The Neo-Ricardian Critique: An Anniversary Assessment*

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CONTENTS

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Introduction

The years between 1998 and 2003 marked several anniversaries related to the controversies on capital theory that climaxed with the debate on reswitching one generation ago. As a consequence, there have been a number of meetings, collective volumes and review articles, all devoted to remember, document, and assess that old quarrel. Yet, they have been almost exclusively by people whose heart is still with the "losing" Sraffian side[1]. What appears slightly strange is not that a few elderly economists may still be clutching to outmoded ideas. Rather, it is the tight silence by the "winning" mainstream side. One might assume that after about forty years there should be a chance to look back at those angry exchanges with some degree of detachment. Indeed, it is hard to believe that an argument of that intensity may have lasted for so long, involving some of the best brains of the profession, without producing any new knowledge at all. The question is thus not just one of historical interest. It is a matter of understanding what exactly is wrong with the idea that reswitching and allied issues undermine the logical consistency of orthodox static theory, as some still believe. There is probably nothing essentially new to be discovered nowadays on this subject. Yet this was clearly not so forty years ago, when Paul Samuelson sponsored Levhari's non-reswitching theorem. And even today much apparent uneasiness may be due to the fact that what has become known between then and now has not percolated effectively outside the specialist field.

So there is perhaps some room for an anniversary assessment of the "Cambridge criticism" intended to clarify in a simple way the technical reasons why that criticism does not actually work. The present paper is an attempt to do just that, using an analytical framework developed for the purpose. But first I must circumscribe the criticism that I propose to criticise.

Most accounts - including recently Cohen and Harcourt (2003) - date the beginning of the Cambridge controversies in capital theory from Joan Robinson's (1953-54) famous *REStud* article on the production function. Yet Pasinetti (2003) - himself a prominent participant - has strongly argued against that view. According to him, the point of departure came only seven years later, with the last chapter of Sraffa's (1960) book, and the real discussion did not erupt until 1966, in a *QJE* symposium devoted to Sraffa's result[2]. Following that, the most prominent role in the subsequent argument was played by Sraffa's followers, with Pasinetti (1969, 1970) and Garegnani (1970, 1976). By contrast, Joan Robinson (1975) eventually concluded that Sraffa's main result was irrelevant. This brought her into collision with Sraffa's followers, particularly Garegnani, and the bitter personal exchange that followed can now be found as an Appendix to Garegnani (1979).

The upshot, it would appear, is that Harcourt's supposed "Anglo-Italian school" - centred on the English Cambridge - never actually existed. There were instead two rather different groups. The first, led by Joan Robinson, Richard Kahn and Nicholas Kaldor, followed the anti-Ricardian tradition of J. M. Keynes. In particular, it argued forcibly against the classical (and neoclassical) assumed "gravitation" to a steady-state path, because - it was maintained - this implied the stability of long-run full-employment equilibrium: see e.g. Keynes (1936) p.190-92 and Robinson (1965) p. 58. Opposed to this, Sraffa (1960) assumed steady-state conditions from the very outset of his book, with constant relative prices and a uniform rate of "profit" on all capital goods. The second group of Cambridge (England) anti-neoclassical economists - aptly called neo-Ricardians - started thus from

[1] See Accademia Nazionale dei Lincei (2004), Bellino (2004), Bidard and Klimovsky (2004), Bortis (2003), Cohen and Harcourt (2003), de Vivo (2003), Foley (2003), Gilbert (2003), Greonewegen (2003), Opocher (2003), Pasinetti (2003), Sen (2003), Bortis (2002), Davis (2002), Kurz (2002), Mongiovi (2002), Parrinello (2002), Sinha (2002), Cozzi and Marchionatti. (2001), Foley (2001), Greonewegen (2001), Kurz and Salvadori (2001), Mongiovi (2001), Vernengo (2001), Kurz (2000), Mongiovi and Petri (1999).

[2] See Pasinetti (1966), Levhari and Samuelson (1966), Morishima (1966), Bruno, Burmeister and Sheshinski (1966), Garegnani (1966) and Samuelson (1966).

axioms that were wholly inconsistent with those of the first. Then, thanks to the reswitching *furore*, the Keynesian voices were gradually overshadowed by the very different neo-Ricardian (or Italian) vision. It was that vision - enshrined in Sraffa's equations - that dominated the furious polemics of the following years. Even now it is the Sraffian set of arguments that comes to mind to most people, when the question is raised of how and when the Cambridge capital controversies ended - if indeed they ended at all.

Most economists today share the belief that all such controversies were in fact concluded long ago, with the final proof that the critique suggested by Sraffa and his followers was mistaken. As a result, all such matters are now routinely ignored not just in the training of new economists, but-more crucially-in most pure and applied research on growth, where aggregate Solow-like models have come back into fashion since a generation or so. Nevertheless, few will give you any precise reason why the Sraffian logical criticism can be treated as dead. As a result, the question of "whatever happened to the Cambridge capital controversies" is sometimes still surrounded by a strange halo of mystery.

Looking back at the literature of the seventies, the *locus* where the Sraffian logical critique was finally put at rest appears to be Bliss' (1975) classic book. Bliss (1970) had already questioned the neo-Ricardian argument based on reswitching (though not of course the possibility of reswitching itself). But it was only with Bliss (1975) that all the main aspects of modern general-equilibrium (GE) capital and distribution theory were addressed with clarity and detail. The book ranged from questions of aggregation to the correct definition, role and meaning of marginal concepts, showing the tight logical consistency of the theory. Since the very first chapters, it highlighted the basic technical misunderstanding that was at the heart of the neo-Ricardian view of marginal productivity theory, as necessarily moving from exogenous input availabilities (including a "given" quantity of capital) to give a causal account of income distribution. Albeit as a special case, Bliss discussed at length these issues also within the classical framework of an economy following a steady-state equilibrium path, where one can have "the" rate of interest that was central to Sraffa's analysis.

In a truly general equilibrium setting, Bliss argued, the concept of a unique interest rate should be dethroned by the formally more robust idea of an intertemporal price system. Still, he drew attention to a fundamental logical fact. If one decides to confine oneself to the special case with constant prices through time and a uniform rate of return, then a neoclassical n-commodity system will share with a Clark-Solow one-commodity model many equilibrium aggregate properties, crucially including the marginal conditions relating to distribution (though not in general a single-valued aggregate production function)[3]. However, Bliss added, one must be careful to notice that all such properties do only involve comparisons between different steady-state equilibria, each already there by assumption. By contrast, the principle of analogy between the n- and the one-commodity worlds will not generally apply to their non-steady-state behaviour, where truly dynamic tools of analysis are required and the Sraffian "paradoxes" arising from steady-state comparisons become irrelevant. Hence the Clark-Solow simplifications, while basically innocuous in the steady-state context chosen by Sraffa, would generally produce arbitrary results when used to predict dynamic paths of accumulation through time, as it was in fact the case with most early neoclassical contenders in the Cambridge controversies.

In conclusion, the neo-Ricardians were shown to be barking under the wrong tree. On their chosen terrain - the logical consistency of (steady-state) equilibria - the prevailing orthodoxy was actually unassailable. The right tree - the extended-accumulation dynamics arbitrarily deduced from

[3] As it is well known, this can be shown provided, among other things, one defines the (endogenous) steady-state-equilibrium quantity of aggregate capital by the appropriate function, which turns out to be Champernowne's index.

Clark-Solow models - was admittedly there, but it was logically out of reach for the neo-Ricardians, owing to their steely adherence to steady-state assumptions. All sides of the original disputation - perhaps with the partial exception of Joan Robinson - were thus shown to have gone astray, but the Sraffians got rather the worse of it.

This damning indictment could reasonably be interpreted as the end of the controversy[4], were it not for the fact that a stubborn group of neo-Ricardians have continued to refuse to take it as such. Apart from a parallel philological controversy on the history of economic thought[5] and a few somewhat doctrinal considerations on pre-analytical issues of "vision"[6], the new neo-Ricardian contributions since the end of the seventies have mostly been attempts to escape precisely from the logical force of Bliss' results. Indeed, one still finds works of this kind in relatively recent years. My reference here will be Petri (1999).

Petri's argument runs in essentially two parts, of which the first is directed to show that modern intertemporal (or Arrow-Debreu) GE theory is not the right neoclassical framework to use in connection with Sraffa's assumptions. The second part attempts then to build a "proper" neoclassical model of long-run equilibrium, in order to show that such a model does indeed necessarily lead to a formal inconsistency, because it "needs" to use the aggregate equilibrium value of capital as a given parameter.

The first claim, based on Garegnani's (1976) authority, leads to a wholesale rejection of Bliss' analysis, following the contention that Arrow-Debreu models are strictly confined to the study of "very-short-period prices", as opposed to the "long-period" or "uniform-profit-rate" [constant] prices that were the object of Sraffa's work. The basis for this contention is that one can use all the initial endowments of the system as arbitrarily given parameters, and work out conclusions on the intertemporal structures of equilibrium prices that are consistent with them. Such price structures - which can be shown to exist under well-known assumptions on technology and tastes - will not be generally "determined" by the model in the way Petri seems to think, because there may be many of them. But it is of course true that they will not in general be like the ones assumed by Sraffa. Indeed, Bliss himself stressed this fact as a criticism to the lack of generality of Sraffa's results.

However, this is not the only possible way to draw inferences from GE models. In particular, it certainly does not mean that a "uniform-profit-rate" equilibrium price system cannot be studied by the theory as a special case. Bliss' Chapter 4 on "Semi-stationary growth" gave an account of the way this is done[7]. The basic analytical tool to be used in this context is a technical notion derived from the optimality properties of competitive equilibria, i.e. the concept of a price system that is supported by the given production set[8]. In the Appendix to his Chapter 4 Bliss gave a formal proof that a constant-rate-of-interest intertemporal price system can in fact be associated in this sense with the production set of any efficient semi-stationary state. And it is of course within such framework that he proved his results relating to Sraffa. One must hence conclude that Petri's first contention is just plainly not true.

4 Some of the implications of Bliss' work were taken up a bit later by Dixit (1977), who also contributed a neat disproof of the once famous neo-Ricardian proposition by Bhaduri (1966, 1969) against the application of marginal productivity to (steady-state) heterogeneous capital. Finally, besides offering a detailed attempt to convince the neo-Ricardians that all of Sraffa's formal results can be obtained in a rather straightforward way — albeit as special cases — from standard general-equilibrium theory, Hahn (1982) was itself but a brief restatement of Bliss' main results relevant to the debate.

