathbf{A}\boldsymbol{w}$, where $\boldsymbol{w} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma}$ is non singular. We then have*

$$E[\exp(Q(\boldsymbol{w}))] = |\boldsymbol{\Sigma}|^{-1/2}|2\mathbf{A} + \boldsymbol{\Sigma}^{-1}|^{-1/2}\times$$
$$\exp\left\{D + \mathbf{b}'\boldsymbol{\mu} + \boldsymbol{\mu}'\mathbf{A}\boldsymbol{\mu} + \frac{1}{2}(\mathbf{b} - \mathbf{A}\boldsymbol{\mu})'(2\mathbf{A} + \boldsymbol{\Sigma}^{-1})^{-1}(\mathbf{b} - 2\mathbf{A}\boldsymbol{\mu})\right\}.$$

Proof of Proposition 1

Notice that in every linear equilibrium, the sequences $\{z^n\}$ and $\{p^n\}$ are informationally equivalent. By induction, in period 1 linearity of the equilibrium price makes p_1 o.e. to z_1. Given that, in period 2 $\{p_1, p_2\}$ is o.e. to $\{z_1, z_2\}$. Therefore in period 2 traders can condition their demands on the sequence $\{z^2\}$.

Next, in period $n = 2$ because of normality of the random variables and CARA utility functions, traders have a mean-variance demand schedule, therefore

$$x_{i,2} = \gamma^I \left(\text{Var}\left[v|s_i^2, z^2\right]\right)^{-1}\left(E\left[v|s_i^2, z^2\right] - p_2\right).$$

Given the fact that $\{z^2\}$ and $\{p^2\}$ are o.e. traders' expectations are given by

$$E\left[v|z^2\right] = \tau_2^{-1}\left(\tau_v\bar{v} + \tau_u\sum_{t=1}^{2}(\Delta a_t)z_t\right)$$

and

$$E\left[v|s_i^2, z^2\right] = \tau_{i2}^{-1}\left(\tau_2 E\left[v|z^2\right] + \sum_{t=1}^{2}\tau_{\epsilon_t}\tilde{s}_{i,2}\right),$$

where $\tau_2 = \tau_v + \tau_u\sum_{t=1}^{2}(\Delta a_t)^2$, and $\tau_{i2} = \tau_2 + \sum_{t=1}^{2}\tau_{\epsilon_t}$. Therefore, one can solve for traders' second period strategies and obtain

$$x_{i,2} = \gamma\left(\sum_{t=1}^{2}\tau_{\epsilon_t}\right)(\tilde{s}_{i,2} - p_2) + \gamma\tau_2(E[v|z^2] - p_2). \tag{8}$$

The second period market clearing equation reads as follows

$$\int_0^1 x_{i,2}\,di + u_1 + u_2 = 0, \tag{9}$$

where u_1 and u_2 are the (uncorrelated) supply increments of the traded asset. Because of the convention that the average signal reveals the *true* value of the asset payoff, (9) becomes

$$a_2(v - p_2) + \gamma\tau_2(E[v|z^2] - p_2) + u_1 + u_2 = 0,$$

adding and subtracting $a_1 v$ gives

$$z_1 + z_2 + \gamma\tau_2 E[v|z^2] = (a_2 + \gamma\tau_2)p_2,$$

and, by using the previously given definitions,

$$p_2 = (1 - \lambda_2 \Delta a_2)(1 - \lambda_{1l} a_1)\bar{v} + (1 - \lambda_2 \Delta a_2)\lambda_{1l} z_1 + \lambda_2 z_2.$$

where $\lambda_2 = (1 + \gamma\tau_u \Delta a_2)/(a_2 + \gamma\tau_2)$ and $\lambda_{1l} = (1 + \gamma\tau_u a_1)/(a_1 + \gamma\tau_1)$. To obtain period 1 strategies, one substitutes period 2 strategy into the objective function of the informed, obtaining

$$E\left[-\exp\left\{-(\gamma)^{-1}\pi_{i,2}\right\}|\tilde{s}_{i,2}, z^2\right] = -\exp\left\{\frac{(x_{i,2})^2}{2(\gamma)^2\tau_{i2}}\right\}.$$

