

An integrated approach to measuring effective tax rates

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Abstract

Nearly twenty years after the publication of King-Fullerton, effective tax rates have grown into a widely accepted tool of public policy. They have been calculated for a large number of countries, applied in different empirical studies of investment behaviour, and employed to back changes in public policies. At the same time, the King Fullerton approach appears to retain a number of weaknesses particularly in respect of company's financing behaviour that have never been fully resolved.

This paper proposes a new method of measuring effective tax rates that explicitly takes account of risk. The measure is based on the modern theory of corporate finance and allows for the pricing of risk. It is more encompassing than those previously proposed and the King-Fullerton measure can be shown to be special limiting case. This new measure has a number of additional features: (a) effective tax rates are uniquely defined as a function of a company's optimal debt/equity ratio which is endogenously determined; (b) this measure is shown to be related to forward looking measures of effective tax rates such as those suggested by Shevlin (1990) and Graham (1996).

1. Introduction ^{*}

Twenty years after the publication of King-Fullerton (1984), effective tax rates have grown into a widely accepted tool of public policy. They have been calculated for a large number of countries (OECD, 1990, Jorgenson and Landau, 1993), applied in different empirical studies of investment behavior (Slemrod, 1990; Devereux and Griffiths, 1998) and employed to support changes in public policies (Ruding Report, 1992, European Commission, 2001).

Nevertheless, the King-Fullerton (KF) model remains open to a number of criticisms that have never been fully addressed from a theoretical and empirical standpoint. First, the KF approach relies on a stripped down model of the functioning of financial markets and on several dubious assumptions, particularly in respect of the relationship between the cost of debt and equity financing. The calculations do not rely on an endogenously determined optimal debt/equity ratio or a rigorous model of equilibrium in the capital markets. Ad hoc arbitrage assumptions are imposed on firm behavior and on the structure of pre and post tax interest rates. The firm is assumed to face a wide spectrum of “effective tax rates” which are then aggregated via exogenous weights that have only an indirect bearing on marginal investment choices. Second, the KF framework explicitly omits risk and the manner risk interacts with corporate valuations, hence precluding a meaningful analysis of the effects of taxes across industries or investments with different structures of returns. Finally, the KF effective tax rates convey a very different picture of the tax system from backward looking measures based on balance sheet data.

This paper has several objectives. The first is to model the financial decisions of firms in an endogenous fashion, allowing for a risky rate of interest for company debt that varies with the debt-equity ratio. The second objective of the paper is to propose a new method of measuring effective tax rates that explicitly takes account of risk. The measure is based on the modern theory of corporate finance and allows for the pricing of risk. It is more encompassing than those previously proposed and the KF measure can be shown to be special limiting case. Since we limit our attention in this paper to the financial dimension of the investment decision the measures derived in this paper cannot be directly compared to the overall effective tax rates derived in the KF framework. However, we compare our measure to the effective rate of tax which would be obtained under assumption that the capital stock was given and firms merely varied their debt-equity ratio. Our measure can also be viewed as the effective tax rate which applies to the weighted cost of capital.

The new measure we propose has a number of additional features:

- a) marginal effective tax rates can be determined from company balance sheet data;
- b) the measure allows for the asymmetries present in tax systems, most notably those arising from the presence of tax losses;
- c) the measures are shown to be related to forward looking measures of effective tax rates such as those suggested by Shevlin (1990) and Graham (1996).

The paper is divided into three sections. The first provides a critical assessment of existing measures of effective tax rate based on the KF approach and more recent contributions. The section

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focuses on the theoretical weaknesses of existing models and on the changes that need to be made to correct these drawbacks. The second section describes a new methodology for computing effective tax rates based on a stochastic theory of a firm and provides base case scenarios for a stylized firm. The third section provides an empirical application to the Italian tax reform based on Montecarlo simulations. The fourth section describes potential extensions of the model and some concluding remarks close the paper.

2. Current Approaches to Effective Tax Rates

Marginal effective tax rates are generally defined as the difference between the internal rates of return on before and after-tax cash flows. The cash flows are derived from simulated investment projects that differ by type of financing (equity vs. debt), by industry and by the specific characteristics of capital goods (equipment vs. buildings). The tax rates and other features of the tax system that are used as inputs in the studies are based on statutory rules. An “average” marginal rate is achieved by applying fixed weights across investments and aggregating the rates across projects.