5 See e.g. Hollander (1998, 2000), Peach (1998, 2001), Blaug (1999, 2002), De Vivo (2001), Kurz and Mongiovi (2002), Kurz and Salvadori (2002), Garegnani (2002).

6 See e.g. the summary given in Foley (2001).

7 A "semi-stationary" path is defined by Bliss as one where "a system reproduces just itself through time", "except perhaps for a change of scale". It is essentially the same as the common notion of a steady state, where one can reasonably assume that all expectations will be fulfilled.

8 Dixit (1977) offered a remarkably brief explanation of this concept.

The fact remains, though, that the Arrow-Debreu approach is not the only conceivable way to spell out a neoclassical theory of steady-state equilibrium in an *n*-commodity world. Petri's own attempt to provide an alternative in the second part of his argument appears to be flawed by a radical confusion between equilibrium conditions and adjustment processes, and by his use of a Wicksellian approach, whose well-known muddle on capital was pointed out long ago by Malinvaud (1953 and 2003). But a consistent alternative can in fact be built, as one can see from some recent literature. The final section of Mandler (1999) and Mandler (2002) discuss an overlapping-generations (OLG) model with the required features and an earlier contribution of the same kind - with a less general but more detailed model - can be found in Ferretti (1989).

OLG specialised models of equilibrium steady growth with incomplete (spot only) markets may be seen as inferior products, lacking the austere generality of standard GE theory. They offer however a few significant advantages. The main point is that they can treat all capital stock variables, in both physical and value terms, as explicit unknowns as indeed they should be in a theory designed to describe the assumed outcome of long-run adaptation in both flows and stocks. Furthermore, the resulting models can be simplified in ways consistent with their algebraic and/or numerical treatment, so that one can establish simple sufficient conditions for existence - and even uniqueness - and directly inspect the comparative statics of the solutions, whenever equilibria exist. The results of the latter exercises can then be compared with the standard neo-Ricardian claims acting, as we shall see, as effective counter-examples.

In what follows, I examine two variants of such a model. In both of them I assume a two-sector competitive system with constant population. Both versions share the same demand side, developed in Section I below, but they differ as to technology. In the first model - developed in Section II and dubbed the "super-neoclassical" case - I assume that within each sector the available technology can be described by a separate well-behaved industrial (or microeconomic) production function. In the second model - set out in Section III and called "Garegnani-neoclassical" - the technological assumptions are instead those introduced by Garegnani (1970), involving both reswitching and capital reversal. The comparative statics of both versions - with given technology and varying parameters for the preference function - offer simple counter-examples to all neo-Ricardian critical claims. Section IV finally summarises these counter-examples and argues that they lead to the very same conclusions reached by Bliss thirty years ago. Any consistent criticism of the prevailing orthodoxy will thus have to rely on different lines.

I. Demand functions

I.1. Household choices

In our two-sector semi-stationary economy there are just two produced goods: a consumption good, which will be our *numéraire*, and a capital good, used together with homogeneous labour to produce both itself and the consumption good. The system as a whole is confidently expected to grow along a steady-state path at the exogenously given rate g, while the productive use of the capital good leads to its "radioactive" decay at the constant and uniform rate h.

There is also a constant population of $3N$ individuals, each with an economic life lasting 3 periods[9]. In any period this population is made up of N "integrated" groups of 3 individuals, each belonging to a different generation: one will be "young" (generation 0), one an "adult" (generation 1) and the third will be "old" (generation 2). These individuals are the consumers of the system and own all its means of production, of which they sell the productive services to firms.
At the beginning of their economic life, our individuals plan all their future transactions maximising an utility function in consumption and leisure, subject to an endowment of 1 unit of time per period, a real-wage path $w(t) = w(0)(1 + g)^t$, a constant price p of the capital good and a constant price s of its productive services. The form of the preference function, assumed to be the same for all, must however obey some additional restrictions, over and above the usual ones, because it must be consistent with the assumed steady-state growth of the system. Specifically, we need individual choices such that - at steady-state equilibrium prices -aggregate labour supply will be stationary and aggregate consumption will grow at the given rate g. Under common assumptions of separability such restrictions can be shown to imply a "period" preference function that is log-linear in consumption. For reasons of symmetry I will hence assume a utility index of the following form

$$\text{(I.1.1)} \qquad U = \sum_{i=0}^{2} \left(\frac{1}{1+\delta} \right)^i \left[a \cdot \ln(c_i) + b \cdot \ln(l_i) \right] \qquad \text{with} \quad a, b > 0$$

where c_i and l_i are quantities of consumption good and leisure in the (1+i)-th period of life, while δ can be interpreted as the rate of time preference.

Turning now to the budget constraints, each period's endowment of time can be either consumed as leisure or sold as labour at the going wage. On the other hand, the income of the first two periods can be either spent on c_i or saved through the purchase of k_i units of physical capital at its market price p. In the latter case, during the next period our individual will be in a position to sell to firms the productive services of k_i at their market price s, getting the net property income $k_i\,(s - hp)$. However, the opportunity to save for the future is not available to the members of generation 2, who are also barred from leaving bequests.

Let us now define the net rate of return on capital (or rate of "profit") as

$$\text{(I.1.2)} \qquad r \equiv \frac{s - hp}{p}$$

To simplify notation, let us further define

$$\text{(I.1.3)} \qquad G \equiv 1 + g, \quad D \equiv 1 + \delta, \quad R \equiv 1 + r$$

Finally, let us indicate by $\theta \equiv t - n$ the point in absolute time where the members of generation $n = 0,1,2$ of period t started their economic life. From the assumptions listed above the budget constraints of our individuals can then be written as

$$\text{(I.1.4)} \qquad \begin{aligned} x_{n,\theta} &\equiv w(0)G^{\theta}(1 - l_0) - pk_0 - c_0 \geq 0 \\ x_{n,\theta+1} &\equiv w(0)G^{\theta+1}(1 - l_1) + p\left[(R - 1 + h)k_0 - k_1\right] - c_1 \geq 0 \\ x_{n,\theta+2} &\equiv w(0)G^{\theta+2}(1 - l_2) + pR\left[(1 - h)k_0 + k_1\right] - c_2 \geq 0 \end{aligned}$$

[9] In the usual setting, with just two OLG, the *stock* of capital at the beginning of each period is identically equal to the investment *flow* carried out by the members of the older generation during the previous period. The slight complication involved in assuming three OLG has thus only the purpose to bring out more clearly the specific role played by capital stocks - as opposed to mere flows - in the conditions of steady-state equilibrium. However, no result is influenced by this assumption.

where for each x_{nj} the first subscript indicates the generation we are talking about and the second stands for the period in absolute time when the constraint becomes effective.

For each generation we can now set up a standard optimisation problem and solve the first-order conditions for the flow decision variables (c_i), (l_i), (k_i). The result is the entire life-cycle transaction plan by each member of each generation, with the consumption, saving and leisure of each period of one's life expressed as explicit algebraic functions of the parameters. Actually, as far as flows are concerned, we are only interested in the section of each plan that is to be carried out during the current period t. Thus, indicating as c_{ni} the consumption planned by each member of current generation n for the (1+i)-th period of his life, and so on for the other flow variables, the only flow functions we need are those where $n = i$ (i, n = 0,1,2).

However, we can use our solutions for the *past* transactions of the members of the older generations to determine the desired *stock* of physical capital of each member of each generation at the beginning of current period t, as given by his past perfect-foresight investments. This desired stock - which I shall indicate as K_i (i = 0,1,2) - is in fact the one that would have been accumulated if past expectations had been correct. For the younger generation we have $K_0 \equiv 0$, whereas for generations 1 and 2 we have the definitions

(I.1.5) $$K_1 \equiv k_{10}\,, \qquad K_2 \equiv k_{20}(1-h) + k_{21}$$

where k_{10}, k_{20} and k_{21} are the past optimal investments determined by the first-order conditions. We thus get the individual asset- (or stock-) demand functions for capital at the beginning of the current period t. By contrast, the individual flow supplies of capital services to firms will be given by the actual stocks, historically inherited from the past. As it is obvious, these actual stocks will coincide with the desired ones only at stock-equilibrium. Indeed, the required equality between actual and desired stocks gives us the explicit conditions that define our equilibrium as a specifically *long-run* one. Such explicit conditions, not generally met in standard GE analysis, are the distinctive feature of the models presented here.

I.2. Market behaviour functions and the Silver Rule

Normalising $N \equiv 1$, the market behaviour functions for households are obtained by simple aggregation over the integrated group. With the notation introduced above, the market demand function for the consumption good during period t is hence:

(I.2.1) $$C(t) = \sum c_{ii} = w(0)\cdot G^{t+2}\cdot\frac{a}{a+b}\cdot\left[D^2 + D\frac{R}{G} + \left(\frac{R}{G}\right)^2\right]\cdot\frac{1+\frac{R}{G}+\left(\frac{R}{G}\right)^2}{1+D+D^2}$$

Similarly, the market labour supply function turns out to be constant through time and equal to:

(I.2.2) $$L_s \equiv 3 - \sum l_{ii} = 3 - \frac{b}{a+b}\cdot\frac{D^2G^2 + DGR + R^2}{R^2}\cdot\frac{1+\frac{R}{G}+\left(\frac{R}{G}\right)^2}{1+D+D^2}$$

Aggregate physical gross investment demand during period t can be shown to be:

(I.2.3) $$I(t) \equiv \sum k_{ii} = (G - 1 + h)\cdot K(t)$$

where $K(t)$ is the aggregate physical stock of desired capital at the beginning of period t, which in turn is given by the market asset-demand function:

(I.2.4) $$K(t) \equiv \sum K_i = \frac{w(0) \cdot G^{t+1}}{p} \cdot \left[\frac{\frac{1}{G^3} \cdot R^3 + \frac{2+D}{G^2} \cdot R^2 - \frac{D \cdot (1+2D)}{G} \cdot R - D^2}{\left(1+D+D^2\right) \cdot R^2} \right]$$

Function (I.2.4) deserves special attention, because it is the distinctive feature of our family of models. Given $w(0)$, $p > 0$ and $G, D \geq 1$, this function is increasing with R, but it is easy to see that for positive values of the rate of return sufficiently close to zero — as R approaches unity from above — $K(t)$ will become negative, because the polynomial in R at the numerator in the square bracket will itself turn negative. Indeed, the only real and positive zero of that polynomial is given by $R = D \cdot G$. It is hence obvious that $K(t)$ will be a positive number, as required, only if $R > D \cdot G$. In other words, for any non-negative value of the rate of time preference δ, the rate of return will have to be greater than the given growth rate of the system. It can further be shown that, when such necessary condition is met, the market supply of labour defined by (I.2.2) above will also be positive.