Going back one step, the function that informed speculators maximize is

$$E\left[-\exp\left\{-(\gamma)^{-1}Q_{i,1}\right\}|s_{i,1}, z_1\right],$$

where

$$Q_{i,1} = (p_2 - p_1)x_{i,1} + \frac{(x_{i,2})^2}{2\gamma\tau_{i2}}.$$

Applying lemma 1 as shown in Holden and Subrahmanyam (1996), the above optimization problem is solved by

$$x_{i,1} = \frac{\gamma(E\left[p_2|z_1, s_{i,1}\right] - p_1)}{G_1} + E\left[x_{i,2}|z_1, s_{i,1}\right]\frac{G_1 - G_2}{G_1}, \tag{10}$$

where G_1 and G_2 are the elements in the first row of the matrix

$$G = \left(\left(\text{Var}\left[p_2, E\left[v|\tilde{s}_{i,2}, z^2\right]|z_1, s_{i,1}\right]\right)^{-1} + \begin{pmatrix} \tau_{i2} & -\tau_{i2} \\ -\tau_{i2} & \tau_{i2} \end{pmatrix}\right)^{-1}.$$

An explicit expression for the matrix $\text{Var}\left[p_2, E\left[v|\tilde{s}_{i,2}, z^2\right]|z_1, s_{i,1}\right]$ (which can be easily checked to be non singular) is

$$\text{Var}[p_2, E[v|\tilde{s}_{i,2}, z^2]|z_1, s_{i,1}] = \begin{pmatrix} \lambda_2^2 \frac{\tau_2 + \tau_{\epsilon 1}}{\tau_{i1}\tau_u} & \frac{\lambda_2 \Delta a_2}{\tau_{i1}} \\ \frac{\lambda_2 \Delta a_2}{\tau_{i1}} & \frac{\tau_{i2} - \tau_{i1}}{\tau_{i2}\tau_{i1}} \end{pmatrix}.$$

Tedious calculations allow to obtain

$$G_1 = \frac{\lambda_2^2}{\tau_{i1}\lambda_2^2 + \tau_u(1-\lambda_2\Delta a_2)^2},$$

$$G_2 = \frac{\tau_u\Delta a_2\lambda_2 + \tau_{\epsilon_2}\lambda_2^2}{\tau_{i2}(\tau_{i1}\lambda_2^2 + \tau_u(1-\lambda_2\Delta a_2)^2)},$$

$$\frac{G_1 - G_2}{G_1} = \frac{(1-\alpha)\tau_2 + \tau_{\epsilon_1}}{\tau_{i2}}.$$

$$E[p_2|s_{i,1}, z_1] = \lambda_2\Delta a_2 E[v|s_{i,1}, z_1] + (1-\lambda_2\Delta a_2)p_{1l}, \tag{11}$$

and

$$E[x_{i,2}|s_{i,1}, z_1] = \gamma\tau_{i2}(1-\lambda_2\Delta a_2)(E[v|s_{i,1}, z_1] - p_{1l}), \tag{12}$$

where $p_{1l} = \lambda_{1l}z_1 + (1-\lambda_{1l}a_1)\bar{v}$.

Identifying parameters, informed first period trading intensity is given by $a_1 = \gamma\tau_{\epsilon_1}$. QED

Proof of Proposition 3

Given that in period 2 the horizon of short and long term traders coincide, $x_{is,2}$ and $x_{il,2}$ are given by (8), where $a_2 = \gamma(\sum_{t=1}^{2}\tau_{\epsilon_t})$. Imposing market clearing,

$$\int_0^\mu x_{is,2}\,di + \int_\mu^1 x_{il,2}\,di + u_1 + u_2 = 0.$$

The second period market clearing price and depth are given by $p_2 = \lambda_2 z_2 + (1-\lambda_2\Delta a_2)p_{1l}$ and $\lambda_2 = (a_2 + \gamma\tau_2)^{-1}(1+\gamma\tau_u\Delta a_2)$.

In the first period, trading horizons differ. Therefore, the market clearing equation reads as follows

$$\int_0^\mu x_{is,1}\,di + \int_\mu^1 x_{il,1}\,di + u_1 = 0.$$

Long term traders' first period strategy is given by (10). For short term traders, consider the following argument. Because of short term horizons,

$$x_{is,1} = \gamma(\mathrm{Var}[p_2|s_{i,1}, p_1])^{-1}(E[p_2|s_{i,1}, p_1] - p_1), \tag{13}$$

where

$$\mathrm{Var}[p_2|s_{i,1}, p_1] = \lambda_2^2\left(\frac{\tau_2 + \tau_{\epsilon_1}}{\tau_{i1}\tau_u}\right).$$

Using the formulas derived in (11–12) and plugging them in (13) gives

$$x_{is,1} = a_{1s}(s_{i,1} - p_1) + \gamma\rho\tau_1(E[v|z_1] - p_1) + \left(\frac{\gamma\rho\tau_{i1}(1-\lambda_2\Delta a_2)}{\lambda_2\Delta a_2}\right)(p_{1l} - p_1), \tag{14}$$

Collecting parameters and identifying the unknown gives $a_{1l} = \gamma\tau_{\epsilon_1}$,

$$a_{1s} = \gamma\alpha(\tau_{\epsilon_1}^{-1} + \tau_2^{-1})^{-1}, \tag{15}$$

$$\begin{aligned} \lambda_1 &= \left(\frac{\lambda_2^2\Delta a_2(1+(\mu\rho+(1-\mu))\gamma a_1\tau_u)}{\lambda_2\left(\mu\rho\gamma\tau_{i1}+(1-\mu)(\gamma\tau_2\phi+a_{1l}\lambda_2\Delta a_2)\right)}\right) \\ &+ \mu\lambda_{1l}\left(\frac{(1-\lambda_2\Delta a_2)\lambda_2(a_{1s}+\gamma\rho\tau_1)}{\lambda_2\left(\mu\rho\gamma\tau_{i1}+(1-\mu)(\gamma\tau_2\phi+a_{1l}\lambda_2\Delta a_2)\right)}\right) \\ &+ (1-\mu)\lambda_{1l}\left(\frac{\gamma\Delta a_2\tau_u(1-\lambda_2\Delta a_2)^2}{\lambda_2\left(\mu\rho\gamma\tau_{i1}+(1-\mu)(\gamma\tau_2\phi+a_{1l}\lambda_2\Delta a_2)\right)}\right), \end{aligned} \tag{16}$$

and

$$\begin{aligned} c_1 &= \bar{v}\left(\frac{\lambda_2^2\Delta a_2(\mu\rho+(1-\mu))\gamma\tau_v}{\lambda_2\left(\mu\rho\gamma\tau_{i1}+(1-\mu)(\gamma\tau_2\phi+a_{1l}\lambda_2\Delta a_2)\right)}\right) \\ &+ \mu(1-a_1\lambda_{1l})\bar{v}\left(\frac{(1-\lambda_2\Delta a_2)\lambda_2(a_{1s}+\gamma\rho\tau_1)}{\lambda_2\left(\mu\rho\gamma\tau_{i1}+(1-\mu)(\gamma\tau_2\phi+a_{1l}\lambda_2\Delta a_2)\right)}\right) \\ &+ (1-\mu)(1-a_1\lambda_{1l})\bar{v}\left(\frac{\gamma\Delta a_2\tau_u(1-\lambda_2\Delta a_2)^2}{\lambda_2\left(\mu\rho\gamma\tau_{i1}+(1-\mu)(\gamma\tau_2\phi+a_{1l}\lambda_2\Delta a_2)\right)}\right), \end{aligned} \tag{17}$$