I shall refer to this condition as the "silver rule of accumulation" since, under the assumptions of the present paper, it must supersede the time-honoured golden rule introduced by Phelps (1961). In the following Sections we shall see that it plays a crucial role in establishing the existence and uniqueness of steady-state equilibrium.

II. The Super-Neoclassical Case

II.1. Technological assumptions and the factor-price frontier

In the "super-neoclassical" case each sector uses the same physical capital good in varying proportions with labour. The available technology is described by two standard well-behaved microeconomic (industrial) production functions in labour and capital:

(II.1.1) $$Q_i(t) = F_i(L_i, K_i; t), \quad i = C, K \quad ,$$

where both outputs and inputs are in physical units. The subscripts refer to the consumption- and capital-goods sector respectively. Both functions exhibit constant returns to scale and satisfy the Inada conditions, with positive but diminishing marginal products. Both display factor-augmenting technical progress, in the manner required to ensure overall Harrod-neutrality.

Firms are assumed to face given factor prices $w(t)$ and $s = p(h + r)$ and to maximise their pure profit per unit of output subject to (II.1.1). The result can be expressed as a Walrasian matrix of optimal (cost-minimising) production coefficients

(II.1.2) $$\mathbf{A}(t) = [a_{i,j}(t)], \quad i = C, K \quad , \quad j = L, K$$

with rows referring to outputs and columns to inputs. The steady-state supply conditions for both sectors are then

(II.1.3) $$\mathbf{A}(t) \cdot \mathbf{v}(t) = \mathbf{p} \quad \text{with} \quad \mathbf{v}(t) = \begin{pmatrix} w(t) \\ p(h+r) \end{pmatrix} \quad \text{and} \quad \mathbf{p} = \begin{pmatrix} 1 \\ p \end{pmatrix}$$

Solving this system for $w(t)$ and p - with r as a parameter - yields the factor-price frontier (FPF) of the system (as a curve shifting in time) and an expression linking the constant steady-state price of the capital good to the rate of return.

To fix ideas, let us suppose in particular that technology (II.1.1) takes the explicit form of two Cobb-Douglas:

$$(\text{II.1.4}) \qquad Q_i(t) = G_i^t \cdot A_i(0) \cdot [K_i(t)]^{\alpha_i} \cdot [L_i(t)]^{1-\alpha_i}$$
$$\text{with} \quad G_i = G^{1-\alpha_i}, \qquad i = C, K$$

The cost-minimising coefficients turn out to be

$$(\text{II.1.5}) \qquad \begin{aligned} a_{i,L}(t) &= \left[A_i(0)\cdot G_i^t\right]^{-1} \cdot \left[\frac{p\cdot(h+r)}{w(0)\cdot G^t}\cdot\frac{1-\alpha_i}{\alpha_i}\right]^{\alpha_i} \\ a_{i,K}(t) &= \left[A_i(0)\cdot G_i^t\right]^{-1} \cdot \left[\frac{w(0)\cdot G^t}{p\cdot(h+r)}\cdot\frac{\alpha_i}{1-\alpha_i}\right]^{1-\alpha_i} \end{aligned}$$

with $i = C, K$. The FPF at $t = 0$ can then be computed as

$$(\text{II.1.6}) \qquad \hat{w}(0) = A_C(0)\cdot(1-\alpha_C)\cdot\left[\alpha_C\cdot\frac{1-\alpha_K}{1-\alpha_C}\cdot\left(\frac{A_K(0)\cdot\alpha_K^{\alpha_K}}{h+r}\right)^{\frac{1}{1-\alpha_K}}\right]^{\alpha_C}$$

and the corresponding expression for the steady-state value of p can be written as

$$(\text{II.1.7}) \qquad \hat{p} = \frac{\hat{w}(0)}{1-\alpha_K}\cdot\left[\frac{(h+r)^{\alpha_K}}{A_K(0)\cdot\alpha_K^{\alpha_K}}\right]^{\frac{1}{1-\alpha_K}}$$

It can now be shown that expression (II.1.6) for the FPF wage at time zero is actually the upper envelope of the family of infinite steady-state $w(0)$–r relationships that obtain when all the alternative sets of production coefficients allowed by the production functions are used as arbitrary parameters, rather than functions of the prevailing factor prices. To this end, let us imagine that producers maximise profits at time zero on the basis of expected values r^e, p^e and $w(0)^e$ which - though in the right relationship among themselves - will coincide with the actual ones only at equilibrium. To get such expectations, we start from an arbitrary (non-negative) value for r^e, and then compute the corresponding $w(0)^e$ and p^e from (II.1.6) and (II.1.7). Using all three expectations, we then get the corresponding "planned" production coefficients from equations (II.1.5). Finally, we substitute such coefficients into conditions (II.1.3) and solve them for $w(0)$ and p. This gives us the specific $w(0)$–r Sraffian relationship, corresponding to the particular technique chosen on the expectation of r^e, which can be written as

$$(\text{II.1.8}) \qquad \overline{w}(0) = \hat{w}(0)\cdot\left(\frac{h+r}{h+r^e}\right)^{\frac{\alpha_C}{1-\alpha_K}}\frac{h(1-\alpha_K)-\alpha_K r+r^e}{h(1-\alpha_K)+\alpha_C(r-r^e)-\alpha_K r+r^e}$$

where $\hat{w}(0)$ is the FPF wage and $r \in \left[0, \frac{h(1-\alpha_K)+r^e}{\alpha_K}\right]$.

Using r^e as a parameter, (II.1.8) represents the entire family of $w(0)$-r Sraffian relationship corresponding to all non-dominated techniques allowed by our industrial production functions at time zero. Within it, the factor applied to the FPF wage $\hat{w}(0)$ is at most unity, which is its equilibrium value, when $r = r^e$. So the FPF is indeed the upper envelope of this family. It can moreover be seen that any two members of the family cannot intersect more than once: our "super-neoclassical" technology effectively rules out reswitching. Finally it should be noticed that the family (II.1.8) turns linear with $\alpha_K = \alpha_C$; Samuelson's (1962) "parable" is but a special case of this model.

II.2. Equilibrium in period zero

Using these results and those of Section I we can now write down the set of steady-state equilibrium conditions at time zero. We begin from the two output-supply conditions (II.1.3), written in the form of their solutions (II.1.6) and (II.1.7) for the wage *w* and the price *p*:

(II.2.1) $$w = \hat{w}(0)$$

(II.2.2) $$p = \hat{p}$$

Next are four conditions for the cost-minimising production coefficients:

(II.2.3 - 6) $$\overset{*}{a}_{i,j} = a_{i,j}(0), \quad i = C, K, \quad j = L, K$$

Then we have the market-clearing conditions for consumption and investment output:

(II.2.7) $$C = C(0)$$

(II.2.8) $$I = (g + h)K(0)$$

The equilibrium conditions relating to labour services may be written as:

(II.2.9) $$L = C \cdot \overset{*}{a}_{C,L} + I \cdot \overset{*}{a}_{K,L}$$

(II.2.10) $$L = L_s$$

Finally, the symmetrical conditions for capital services are:

(II.2.11) $$K = C \cdot \overset{*}{a}_{C,K} + I \cdot \overset{*}{a}_{K,K}$$

(II.2.12) $$K = K(0)$$

where *K* is the historical capital stock at the beginning of period zero and the latter equation is of course the asset-equilibrium condition of the system.

We have thus 12 conditions in 11 unknowns (the four production coefficients, *r, w, p, C, I, L* and *K*) but only 11 conditions are independent of the others, thanks to Walras' law. We thus drop equation (II.2.11) and reduce the others by substitution, until we are left with just two equations in two unknowns, *r* and *w*. Of these the first is (II.12.1) above, i.e. the FPF of the system, while the second will be the labour market-clearing condition (II.2.10), after substituting into it all the remaining relationships of the model. The final result of this substitution can itself be written in the form of a *w-r* relationship, as:

(II.2.13) $$\tilde{w} = \frac{\hat{w}(0)}{1 - \alpha_C} \cdot P(R),$$

where $\hat{w}(0)$ is the FPF wage at time zero and $P(R)$ is a fourth-degree polynomial in $1 + r$, whose parameters depend on all the technological parameters (bar the absolute value of total factor productivities), the ratio $\gamma = a/b$ of the elasticities of the preference function and the rate δ of time preference. Function (II.2.13) gives the set of (w, r) combinations consistent with overall market clearing, and so I shall indicate it as the SD curve of the model, from supply equals demand.

Each equilibrium - if any exist - can now be represented on the (w, r) plane as an intersection of the FPF and SD curves in the subset of the positive quadrant defined by the silver rule of Section I. We hence write:

(II.2.14) $$\frac{P(R)}{1-\alpha_C} - 1 = 0 \qquad \text{with} \qquad R > G \cdot D$$

as the condition required by the equilibrium rate of return r^*. Applying Budan-Fourier's theorem to the polynomial on the left-hand side of this expression, one finds that - given G, $D \geq 1$, ´$0 \leq \alpha_C$, α_K, $h \leq 1$ and $\gamma > 0$ - it has always one and only one real positive root within the stated interval. This proves that in the "super-neoclassical" case of this Section equilibrium not only always exists, but is also unique.

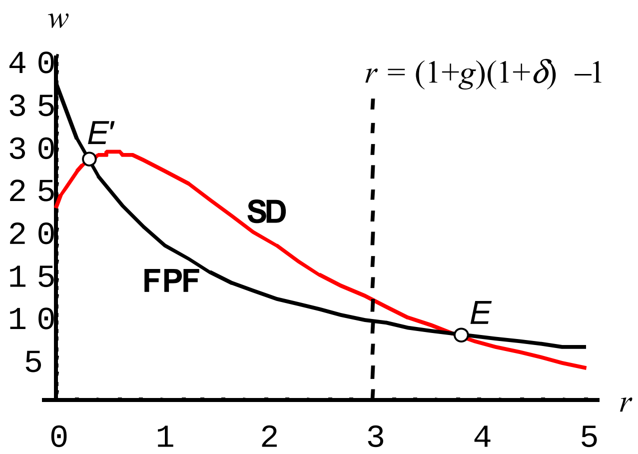

Figure 1

Figure 1 illustrates a numerical example where the real positive roots are actually two, given by the r values corresponding to the intersections E and E'. It is however apparent that the second intersection falls outside the region of the silver rule. Hence the only meaningful equilibrium is the one corresponding to intersection E, with an equilibrium rate of return equal to slightly more than 3.8 and an equilibrium wage at time zero of about 10.