where $\phi = \lambda_2\Delta a_2(1-\alpha) + (1-\lambda_2\Delta a_2)\alpha$. Existence of an equilibrium, depends on the existence of a real solution to (15). Suppose that at equilibrium

$$a_{1s} \neq \frac{1}{\gamma\mu\tau_u} + \frac{a_2-(1-\mu)a_{1l}}{\mu},$$

then solving (15) is equivalent to solve

$$F(a_{1s}) = a_{1s}(\tau_2+\tau_{\epsilon_1})(1+\gamma\tau\Delta a_2) - \gamma\tau_{\epsilon_1}(a_2+\gamma\tau_2) = 0.$$

From here, one can check that

$$F\left(\frac{1}{\gamma\mu\tau_u} + \frac{a_2-(1-\mu)a_{1l}}{\mu}\right) = \frac{\tau_{\epsilon_1}(2+3a_2\gamma\tau_u+a_2^2\gamma^2\tau_u^2+\gamma^2\tau_u\tau_v)}{\gamma\tau_u^2} > 0.$$

Next,

$$\begin{aligned} F(0) = -\gamma\tau_{\epsilon_1}(a_2-(1-\mu)\tau_{\epsilon_1})\times \\ (a_2+\gamma(\gamma^2(1-\mu)^2\tau_{\epsilon_1}^2\tau_u+(a_2-\gamma(1-\mu)\tau_{\epsilon_1})^2\tau_u+\tau_v)) < 0. \end{aligned}$$

Given that $F(\cdot)$ is continuous, there exists a a_{1s}^* in the interval $(0, 1/\gamma\mu\tau_u + (a_2-(1-\mu)a_{1l})/\mu)$, such that $F(a_{1s}^*) = 0$.

QED

Proof of proposition 4

Part 1 and 2 directly follow from the proof of proposition 3. For part 3 consider the following argument. The equilibrium condition in the short term trading case can be rewritten as follows

$$a_{1s} = \frac{\gamma \Delta a_2 \tau_u (\gamma \tau_2 + a_2) \tau_{\epsilon_1}}{(\tau_2 + \tau_{\epsilon_1})(1 + \gamma \tau_u \Delta a_2)}.$$

Assume that the denominator in the r.h.s. of the previous equation is non null (at equilibrium), then rearranging we obtain the following equilibrium condition

$$a_{1s}(\tau_2 + \tau_{\epsilon_1})(1 + \gamma \tau_u \Delta a_2) - \gamma \Delta a_2 \tau_u \tau_{\epsilon_1}(a_2 + \gamma \tau_2) = 0,$$

which is a quartic equation in a_{1s}. If $\tau_{\epsilon_2} = 0$, the equilibrium condition becomes

$$- \underbrace{(a_{1s}^2 \gamma \tau_u - a_{1s}(1 + 2\gamma^2 \tau_{\epsilon_1} \tau_u) + \gamma^3 \tau_{\epsilon_1}^2 \tau_u)}_{(a)} \times \underbrace{(2a_{1s}^2 \tau_u - 2a_{1s} \gamma \tau_{\epsilon_1} \tau_u + \tau_v + \gamma^2 \tau_{\epsilon_1} \tau_u + \tau_{\epsilon_1})}_{(b)} = 0.$$

Checking the solution of the two quadratic equations in parenthesis, shows that while equation (a) has two real roots, equation (b) only possesses imaginary solutions. In particular

$$a_{11s}, a_{12s} = \frac{(1 + 2\gamma^2 \tau_{\epsilon_1} \tau_u) \pm \sqrt{1 + 4\gamma^2 \tau_{\epsilon_1} \tau_u}}{2\gamma \tau_u}.$$

This solution clearly satisfies the condition $1 + \gamma \tau_u \Delta a_2 \neq 0$.