II.3. Comparative statics

From equilibrium conditions (II.2.1-12) we determine a unique solution for every one of our unknowns, expressed as a continuous algebraic function of all our technological parameters ($A_C(0)$, $A_K(0)$, α_C, α_K, g and h), of the preference-elasticity ratio γ and of the time-preference rate δ. Treating the technological parameters as constants, I shall now use such functions to compare same-technology equilibria as the preference parameters γ and δ take on different values, so that the SD curve of Figure 1 is shifted and the intersection E slides along a given FPF curve. To simplify the exercise, I shall assume a stationary path ($g = 0$) with circulating capital ($h = 1$). Reporting on a numerical example with $A_C(0) = A_K(0) = 10$, $\alpha_C = 1/5$ and $\alpha_K = 4/5$, Table 1 lists the equilibrium values (indicated by an asterisk) of the main unknowns for selected values of γ and δ.

Table 1. Selected equilibrium values
with $g = 0$, $h = 1$, $A_C(0) = A_K(0) = 10$, $\alpha_C = 1/5$, $\alpha_K = 4/5$

	$\gamma = 0.5$			$\gamma = 1$			$\gamma = 2$		
	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$	$\delta = 0$	$\delta = 0.5$	$\delta = 1$
r^*	0.276073	0.743653	1.25205	0.376653	0.855666	1.37629	0.471619	0.96597	1.50159
w^*	28.8058	21.0812	16.3222	26.7012	19.8087	15.4688	24.9782	18.6973	14.694
p^*	0.00932378	0.0237874	0.0512506	0.0117068	0.0286725	0.0602096	0.0143005	0.0340955	0.070245
C^*	29.9643	22.2502	17.3177	42.8508	32.3339	25.4229	55.1168	42.0383	33.3069
L^*	0.919571	0.889099	0.87802	1.39518	1.36769	1.3565	1.8967	1.87582	1.86665
K^*	1350.11	198.246	46.5414	1269.51	213.646	53.5737	1147.73	211.49	55.7306

From the comparative-static functions sampled in Table 1 one can obviously get equilibrium aggregate net output per unit of labour $y^* = C^*/L^*$ and two alternative measures of the equilibrium aggregate capital/labour ratio: $c^* = K^*/L^*$, which is in physical units, and $k^* = p^*K^*/L^*$, where capital is measured in value at current equilibrium prices. Differentiating now y^*, c^* and k^* relative to γ and δ, one finds that they share among themselves (and with r^*, w^* and p^*) a common value for the ratio of their first partial derivatives, so that:

$$\text{(II.3.1)} \qquad \frac{r^*_\gamma}{r^*_\delta} = \frac{w^*_\gamma}{w^*_\delta} = \frac{p^*_\gamma}{p^*_\delta} = \frac{y^*_\gamma}{y^*_\delta} = \frac{c^*_\gamma}{c^*_\delta} = \frac{k^*_\gamma}{k^*_\delta} = \rho$$

where the common ratio ρ is itself a function of γ and δ.

Property (II.3.1) implies that r^*, w^*, p^*, y^*, c^* and k^* share a common map of equal-level curves in γ and δ, which in turn means that there is a one-to-one correspondence between any two of them, as γ and δ vary. In particular, there will hence be the following two single-valued functions:

$$\text{(II.3.2)} \qquad y^* = \phi(k^*)$$

$$\text{(II.3.3)} \qquad y^* = \zeta(c^*)$$

Expression (II.3.2) gives equilibrium aggregate net output per unit of labour as a function of the value of average equilibrium capital per unit of labour, measured at current equilibrium prices. Expression (II.3.3) gives the same output as a function of average equilibrium physical capital per unit of labour. Both these functions can be interpreted as aggregate pseudo-production functions in intensive form, and as such both are *prima facie* well behaved, i.e. single-valued, increasing and with falling first derivatives. Their existence suggests a first analogy with one-commodity models[10], though not a very important one in itself. So we now turn to the more relevant issue of marginal conditions. As we shall see, functions (II.3.2) and (II.3.3) can in fact lead to three different definitions of the comparative-static marginal product of capital, not all of them equally relevant for competitive distribution theory.

The first definition starts from (II.3.2) and computes the marginal product in value terms at current equilibrium prices, simply as the ratio of differentials dy^* and dk^*. Following a well known approach introduced by Bhaduri (1969), both differentials can be simply calculated as functions of r^* from the family (II.1.8) of individual-technique w-r relationships. The result is:

[10] It should however be kept in mind that function (II.3.3) - in *physical* capital per head - owes its existence to the fact that in our two-sector model there is just one (perhaps composite) capital good. To "collapse" in such a way a (n + 1)-commodity model - with one consumption good and n different capital goods - one has to restrict the production technology of the ("non-basic") consumption good, assuming that under all competitive techniques the n capital goods enter into its production in the same proportions as they enter into Sraffa's "basic commodity": see Ferretti (1989).

(II.3.4) $$\frac{dy^*}{dk^*} = \frac{(1+r^*)(1-\alpha_C)+\alpha_C(\alpha_C-\alpha_K)}{(1+r^*)(1-\alpha_K)-r^*\alpha_C(\alpha_C-\alpha_K)}r^*$$

As it is easily seen, this definition will lead to equality with r^* in the special case with $\alpha_C = \alpha_K$ (i.e. Samuelson's "surrogate" case), but not in general. The simple reason is that with $\alpha_C \neq \alpha_K$ the differential $dk^* = p^*{\cdot}dc^* + c^*{\cdot}dp^*$ is affected by a generally non-zero dp^*, and it is thus the wrong one for a price-taking firm. Hence expression (II.3.4.) – while perfectly orthodox in itself – is unfit to represent marginal-productivity theory.

The second definition starts from (II.3.3) and computes the comparative-static marginal product of physical capital, given by the ratio of the differentials dy^* and dc^*. Following a similar approach to the previous one to express both differentials as functions of r^*, one gets:

(II.3.5) $$\frac{dy^*}{dc^*} = r^*{\cdot}p^*$$

Remembering that with $h = 1$ the definition (I.1.2) of the rate of return is $r \equiv (s-p)/p$, expression (II.3.5) gives the equilibrium marginal product of physical capital as equal to $s^* - p^*$, i.e. the equilibrium rental price of capital net of depreciation. We thus have a standard result of marginal productivity theory.

Finally, a third definition could again be obtained from (II.3.2) in value terms, only correcting the Wicksellian mistake we have found in (II.3.4) above. The idea is to have a differential of k^* computed in a way consistent with the optimisation problem of price-taking firms, i.e. at (locally) constant equilibrium prices. Treating p^* as a constant, we have :

(II.3.6) $$d\hat{k} \equiv (dk^* | dp^* = 0) = p^* dc^*$$

and using this definition it follows immediately from (II.3.5) that

(II.3.7) $$\frac{dy^*}{d\hat{k}} = r^*$$

which is of course "the" orthodox statement of marginal-productivity distribution theory as applied to capital and the real rate of interest.

This completes our positive results on the comparative statics of the "super-neoclassical" case, but a further negative – and perhaps less obvious – result deserves finally to be pointed out. Expressions (II.3.4) and (II.3.5) above rely on the fact that – thanks to property (II.3.1) – our equilibrium solutions imply the existence of two comparative-static functions:

(II.3.8) $$k^* = \Phi(r^*)$$

(II.3.9) $$c^* = \Xi(r^*)$$

relating the *equilibrium* capital/labour ratio (in value and physical units) to the *equilibrium* rate of return, as the parameters of the preference function vary.

It should however be clear that neither of these comparative-static functions can be confused with a demand function for capital, describing actors' *out-of-equilibrium* choices (within *given* preferences and technology) at varying *out-of-equilibrium* prices. As a matter of fact, our model (II.2.1-12) includes two such behaviour functions – the asset-demand by households defined by (I.2.4) in Section I and the flow-demand (for capital services) by firms on the right-hand side of (II.2.11) in this Section – and it is readily seen that their form is quite different from either (II.3.8) or (II.3.9). Thus, any attempt to deduce from $\Phi(r^*)$ the shape of an excess-demand function for capital – in order to draw conclusions on the dynamic stability of equilibrium – would involve a basic conceptual confusion, leading in particular to misleading results on the implications of reswitching, as we shall now see in the next Section.

III. The Garegnani-Neoclassical Case

III.1. Technological assumptions and the factor-price frontier

In the Garegnani-neoclassical case we mix the demand conditions of Section I with the special technology assumed in Garegnani (1970), implying both reswitching and capital reversal. We still have a two-sector (stationary) economy using circulating capital with homogeneous labour, but the available production coefficients are now described by the following relationships:

(III.1.1)
$$
\begin{aligned}
a_{C,L} &= \frac{30 + 11 \cdot u^{\frac{11}{10}} + u^{\frac{11}{5}} - 27 \cdot e^{-2u}}{6 + u^{\frac{11}{10}}} \\
a_{C,K} &= \frac{27 \cdot e^{-2u}}{\left(6 + u^{\frac{11}{10}}\right)^2} \\
a_{K,L} &= 1 \\
a_{K,K} &= \frac{5 + u^{\frac{11}{10}}}{6 + u^{\frac{11}{10}}}
\end{aligned}
$$

where u is a non-negative parameter which identifies every individual technique. We substitute (III.1.1) into system (II.1.3) of the previous Section – with time dropped, owing to the assumption of a stationary economy – and solve for w and p, getting the following solution for the wage rate:

(III.1.2)
$$
w(u,r) = \frac{1}{5 + u^{\frac{11}{10}} - \dfrac{27 \cdot e^{-2u} \cdot r}{\left(5 + u^{\frac{11}{10}}\right) \cdot r - 1}}
$$

This expression gives the entire family of w-r relationships of the model. The corresponding FPF is the upper envelope of this family, defined by choosing the value of u that maximises $w(u, r)$ at any given r. The first-order condition for such a maximum can be written as:

(III.1.3)
$$
11 e^{4u} u^{\frac{11}{10}} \left[\left(5 + u^{\frac{11}{10}}\right) r - 1\right]^2 + 27 e^{2u} r \left\{\left[100 + u^{\frac{1}{10}}(11 + 20u)\right] r - 20\right\} = 0
$$

and although it cannot be solved algebraically for u, one can compute its solutions $u^* = u^*(r)$ by numerical methods. The results are illustrated by the curve of Figure 2, with the optimal (return-maximising) u^* on the vertical axis and the corresponding rate of return on the horizontal one. Its bell shape – with the same u^* (bar the maximum one $\bar{u}$) associated with *two* different r-values – reflects of course the assumption of reswitching.