Direct comparison of the solutions found with the long term case, gives that

$$a_{12s} < a_{1l} < a_{11s},$$

and this, since $(\tau_{\epsilon_1}^{-1} + \tau_2^{-1})^{-1} < \tau_{\epsilon_1}$, immediately implies that $\alpha(a_{11s}) > 1$. To see that $\alpha(a_{12s}) < 1$, consider that because of the definition of α

$$\alpha < 1 \Leftrightarrow 1 - \frac{\Delta a_2 a_1 \tau_u}{\tau_1} > 0,$$

plugging a_{12s} into the above condition and rearraging, gives the result.

QED

Proof of proposition 5

From the price formula obtained in proposition 4, it is easy to see that

$$\lambda_{1s} = (1 - \lambda_2 \Delta a_2)\lambda_{1l} + \lambda_2 \Delta a_2 \left(\frac{1 + \gamma\rho a_{1s}\tau_u}{\gamma\rho\tau_{i1}} \right).$$

Given this, it is possible to express $\lambda_{1s} - \lambda_{1l}(a_{1l})$ as follows

$$\begin{aligned}\lambda_{1s} - \lambda_{1l}(a_{1l}) &= \frac{a_{1l}(1-\rho)(1+\gamma\tau_u a_{1l}(1+\rho)) + \gamma^2\tau_u(1-\rho)a_{1l}(a_{1s}a_{1l}\tau_u - \tau_v)}{(a_{1s}+\gamma\tau_{1s})(a_{1l}+\gamma\tau_{1l})} \\ &+ \frac{\lambda_2\Delta a_2\gamma\tau_v(1-\rho)}{(a_{1s}+\gamma\rho\tau_{1s})(a_{1s}+\gamma\tau_{1s})}.\end{aligned}$$

Now, if

$$a_{1s} > \frac{1 - \beta_{v|z_{1l}}}{\beta_{v|z_{1l}}},$$

the above expression is positive iff $\rho > 1$. Hence, the result follows.

To see that market reaction to information arrival is negative in the second period when $a_{1s} = a_{11s}$, notice that the numerator of $\lambda_2(a_{11s})$ is given by

$$1 + \gamma\tau_u(a_{1l} - a_{11s}),$$

rearranging it is easy to obtain that

$$\lambda_2(a_{11s}) < 0 \Leftrightarrow 2\gamma^2\tau_{\epsilon_1}\tau_u > 0,$$

which is always satisfied.

Finally, given that $a_{12s} < a_{1l}$, the numerator of $\lambda_2(a_{12s})$, is positive, hence $\lambda_2(a_{12s}) > 0$.

Finally, to see that market depth is higher along the *high* trading intensity equilibrium, substitute equilibrium values in the expression for λ_2, then

$$|\lambda_2(a_{11s})| < \lambda_2(a_{12s}) \Leftrightarrow \lambda_2(a_{11s})^2 < \lambda_2(a_{12s})^2.$$

Clearly, the denominator of $\lambda_2(a_{11s})^2 - \lambda_2(a_{12s})^2$, is positive, therefore one only needs to check the numerator of this expression which is given by

$$- \sqrt{1 + 4\gamma^2\tau_{\epsilon_1}\tau_u} \times$$
$$(4 + 5\gamma^8\tau_{\epsilon_1}^4\tau_u^4 + 4\gamma^2\tau_u(7\tau_{\epsilon_1} + \tau_v) + 2\gamma^6\tau_{\epsilon_1}^2\tau_u^3(17\tau_{\epsilon_1} + 3\tau_v) + \gamma^4\tau_u^2(57\tau_{\epsilon_1}^2 + 14\tau_{\epsilon_1}\tau_v + \tau_v^2)),$$

which is always negative. QED

Proof of proposition 6

For part 1, if $\tau_{\epsilon_2} = 0$, then $\tau_2(a_{1l}) = \tau_1(a_{1l})$. If $\rho > 1$ direct substitution of the equilibrium result gives $\tau_1(a_{11s}) > \tau_1(a_{1l})$. Since $\tau_2(a_{11s}) = \tau_1(a_{11s}) + (\Delta a_{2s})^2\tau_u$, it follows that $\tau_1(a_{1l}) < \tau_2(a_{11s})$. Next, if $\rho < 1$, then since $(a_{1l}+\Delta a_{2l})^2 \geq a_{1l}^2+(\Delta a_{2l})^2 > a_{1s}^2 + (\Delta a_{2s})^2$, it follows that $\tau_2(a_{1l}) > \tau_2(a_{1s})$.