We now substitute u^* into expressions (III.1.1) to get our new Walrasian matrix $\mathbf{A} = \left[\hat{a}_{i,j}(r)\right]$ of optimal coefficients as functions of r. Solving $\mathbf{A} \cdot \mathbf{v} = \mathbf{p}$ for w and p we thus get the function

$\hat{w} = \hat{w}(r)$ for the FPF wage and the parallel function $\hat{p} = \hat{p}(r)$ for the competitive price of the capital good. The curve representing $\hat{w}(r)$ - not reproduced here - has the slanted-bell shape depicted in Garegnani's (1970) article.

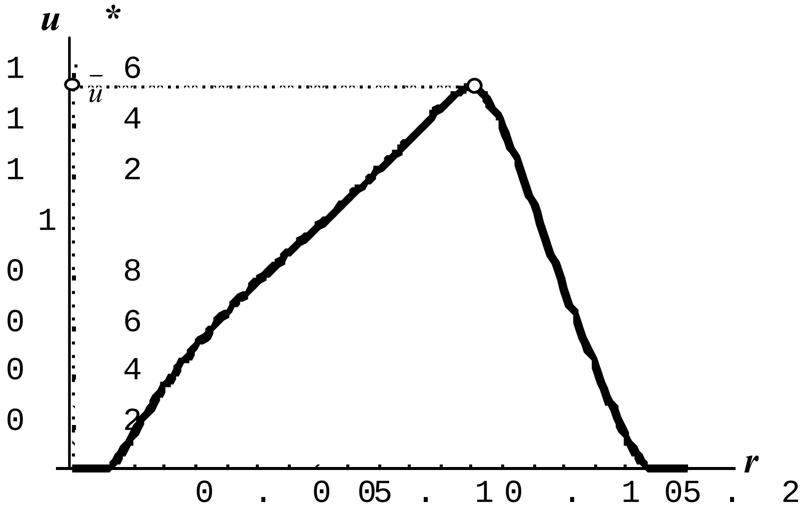

Figure 2

III.2. Equilibrium conditions and solutions

The steady-state equilibrium conditions for the Garegnani-neoclassical economy parallel closely those of the super-neoclassical case and can now be written as:

(III.2.1) $\quad w = \hat{w}$

(III.2.2) $\quad p = \hat{p}$

(III.2.3 - 6) $\quad \overset{*}{a}_{i,j} = \hat{a}_{i,j}(r), \quad i = C, K, \quad j = L, K$

(III.2.7) $\quad C = \hat{C}$

(III.2.8) $\quad I = \hat{K}$

(III.2.9) $\quad L = C \cdot \overset{*}{a}_{C,L} + I \cdot \overset{*}{a}_{K,L}$

(III.2.10) $\quad L = L_s$

(III.2.11) $\quad K = C \cdot \overset{*}{a}_{C,K} + I \cdot \overset{*}{a}_{K,K}$

(III.2.12) $\quad K = \hat{K}$

where $\hat{C}$ an $\hat{K}$ are the consumption- and asset-demand functions of Section I with time dropped and $G = 1$. We have again 11 independent conditions in 11 unknowns and proceed to their solution in a similar way, dropping equation (III.2.11) and reducing the others by substitution to just two equations in r and w. The final result is once again a FPF-SD curve diagram, where the SD curve can be written as

$$\text{(III.2.13)} \quad \overline{w}(r) = \frac{R^2\left(1 + D + D^2\right)\left[3 - \frac{\left(1+R+R^2\right)\left(D^2+DR+R^2\right)}{R^2\left(1+D+D^2\right)(1+\gamma)}\right]}{\overset{*}{a}_{C,L}\left(1 + R + R^2\right)\left(D^2 + DR + R^2\right)\frac{\gamma}{1+\gamma} + \frac{1}{p}(R - D)\left[D + 2DR + R(2 + R)\right]}$$

with $R = 1 + r$, $D = 1 + \delta$ and $\gamma = a/b$, while $\overset{*}{a}_{C,L}$ and $\hat{p}$ are the functions of r computed in (III.2.2) and (III.2.3).

The equilibrium level(s) of the rate of return, r^*, will thus emerge from the solution of the intersection condition:

(III.2.14) $$\hat{w}(r) = \overline{w}(r) \quad \text{with} \quad r > \delta$$

where the restriction on r follows from the silver rule.

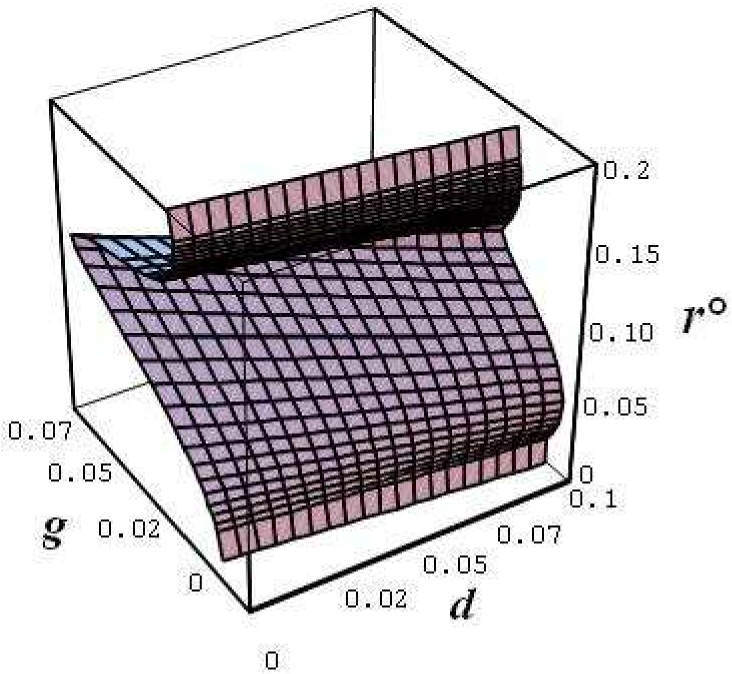

Figure 3

Given the functions in **A**, the existence and number of solutions depend entirely on the parameters γ and δ of the preference function. Since the FPF function $\hat{w}(r)$ sets a finite upper level r_{MAX} to the competitive r consistent with a non-negative wage, it is first of all obvious that one can always find a time-preference rate δ high enough to make equilibrium impossible for any elasticity ratio γ. However, equilibrium may be impossible even with $\delta < r_{MAX}$, if γ takes on a value sufficiently close to zero. Thus – contrary to what happened in the super-neoclassical case – a Garegnani-neoclassical equilibrium may not exist at all. In second place, if for some combination (γ_0, δ_0) one equilibrium exists, there may be many, as the SD curve may intersect the FPF more than once in the relevant range of r. Figure 3 shows how the intersection rates of return $r°$ vary in fact with γ and δ. Each point on this surface with $\gamma > 0$ and $r° > \delta$ represents a possible equilibrium solution and, since the surface folds over itself four times, there may easily be two or three equilibrium values of r for any given set of preference parameters. It is interesting to note that the major fold, responsible for most cases of multiple equilibria, corresponds to the maximum u^* of Figure 2, and is thus a direct consequence of the reswitching assumption.

III.3. Comparative statics and the demand for capital

Multiple equilibria make for cumbersome comparative statics, since any slide of the SD curve along the FPF – as tastes change with the given technology – may at the same time alter the number of alternative solutions. Moreover, even when that number does not change, the different equilibria consistent with given tastes will generally be affected differently by the same increment in γ or δ. As a result, comparing equilibria will not by itself tell much, leading to no clear-cut general result on the static implications of differences in tastes.

However, Figure 3 makes it clear that any point of the FPF curve within the interval $r \in [0, r_{MAX}]$ can in fact become an equilibrium combination (r^*, w^*), given appropriate non-negative values for γ and δ. Thus, from a purely formal point of view one can certainly use the properties of the FPF and its generating family (III.1.2), in order to deduce comparative-static functions between the possible equilibrium levels of some variables. In particular, one can get the relationship:

$$k^* = \chi(r^*) \tag{III.3.1}$$

where k^* is the equilibrium level of capital per unit of labour, measured in value at equilibrium prices. And it is of course easy to see that the reswitching assumption leads to a non-monotonic shape of (III.3.1), giving rise to what has been called "capital reversal".

However, in the light of the highly dubious significance of equilibrium comparisons in a context of likely multiple equilibria, we must now pause and ask why capital reversal should be considered at all important. In particular, we shall consider whether function (III.3.1) can be interpreted as the market demand function (or "curve") for capital by firms, as some has suggested, so that a positive slope of it could mean an upward-sloping excess-demand function in the market for capital services, with possible effects on the stability of equilibrium.

As it is well known, $\chi(r^*)$ can be easily computed from the family (II.1.2) above as:

$$\chi(r^*) = \frac{w(u^*, 0) - w(u^*, r^*)}{r^*} \tag{III.3.2}$$

On the other hand, the market excess-demand function for capital services is defined by the difference between the right-hand sides of equations (III.2.11-12) above:

$$XD_K(r) = \hat{C} \cdot \hat{a}_{C,K}(r) - \hat{K}\left[1 - \hat{a}_{K,K}(r)\right] \tag{III.3.3}$$

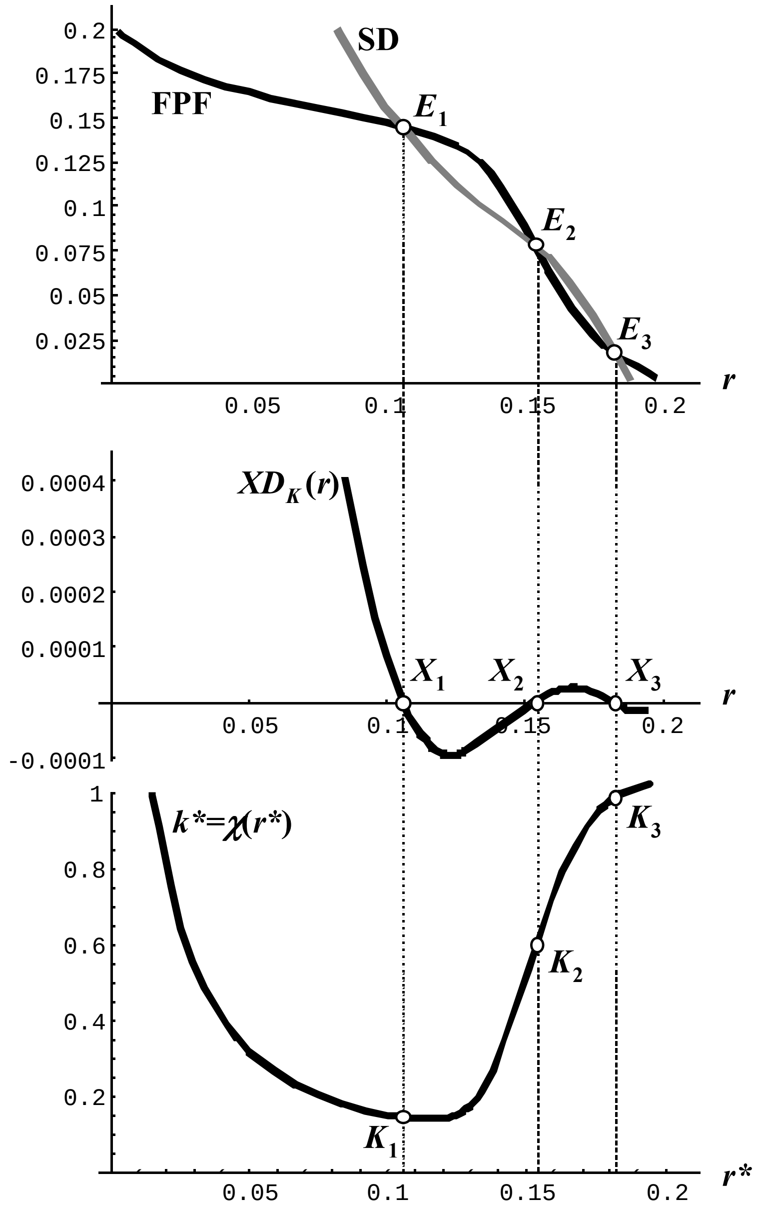

Figure 4

It should be noted that in (III.3.3) w and p (which enter into functions $\hat{C}$ and $\hat{K}$ besides r) are independent parameters. In what follows – to get as close as possible to (III.3.2) – I shall assume that, as r varies, w and p will adapt too, in accordance with the relationships defining the FPF. Even so, however, $XD_K(r)$ will also depend on the preference parameters γ and δ, while $\chi(r^*)$ does not. It is hence clear that $XD_K(r)$ cannot in general be treated as equivalent to $\chi(r^*)$, not even up to a constant. To show that there is no general relationship even among the sign of their slopes, I will now report on a numerical example where I assume $\gamma = 0.01$ and $\delta = 0.09$. The results are illustrated in the three panels of Figure 4.

In the upper panel of Figure 4 we measure the rate of return on the horizontal axis and the wage rate on the vertical one. The SD market-clearing locus, drawn in grey, intersects Garegnani's FPF curve three times, determining three equilibria at E_1, E_2 and E_3. The middle panel plots $XD_K(r)$ against r. Given the assumed behaviour of w and p this function is zero at X_1, X_2 and X_3, corresponding to the three equilibrium levels of the rate of return determined in the upper panel. Finally, the lower panel depicts $\chi(r^*)$, with the three equilibrium levels of the rate of return determining points K_1, K_2 and K_3 on it. Of these points, the latter two fall within the upward-sloping section of $\chi(r^*)$, where capital reversal is taking place. Yet the corresponding slope of $XD_K(r)$ varies in sign, being positive at X_2 and negative at X_3.

III.4. Marginal conditions

It can be shown that along the downward-sloping section of $\chi(r^*)$ – where no capital reversal occurs – an aggregate marginal condition relating to capital holds in the form

(III.4.1) $$\frac{dy^*}{dc^*} = r^* \cdot p^*$$

where y^* is net equilibrium labour productivity and c^* is the equilibrium average physical quantity of capital per unit of labour[11], computed as k^*/p^*.

Although (III.4.1) ceases to hold when k^* becomes an increasing function of r^*, the (endogenous) equilibrium quantity of capital per labour unit can be defined by a quantity index $\kappa = \kappa(r^*)$ – with the corresponding price index $\pi = \pi(r^*)$ – such that the neoclassical equality between its aggregate marginal product and its rental rate holds at any equilibrium, irrespective of the slope of $\chi(r^*)$. The required form for $\kappa(r^*)$ turns out to be Champernowne's (1953) chain index, modified in the way required by the assumption of a continuum of techniques. We shall now turn to this problem in a framework slightly more general than the special continuum assumed by Garegnani.

Let our continuum be defined by the Walrasian matrix

(III.4.2) $$\mathbf{A}(u) = \begin{pmatrix} a_{C,L}(u) & a_{C,K}(u) \\ a_{K,L}(u) & \frac{u}{1-u} \end{pmatrix}$$

where the parameter $u \in \,]\,0, 1[\,$ is the reciprocal of the maximum rate of return allowed by each technique. The family of *w-r* relationships determined by the steady-state supply conditions with technology $\mathbf{A}(u)$ can be written as the implicit function

(III.4.3) $$\varphi(w, r, u) = 0$$

which is assumed to be continuous and twice differentiable relative to all three variables at all its points. The FPF is defined by condition $\varphi_u = 0$ that leads to the frontier rate of return $r^* = r(u)$ and frontier wage $w^* = w(r^*)$. One has also net output per unit of labour $y = y(u)$ and the value of capital per unit of labour $.k = k(r, u)$.

Now, the original index introduced by Champernowne (1953-54) refers to cases where u varies discontinuously. Without reswitching and capital reversal, and ordering our discrete techniques by the wage at which they become competitive, the definition is:

(III.4.4) $$\kappa_n = \kappa_0 \prod_{i=1}^{n} \frac{k_i}{k_{i-1}}\bigg|_{r=r^*_{i,i-1}}$$

where κ_0 is an arbitrary base and $r^*_{i,i-1}$ is the rate of return at which the i-th and $(i-1)$-th techniques are equally competitive. Setting now $\kappa_0 = y_0 - w^*_{1,0}$, this definition can be written as

[11] The change in the quantity of physical capital per labour unit from one technique to another appears to be unambiguously defined in the common units implied by (III.1.1).

(III.4.5) $$\kappa_n = \left(y_n - w^*_{n,n-1}\right)\prod_{i=1}^{n-1}\left(1 - \frac{w^*_{i+1,i} - w^*_{i,i-1}}{y_i - w^*_{i+1,i}}\right).$$

Now, as n tends to infinity, it can be shown that the limit of (III.4.5) is

(III.4.6) $$\kappa(v) = \{y(v) - w[r(v)]\}e^{\int_0^v \frac{\partial w}{\partial r}\frac{dr^*}{du}\frac{1}{y-w^*}du}.$$

where v is a particular value of u and $\kappa(u)$ is our chain index for the continuous case. Under the assumptions of no reswitching and capital reversal made so far, $y(u)$ and $r(u)$ are both single-valued, with $y(u)$ monotonically increasing and $r(u)$ monotonically decreasing. Then y can be expressed as an increasing single-valued function of $\kappa(u)$ and it is easy to prove that for all values of u and r^* it is the case that

(III.4.7) $$\frac{dy}{d\kappa} = \pi \cdot r^* \quad \text{with} \quad \pi \equiv \frac{k}{\kappa},$$

which is of course the standard form of the aggregate marginal condition in the quantity of capital given by index κ.

When reswitching and capital reversal do occur, as in Garegnani's example, $y(u)$ is decreasing and $r(u)$ is a two-valued function of u, with a decreasing upper branch $r_1^* \geq r(\bar{u})$ and an increasing lower branch $r_2^* \leq r(\bar{u})$. Hence definition (III.4.6) will itself become a two-valued function:

(III.4.8) $$\kappa_i(v) = \{y(v) - w[r_i(v)]\}e^{\int_0^v \frac{\partial w}{\partial r}\frac{dr_i^*}{du}\frac{1}{y-w^*}du} \qquad (i = 1, 2).$$

Definition (III.4.8) implies of course a pseudo-production function of y in κ that is also two-branched. Yet it is easily shown that – on each separate branch – aggregate marginal condition (III.4.7) still holds true.

The point is of course that the chain index κ is built for local equilibrium-quantity comparisons – in the neighbourhood of any given intersection between the FPF and the SD market-clearing curve. It is thus the right definition for the marginal notions which are relevant to neoclassical static distribution theory, where in fact it yields results consistent with the micro-economic marginal conditions implicit in competitive cost minimisation. Reswitching does not affect this role of κ. However it does affect the index' ability to obtain consistent non-marginal comparisons between equilibria associated with different branches of $r(u)$ along the given FPF. Thus within the Garegnani-neoclassical case a neoclassical (static) theory of distribution still holds true, although the model does not possess a well-behaved (single-valued) aggregate pseudo-production function even in intensive form.

IV. Conclusions

The object of the present paper has been to show that all neo-Ricardian arguments questioning the logical consistency of neoclassical equilibrium (or static) capital theory run counter a set of simple counter-examples, drawn from two versions of a static model of long-run equilibrium for a two-sector competitive economy. In both versions, equilibrium has been defined by market-clearing conditions based on demand and supply functions obtained from assumptions of perfect competition and maximising behaviour, so that the basic model can be described as strictly neoclassical. Equilibrium has however been restricted to the subset of long-run steady-state equilibria, with the market-clearing conditions embedding the further assumptions of a steady (exogenous) growth of outputs and real wage, with all the other relative prices – and hence "the" rate of return on capital – remaining constant through time. A consequence of such restriction has been that the capital endowment of the system has become an unknown variable, rather than a parameter.