For part 2. Substitute the equilibrium values found in proposition 4 and compute $\text{Var}[p_2|p_1; a_{11s}] - \text{Var}[p_2|p_1; a_{12s}]$. Given that that the denominator of the difference is positive, one only need to check the numerator which is given by

$$\begin{aligned}
&-\sqrt{1+4\gamma^2\tau_{\epsilon_1}\tau_u}\,\big(5\gamma^{14}\tau_{\epsilon_1}^8\tau_u^7 + 4\tau_v + 8\gamma^{12}\tau_{\epsilon_1}^6\tau_u^6(5\tau_{\epsilon_1}+2\tau_v) + \\
&\qquad \gamma^6\tau_u^3(113\tau_{\epsilon_1}^4 + 360\tau_{\epsilon_1}^3\tau_v + 132\tau_{\epsilon_1}^2\tau_v^2 + 18\tau_{\epsilon_1}\tau_v^3 + \tau_v^4) + \\
&(36\tau_{\epsilon_1}^3 + 185\tau_{\epsilon_1}^2\tau_v + 58\tau_{\epsilon_1}\tau_v^2 + 5\tau_v^3) + \gamma^8\tau_{\epsilon_1}^2\tau_u^4(156\tau_{\epsilon_1}^3 + 309\tau_{\epsilon_1}^2\tau_v + 92\tau_{\epsilon_1}\tau_v^2 + 8\tau_v^3) \\
&\quad +4\gamma^2\tau_u(\tau_{\epsilon_1}^2 + 11\tau_{\epsilon_1}\tau_v + 2\tau_v^2) + 2\gamma^{10}\tau_{\epsilon_1}^4\tau_u^5(55\tau_{\epsilon_1}^2 + 57\tau_{\epsilon_1}\tau_v + 9\tau_v^2) + \gamma^4\tau_u^2\big),
\end{aligned}$$

which can be easily checked to be negative. QED

Proof of corollary 2

Suppose that the denominator in the equation defining the first period trading intensity of short term traders is non null. This implies assuming that at equilibrium $\mu \neq 1/(\gamma\tau_u(a_{1l} - a_{1s}))$. Then, the equilibrium condition can be rewritten as follows:

$$a_{1s}(1+\gamma\tau_u\Delta a_2)(\tau_2+\tau_{\epsilon_1}) - \gamma\Delta a_2\tau_u\tau_{\epsilon_1}(a_2+\gamma\tau_2) = 0, \tag{18}$$

under the assumption that $\tau_{\epsilon_2} = 0$, this equation can be factored in the following way

$$\begin{aligned}
&-\underbrace{(a_{1s}^2\gamma\mu\tau_u - a_{1s}(1+2\gamma^2\mu\tau_{\epsilon_1}\tau_u) + \gamma^3\mu\tau_{\epsilon_1}^2\tau_u)}_{(a)} \\
&\times\ \underbrace{(2a_{1s}^2\mu^2\tau_u + 2a_{1s}\gamma\tau_{\epsilon_1}\tau_u(1-2\mu)\mu + \tau_{\epsilon_1} + \gamma^2(1-2\mu+2\mu^2)\tau_{\epsilon_1}^2\tau_u + \tau_v)}_{(b)} = 0.
\end{aligned}$$

The discriminant of the quadratic equation (a) is positive, while the one of equation (b) is negative. Therefore, there exists two real roots of equation (18), i.e. there are two equilibria in this case.

It is easy then to see that

$$a_{11s}, a_{12s} = \frac{(1+2\gamma^2\mu\tau_{\epsilon_1}\tau_u) \pm \sqrt{1+4\mu\gamma^2\tau_{\epsilon_1}\tau_u}}{2\gamma\mu\tau_u},$$

and checking the existence condition, we see that $\mu \neq 1/(\gamma\tau_u(a_{1l} - a_{1s}))$ if and only if $\mu > 0$, which will always be the case in this model. Finally, direct comparison of a_{11s}, a_{1l} and a_{12s} show that $a_{12s} < a_{1l} < a_{11s}$.

To check the sign of second period market depth, remember that $a_{12s} < a_{1l}$. Therefore, since in this case $\Delta a_2 = \mu(a_{1l} - a_{12s})$, the result follows. If $\rho > 1$, it is easy to see that the numerator of $\lambda_2(a_{11s})$ will be positive if and only if $4\gamma^2\mu\tau_{\epsilon_1}\tau_u \leq 0$ which can happen if and only if $\mu = 0$, a condition ruled out by the solution of the equilibrium equation.

The last part of the proposition follows from the proof of proposition 6.

QED

Proof of Proposition 7

Notice that $p_{1l} - p_1 = (\lambda_{1l} - \lambda_1)z_1 + \psi(\bar{v})$, where $\psi(\bar{v}) = (1 - a_1\lambda_{1l})\bar{v} - c_1$. Computing

$$\lambda_{1l} - \lambda_1 = \frac{\lambda_{1l}}{D}\Big(D - \lambda_2^2\Delta a_2(a_1 + \gamma\tau_1) - \mu\lambda_2(1 - \lambda_2\Delta a_2)\gamma\rho\tau_{i1} - (1-\mu)\gamma\Delta a_2\tau_u(1 - \lambda_2\Delta a_2)^2\Big) - \frac{\lambda_2^2\Delta a_2(\rho - 1)\gamma a_1\tau_u}{D},$$

where $D = \lambda_2(\mu\rho\gamma\tau_{i1} + (1-\mu)(\gamma\tau_2\phi + a_{1l}\lambda_2\Delta a_2))$. Manipulating the above formula,

$$\lambda_{1l} - \lambda_1 = \frac{\lambda_2^2\Delta a_2\mu\rho(\rho - 1)\tau_v}{D(a_1 + \gamma\tau_1)},$$

and similarly one can show

$$\psi(\bar{v}) = -\frac{\lambda_2^2\Delta a_2\mu\rho(\rho - 1)\tau_v a_1\bar{v}}{D(a_1 + \gamma\tau_1)}.$$

Therefore,

$$p_{1l} - p_1 = \frac{\lambda_2^2\Delta a_2(1 - \lambda_{1l}a_1)}{D}\mu\rho(\rho - 1)(z_1 - E[z_1]).$$

This establishes point 1. The second part is a straigthforward application of this result to (2) and (3). Finally,

$$K_1 = \frac{\rho\gamma\Delta a_2\tau_u(1 - \lambda_2\Delta a_2)^2(1 - \lambda_{1l}a_1)}{D},$$

$$K_2 = \frac{\rho^2\gamma\tau_{i1}\lambda_2(1 - \lambda_2\Delta a_2)(1 - \lambda_{1l}a_1)}{D},$$

and it is easy to check that both K_1 and K_2 are positive. QED

Proof of Proposition 8

Differentiating a_{11s} w.r.t. μ,

$$\frac{\partial a_{11s}}{\partial\mu} = \frac{\gamma\tau_{\epsilon_1}}{\sqrt{1 + 4\mu\gamma^2\tau_{\epsilon_1}\tau_u}} > 0,$$

and in a similar way it can be proven that $(\partial a_{11s}/\partial\mu) < 0$.