The model, built so that it could be explicitly computed, has been used to study three main problems. First, I wanted to know if a demand-and-supply steady-state equilibrium could be shown to exist at all. Secondly, I asked whether – assuming existence – equilibrium would imply aggregate marginal-productivity conditions regarding capital. And, thirdly, I tried to establish whether one could deduce the form of the excess-demand function for capital from the comparative-static relationship linking r^* and k^*, so that capital reversal could lead to restrictions on dynamic stability. The general answers have turned out to be 'yes' for the two former questions and 'no' for the third one. But the details depend on the assumed technology.

On the existence issue, the "circular reasoning" alleged by the neo-Ricardian writers should produce – if true – a lack of existence in all circumstances. However, I have shown that in the "super-neoclassical" case – where physical capital and labour are well-behaved substitutes for each other in each industry – equilibrium not only always exists, but is unique. Assuming a reswitching technology instead, as in Garegnani (1970), one gets situations where equilibrium may be unique, multiple or non-existent at all, depending on tastes. However, given appropriate ranges for the preference parameters, all points on Garegnani's FPF can lead to existing equilibria. The "circular reasoning" issue is then the first neo-Ricardian proposition for which our models provide direct counter-examples. And the fallacy of this first contention is strictly related to that of the other neo-Ricardian claim, according to which a neoclassical theory necessarily involves a "given" amount of capital. Here too our models produce direct counter-examples; Table 1 above, in particular, illustrates numerically how both the quantity and value of capital in the "super-neoclassical" economy are determined uniquely by the equilibrium conditions and vary with the preference parameters, being in this sense endogenous.

The second question – of marginal conditions – involves the use of comparative-static relationships linking the equilibrium values of different unknowns, as tastes shift with a given technology. Within the "super-neoclassical" case – thanks to the uniqueness of equilibrium – such comparisons are straightforward and – provided one avoids Wicksellian pitfalls – the results are perfectly orthodox. The same kind of comparative-static relationships becomes of more dubious significance within the reswitching technology of Garegnani (1970), owing to the presence of multiple equilibria. Nevertheless, ignoring all interpretation problems and sticking to the formal properties of the model, Section III has shown that all conceivable equilibria along Garegnani's FPF do imply standard neoclassical marginal conditions, if only the endogenous quantity of capital is defined by the appropriate version of Champernowne's chain index – although the aggregate pseudo-production function in intensive form ceases to be single-valued (but not to exist) as a consequence of capital reversal.

Here too, then, we have two direct counter-examples that refute a well-known set of neo-Ricardian propositions, alleging the logical fallacy of marginal-productivity theory. Two related points deserve however to be made in this connection. First, the endogenous nature of the quantity of capital shows that the traditional neo-Ricardian argument against the use of Champernowne's index – its being a function of r^* – is actually spurious. Since κ and r^* are both unknowns of the simultaneous equilibrium conditions, it would be an obvious logical mistake to require that κ be computed before r^* is known through the solution of the system.

The second related point is that, although standard marginal conditions are found to obtain in all its versions, in no case – not even when equilibrium is unique – can it be said that in our model marginal products determine the distribution of income. That is probably "the" basic neo-Ricardian misunderstanding, and one suspects that it has provided much of the ideological fuel behind the Sraffian attempt to undermine the logic of marginal-productivity theory. The simple point, once again, is that all marginal products are themselves among the unknowns of the system, together with the quantities of labour and capital and the return and wage rates. In no case the model implies a causal chain starting from given inputs, passing through marginal products and leading on to income distribution – as Sraffa and his followers have imagined. Even when equilibrium is unique, all these quantities will be determined collectively by the simultaneous equilibrium conditions. And even then, it would be a mistake to think – along the lines popularised a century ago by Pareto – that marginal products and income distribution are mutually determining each other in some obscure causal sense. One must not confuse between the mathematical and the causal determination of things. From a mathematical point of view, all the elements of a unique solution are determined by the set of simultaneous equations – not by each other. From a causal point of view, at least in the ordinary meaning, what determines what will depend on the dynamic process – if any – that leads to the equilibrium described by the model. But the model itself is utterly dumb on its own dynamic stability, which in fact may not exist at all. All that the model says is what its unknowns must be in order to satisfy a set of conditions assumed to be required for the vanishing of the time-derivatives of prices within an unspecified dynamic system. The upshot is that our model has no causal implication whatever. There is just no capitalist causal theory of distribution to fight against in the interest of the working classes; not at least until someone specifies the details of the model's dynamic stability.

The subject of dynamic stability brings us finally to the implications of reswitching. The neoclassical model of Section III has shown that – apart from a two-valued pseudo-production function in Champernowne's index – the main effect of the capital reversal implied by Garegnani's (1970) reswitching technology is to lead to multiple equilibria. Multiple equilibria are of course relevant for dynamic stability, but this is not the point raised by the neo-Ricardians, who – having ignored the demand side of the model – seem actually unaware of it. The question raised by them is the totally different one of the implications for stability of capital reversal *per se*. The idea, first explicitly stated in Garegnani (1970), is that the comparative-static relationship between the equilibrium rate of return r^* and the equilibrium value of capital per worker k^* should be interpreted as the demand "curve" for capital by firms, to be used in the analysis of dynamic, out of equilibrium adaptations of r. If this idea was correct, then capital reversal would imply an upward-sloping demand "curve" which – combined with a supposedly "given" supply of capital – would lead to an excess-demand function which was itself increasing in r. Starting from this and reasoning along standard *tâtonnements* lines, Garegnani and his followers have then argued that any initial non-zero excess demand would cause r to adapt dynamically in the wrong direction, away from equilibrium. Hence – they concluded – a supply-and-demand theory of capital could not survive capital reversal, because equilibrium would then necessarily be dynamically unstable[12].

We are now in a position to see why this argument appears to be entirely mistaken. First of all, we have already noticed that the models of Sections II and III include explicit formulations for

[12] In recent times this approach has been closely followed by Kurz and Salvadori (1995 and 2001); but some of its features appear to be shared even by the criticism offered in Potestio (1999).

both the demand function for capital and the comparative-static relationships between r^* and k^*. It has thus been easy to see that – both formally and conceptually – they are two quite distinct relationships. In particular, the numerical example of Figure 4 above has made it clear that there is in fact no general relationship between capital reversal and the slope of the (static) excess-demand "curve" for capital. Although this point, as far as I know, has never been made before, it appears sufficient to conclude that Garegnani's argument has its very roots in a rather basic confusion.

This is not however the only error, and perhaps not even the main one. A second fundamental misconception follows from the idea that the (static) excess-demand function should involve the supply of capital as a "given" parameter. Yet the capital stock – determined by past accumulation – is actually endogenous in the model, and we can now assess some implications of this basic fact for stability analysis. In particular, we can see that the (static) excess-demand function for capital by firms – given by (III.3.3) above – is affected by the asset-demand function $\hat{K}$, from which the equilibrium flow-supply of capital (services) to firms is derived. Households' asset demand $\hat{K}$ – defined by function (I.2.4) above with G = 1 to express stationary conditions – is given by the past perfect-foresight investments of the two elder generations, assuming constant prices (and thus r) from t – 2 to t. This means, inter alia, that any two points on this function do actually refer to different economic systems, each one with its distinctive past history of investments. Going back to Figure 4, it would thus be entirely misleading to suggest that, of the three equilibria shown there, E1 and E3 would be stable, and E2 unstable, owing to the corresponding slope of the XDK(r) curve. There is no way for a given economy to move along this function, in response to price changes. So, even the properly formulated static excess-demand function XDK(r) cannot be used as an element of a tâtonnement process, describing the dis-equilibrium dynamics of a given economy.

To put the same thing in another way, the whole point of the tâtonnements idea – given the auctioneer's rule – is to describe dynamic adaptations through the very same (static) behaviour functions that enter the equilibrium conditions, thanks to some sort of re-contracting like Walras' bons. However, there is no practical way to re-contract one's past. Thus, while a tâtonnement process is at least conceivable in a pure flow (short-run) equilibrium, this cannot be the case with (long-run) equilibrium stocks emerging from the past. Long-run dis-equilibrium dynamics consists by necessity of some non-negligible amount of false trading, and long-run stability analysis has thus no way to be built on the same (static) relationships used to define equilibrium.

Garegnani's attempt to draw dynamic inferences from capital reversal can thus be described as a misunderstanding within a misunderstanding. It was however, when it took place, a useful mistake, because the confusions leading to it were in fact shared at the time by the very authors he intended to criticise, and in particular by a great neoclassical economist like Paul Samuelson. The reswitching furore of the sixties and seventies proved thus eventually to be the path to new knowledge. Nevertheless, the contributions made by all the original participants to the dispute are perhaps nowadays best forgotten, because the new knowledge had to be gained by clarifying the common mistakes they were all making – and this, understandably, was the task of others. As a matter of fact, all the counter-examples listed above add very little to what was shown thirty years ago by Bliss (1975). While their simplicity may help a better understanding of the underlying issues, their being drawn from a different framework merely testifies to the logical robustness of Bliss' results.

Thus, the overall message has not changed much since then. The prevailing orthodoxy on growth does have problems, mainly relating to individual behaviour assumptions and – even more crucially – the dynamics of the assumed stability of steady-state paths. The non-steady-state behaviour of market economies is still not well understood. Orthodox growth theory continues to be based on the classical notion that such economies "gravitate" towards a steady state, but the main argument behind it is observed trends rather than theoretical prediction. The reswitching controversy has helped to see that in an n-commodity world one cannot generally base the dynamics of stability on the same aggregate relationships which can be used to define steady-state equilibrium, as in Clark-Solow models. Yet the microeconomics of the assumed convergence is still virtually non-existent. All available exercises in "transition dynamics" continue to be carried out in a one-commodity world, begging a host of well-known dynamic questions: from the aggregation problems pointed out long ago by F.M. Fisher to the fundamental issue of how price flexibility can be the outcome of rational behaviour by individual competitive agents.

However, none of these things amounts to a formal difficulty in the definition of long-run supply-and-demand equilibrium. Rather, they are problems of empirical content, making for arbitrary and essentially untestable predictions on historical events whose context is that of non-static conditions, where all the Sraffian (and static neoclassical) relationships become irrelevant. The neo-Ricardian onslaught of forty years ago did help to identify the issues, but essentially in a negative way, as Sraffa's "prelude" proved to be a dead end. Any useful criticism of mainstream economics will hence have to follow the radically different and much less simple lines of explicit dynamics.

References

Accademia Nazionale dei Lincei (2004) *Piero Sraffa,* Rome: Collana Atti Convegni Lincei (ACL-200).

Bellino, E. (2004) 'On Sraffa's standard commodity' *Cambridge Journal of Economics* 28 (1) 121-32.

Bhaduri, A. (1966) 'The concept of marginal productivity of capital and the Wicksell effect' *Oxford Economic Papers* 18 (3) 284-88.

Bhaduri, A. (1969) 'On the significance of recent controversies on capital theory: A Marxian view' *Economic Journal* 79 (September) 532-39.

Bidard, C. and Klimovsky, E. (2004) 'Switches and fake switches in methods of production' *Cambridge Journal of Economics* 28 (1) 89-97.

Blaug, M. (1999) 'Misunderstanding classical economics: the Sraffian interpretation of the surplus approach' *History of Political Economy* 31 (2) 213-36.

Blaug, M. (2002) 'Kurz and Salvadori on the "Sraffian interpretation of the surplus approach"' *History of Political Economy* 34 (1) 237-40.

Bliss, C. J. (1970) 'Comment on Garegnani' *Review of Economic Studies* 37 (3) 437-38.

Bliss, C. J. (1975) *Capital Theory and the Distribution of Income*, Amsterdam: North-Holland.

Bortis, H. (2002) 'Piero Sraffa and the revival of classical political economy: A review article' *Journal of Economic Studies* 29 (1) 74-97.

Bortis, H. (2003) 'Marshall, the Keynesian Revolution and Sraffa's significance' *Journal of Economic Studies* 30 (1) 77-97.

Bruno, M., Burmeister, E. and Sheshinski, E. (1966) 'Nature and implications of the reswitching of techniques' *Quarterly Journal of Economics* 80 (4) 526-53.

Champernowne, D. G. (1953-54) 'The production function and the theory of capital: Comment' *Review of Economic Studies* 21 (2) 112-35.

Cohen, A. J. and Harcourt, G. C. (2003) 'Whatever happened to the Cambridge capital theory controversies?' *Journal of Economic Perspectives* 17 (1) 199-214.

Cozzi,T. and Marchionatti, R. (2001) eds. *Piero Sraffa's Political Economy,* London: Routledge.

Davis, J. B. (2002) 'Gramsci, Sraffa, Wittgenstein: philosophical linkages' *European Journal of the History of Economic Thought* 9 (3) 384-401.

De Vivo, G. (2001) 'On Torrens's theory of profits' *Cambridge Journal of Economics* 25 (5) 697-703.

De Vivo, G. (2003) 'Sraffa's path to Production of Commodities by Means of Commodities: an interpretation' *Contributions to Political Economy* 22 (1) 1-25.

Dixit, A. (1977) 'The accumulation of capital theory' *Oxford Economic Papers* 29 (1) 1-29.

Ferretti, M. (1989) 'Un modello walrasiano d'equilibrio di lungo periodo' *Giornale degli economisti e Annali di economia* 48 415-447.

Foley, D. K. (2001) 'Value, distribution and Capital: a review essay' *Review of Political Economy* 13 (3) 365-381.

Foley, D. K. (2003) 'Sraffa's legacy' *Cambridge Journal of Economics* 27 (2) 225-38.

Garegnani, P. (1966) 'Switching of techniques' *Quarterly Journal of Economics* 80 (4) 554-67.

Garegnani, P. (1970) 'Heterogeneous capital, the production function and the theory of distribution' *Review of Economic Studies* 37 (3) 407-36.

Garegnani, P. (1976) 'On a change in the notion of equilibrium in recent work on value and distribution', in M. Brown, K. Sato e P. Zarembka (eds.), *Essays in Modern Capital Theory*, Amsterdam: North-Holland 25-45.

Garegnani, P. (1979) *Valore e domanda effettiva*, Torino.

Garegnani, P. (2002) 'Misunderstanding classical economics? A reply to Blaug' *History of Political Economy* 34 (1) 241-54.

Gilbert, G. (2003) 'The equations unveiled: Sraffa's price equations in the making' *Contributions to Political Economy* 22 (1) 27-40.

Greonewegen, P.(2001) Review of Kurz (2000) *Journal of Economic Literature* 39 (4) 1228-29.

Greonewegen, P (2003) 'On re-reading Joan Robinson's "On re-reading Marx"' *Review of Political Economy* 15 (4) 509-19.

Hahn F., (1982) 'The neo-Ricardians' *Cambridge Journal of Economics* 6 353-374.

Hollander, S. (1998) 'Ricardo, Torrens and Sraffa: the untenability of De Vivo's summing up' *Cambridge Journal of Economics* 22 (5) 617-622.

Hollander, S. (2000) 'Sraffa and the interpretation of Ricardo: the Marxian Dimension' *History of Political Economy* 32 (2) 187-232.

Keynes, J. M. (1936) The General Theory of Employment, Interest and Money, reprinted in The Collected Writings of John Maynard Keynes VII 1973 London: Macmillan.

Kurz, H. D. (ed.) (2000) Critical Essays on Piero Sraffa's Legacy in Economics Cambridge: CUP.

Kurz, H. D. (2002) 'Professor Roncaglia's Piero Sraffa. His life, thought and cultural heritage' History of Economic Ideas 10 (1) 119-26.

Kurz, H. D. and Mongiovi, G. (2002) 'Interpreting Ricardo: a rejoinder to Peach' Cambridge Journal of Economics 26 (3) 371-80.

Kurz, H. D. and Salvadori, N. (1995) Theory of production. A long-period analysis Cambridge: CUP.

Kurz, H. D. and Salvadori, N. (2001) 'The aggregate neoclassical theory of distribution and the concept of a given value capital: a reply' Structural Change and Economic Dynamics 12 479-485.

Kurz, H. D. and Salvadori, N. (2002) 'Mark Blaug on the "Sraffian interpretation of the surplus approach" 'History of Political Economy 34 (1) 225-36.

Levhari, D. and Samuelson, P. A. (1966) 'The nonswitching theorem is false' Quarterly Journal of Economics 80 (4) 518-19.

Malinvaud, E. (1953) 'Capital accumulation and the efficient allocation of resources' Econometrica 21 (3) 233-68.

Malinvaud, E. (2003) 'The legacy of Knut Wicksell to capital theory' Scandinavian Journal of Economics 105 (4) 507-25.

Mandler, M. (1999) 'Sraffian indeterminacy in general equilibrium' *Review of Economic Studies*, 66 (3) 693-711.

Mandler, M. (2002) 'Classical and neoclassical indeterminacy in one-shot versus ongoing equilibria' *Metroeconomica* 53 (3) 203-222.

Mongiovi, G. (2001) 'The Cambridge tradition in economics: An interview with G. C. Harcourt' *Review of Political Economy* 13 (4) 503-21.

Mongiovi, G. (2002) 'Classics and moderns: Sraffa's legacy in economics' *Metroeconomica* 53 (3) 223-241.

Mongiovi, G. and Petri, F. (1999), eds., *Value, Distribution and Capital, Essays in honour of Pierangelo Garegnani* London: Routledge.

Morishima, M (1966) 'Refutation of the nonswitching theorem' *Quarterly Journal of Economics* 80 (4) 520-25.

Opocher, A. (2003) 'Interrelated prices and Sraffa's critique of partial equilibrium' *European Journal of the History of Economic Thought* 10 (3) 479-96.

Parrinello, S. (2002) 'Sraffa's legacy in Economics: some critical notes' *Metroeconomica* 53 (3) 242-260.

Pasinetti, L. L. (1966) 'Changes in the rate of profit and switches of techniques' *Quarterly Journal of Economics* 80 (4) 503-17.

Pasinetti, L. L. (1969) 'Switches of technique and the «Rate of Return» in capital theory' *Economic Journal* 79 (September) 508-531.

Pasinetti, L. L. (1970) 'Again on capital theory and Solow's «Rate of Return»" *Economic Journal* 80 (June) 428-30.

Pasinetti, L. L. (2003) 'Comment' *Journal of Economic Perspectives* 17 (4) 227-28.

Peach, T. (1998) ‘On interpreting Ricardo: A reply to Sraffians’ *Cambridge Journal of Economics* 22 (5) 597-616.

Peach, T. (2001) ‘Hollander, de Vivo and the «further evidence» for the corn model: A conspiracy of silence?’ *Cambridge Journal of Economics* 25 (5) 685-692.

Peach, T. (2002) ‘Interpreting Ricardo: A further reply to Sraffians’ *Cambridge Journal of Economics* 26 (3) 381-91.

Petri, F. (1999) ‘Professor Hahn on the “neo-Ricardian” criticism of neoclassical Economics’ in Mongiovi and Petri (1999) 19-68.

Phelps, E. S. (1961) 'The golden rule of accumulation: a fable for growthmen', *American Rconomic Review* 51 (3) 638-43

Potestio, P. (1999) ‘The aggregate neoclassical theory of distribution and the concept of a given value of capital: towards a more general critique’ *Structural Change and Economic Dynamics* 10 381-394.

Robinson, J. (1953-54) ‘The production function and the theory of capital’ *Review of Economic Studies* 21 (2) 81-106, reprinted with a ‘Postscript’ in Robinson, J., *Collected Economic Papers* II Oxford: Blackwell, 1964, 114-131.

Robinson, J. (1965) *The Accumulation of Capital*, 2nd ed. London: Macmillan.

Robinson, J. (1975) ‘The unimportance of Reswitching’ *Quarterly Journal of Economics* 89 (1) 32-39.

Samuelson, P. A. (1962) ‘Parables and realism in capital theory: the surrogate production function’ *Review of Economic Studies* 29 (2) 193-206.

Samuelson, P. A. (1966) ‘A summing up’ *Quarterly Journal of Economics* 80 (4) 568-583.

Sen, A. (2003) ‘Sraffa, Wittgenstein, and Gramsci’ *Journal of Economic Literature* 41 (4) 1240-55.

Sinha, A. (2002) Book review of Mongiovi and Petri (1999), *Review of Radical Political Economics* 34 499-502

Sraffa, P. (1960) *Production of Commodities by Means of Commodities* Cambridge: CUP.

Vernengo, M. (2001) ‘Sraffa, Keynes and “The years of high theory”’ *Review of Political Economy* 13 (3) 343-54.