Next, to check that first period price informativeness increases in μ along the *high* short-term trading equilibrium, notice that

$$\frac{\partial\tau_1}{\partial\mu} = 2a_1\tau_u\frac{\partial a_1}{\partial\mu},$$

differentiating a_1,

$$\frac{\partial a_1}{\partial\mu} = (a_{1s} - a_{1l}) + \mu\frac{\partial a_{1s}}{\partial\mu},$$

and substitution of the derivatives determined above, gives the result. Turning attention to second period price informativeness,

$$\frac{\partial\tau_2}{\partial\mu} = \frac{\partial\tau_1}{\partial\mu} - 2\tau_u\Delta a_2\left(\frac{\partial a_{1s}}{\partial\mu}\right),$$

this expression is clearly positive if $\rho > 1$, since in this case, both first period price informativeness and short term traders trading intensity increase in μ while $\Delta a_2 < 0$; however, in the case $\rho < 1$, the result is not evident. Computing the derivative,

$$\frac{\partial\tau_2}{\partial\mu} = -\frac{2\tau_{\epsilon_1}\left(1 + \gamma^2\tau_{\epsilon_1}\tau_u - \sqrt{1 + 4\mu\gamma^2\tau_{\epsilon_1}\tau_u}\right)}{\sqrt{1 + 4\mu\gamma^2\tau_{\epsilon_1}\tau_u}},$$

which is positive if and only if $\mu \in (0, 1/2 + \gamma a_{1l}\tau_u/4)$ and negative otherwise. QED

References

Admati, A. and P. Pfleiderer (1991). Sunshine trading and financial market equilibrium. *Review of Financial Studies 4*, 443–481.

Admati, A. R. (1985). A noisy rational expectations equilibrium for multi-asset securities markets. *Econometrica 53*, 629–657.

Brown, D. P. and R. H. Jennings (1989). On technical analysis. *Review of Financial Studies 2*(4), 527–551.

Cespa, G. (1999). *Three Essays in Market Mictrostructure Theory.* Ph. D. thesis, UAB.

Danthine, J. and S. Moresi (1992). Volatility, information and noise trading. *European Economic Review 37*, 961–982.

Dennert, J. (1991). Insider trading and the cost of capital in a multi-period economy. *LSE Financial Market Group Discussion Paper 128*.

Dow, J. and R. Rahi (1999). Should speculators be taxed? *Journal of Business*. forthcoming.

Friedman, B. M. (1996, Spring). Economic implications of changing share ownership. *Journal of Portfolio Management*, 59–70.

Gompers, P. A. and A. Metrick (1998). Institutional investors and equity prices. *NBER Working Paper 6723*.

Grundy, B. D. and M. M. Nichols (1989). Trade and the revelation of information through prices and direct disclosure. *Review of Financial Studies 2*(4), 495–526.

He, H. and J. Wang (1995). Differential information an dynamic behavior of stock trading volume. *Review of Financial Studies 8*(4), 919–972.

Holden, C. W. and A. Subrahmanyam (1996). Risk aversion, liquidity and endogenous short horizons. *Review of Financial Studies 2*, 691–722.

Jacobs, M. (1991). *Short-Term America: The Causes and Cures of Our Business Myopia*. Boston: Harvard Business School Press.

Kahn, C. and A. Winton (1998). Ownership structure, speculation and shareholder intervention. *Journal of Finance 53*.

Pagano, M. (1989). Endogenous market thinnes and stock-price volatility. *Review of Economic Studies 56*, 269–288.

Sias, R. (1996). Volatility and the institutional investor. *Financial Analyst Journal*, 13–20. March-April.

Subrahmanyam, A. (1991). Risk aversion, market liquidity, and price efficiency. *Review of Financial Studies 4*, 417–441.

Tirole, J. (2000). Corporate governance. *Working Paper*.

Vives, X. (1995). Short-term investment and the informational efficiency of the market. *Review of Financial Studies 8*, 125–160.

Wermers, R. (1998). Mutual fund herding and the impact on stock prices. *Journal of Finance*. forthcoming.