This basic approach to measuring “effective tax rates”, which is derived from the cost of capital measures first used by Hall and Jorgenson (1967) (henceforth, HJ) for estimating investment demand equations, was inaugurated by King and Fullerton (1984). This study was largely descriptive in nature and the measures of effective tax rates were intended to be suggestive of the workings of various elements of the tax system. The “average marginal effective tax rates” were derived as summary measures of how different elements of the tax system interacted with one another and of how exogenous factors might affect the measurement of tax burdens. In particular, at the time the study was carried out, high levels of inflation appeared to distort the measurement of tax burdens based on statutory rates or accounting profits. KF described these distortions in great detail and focused on the wide dispersion of rates resulting from high levels of inflation. The model had a very stripped down version of the capital markets and KF were admittedly non-committal as to the interaction between tax rates, inflation and the structure of interest rates. A number of different capital market equilibrium conditions were shown to produce significantly different level of effective tax rates.

Since the mid-1980s, the KF methodology has been utilized in the study of a wide range of countries (OECD, 1990; Jorgenson and Landau, 1993), applied to multinationals and implemented for a range of other financing arrangements (OECD, 1990, Alworth, 1988). The notion of “marginal effective rates” has also been extended to allow for the taxation of “pure profits” (Devereux and Griffiths, 1998). The effective tax rates derived using the KF methodology have also been used as both dependent and independent variables in econometric studies (Slemrod, 1990). Moreover, both the EU and OECD have based a number of policy pronouncements regarding the level of taxation across countries on these effective tax rate measures.

Bradford and Fullerton (1981) and Bradford and Stuart (1986) provided a critical assessment of the KF and examined a number of the simplifying assumptions adopted by KF. Since then, the basic KF methodology has remained unchallenged and no work has been carried out to assess the appropriateness of the framework itself, which the authors had originally been very careful to circumscribe. No alternative to the KF approach has been proposed although some extensions for special features of individual countries have drawn attention to the limited generality of the KF approach.

In our view, the KF approach and its subsequent embellishments can be faulted on three major grounds:

- a) the effective tax rates measures are driven by “corner solutions” that have no empirical basis and tend to provide a distorted view of the tax advantages to various types of financing policies in the presence of uncertainty;
- b) the theoretical underpinnings of the workings of the capital market are not internally consistent;
- c) the KF “effective tax rates “ cannot be related to observable policies followed by companies (i.e. the measures of effective tax rates cannot be meaningfully considered to reflect policy choices faced by individual companies). Indeed KF "effective tax rates apply only to stripped down stylized models of companies.

2.1 “Corner Solutions” and effective tax rates

One of the most serious issues regarding the King-Fullerton approach is the reliance on “extreme” valuations. In essence firms are assumed to follow corner solution financial strategies: investments are financed at the margin out of debt, retained earnings or the issuance of new shares. In addition, the cost of each form of financing is assumed by hypothesis to be identical, save for the potential interactions with the tax code (and inflation). The values for the cost of capital and the effective tax rates under each of these three forms of finance is aggregated upward by weighting each marginal investment by *average* measures of firm financing typically taken from backward looking nationwide financial statistics or aggregate corporate balance sheets.

This procedure suffers from two major drawbacks. Firstly, there is no connection between the cost of finance at the margin and the risk of individual firms or projects. In other words the KF approach eschews any analysis between the risk of a project/firm and the cost of capital. As is well known, the interaction between taxation and risk taking can be quite complex and depends, amongst other things, on the nature of loss offsets. With imperfect loss offsets, the ex-ante marginal tax rates of riskier companies tend to be higher (Majd and Myers, 1985).

Secondly, in the absence of risk there is no connection between leverage and the cost of debt financing. Even in the simple Modigliani-Miller framework without taxes the cost of debt finance rises with company indebtedness as bondholders increasingly share in the variable component of company earnings (i.e. higher interest rates must be paid for higher leverage to compensate for the increasing risk of default). By assuming a constant rate of interest the King-Fullerton approach implicitly assumes that bondholders do not bear any risk of default and that the tax benefits of debt finance accrue even if the firm is totally financed by debt and the company is tax exhausted. The tax advantage to debt in the KF framework is a constant given by the corporate marginal tax rate or by the difference between the marginal corporate tax rate and the marginal personal tax rate. Consequently, the KF approach fails to capture important differences in marginal tax rates across firms and the relationship between leverage and marginal tax rates. The tax benefits from increasing leverage for a company that has not issued any debt are very different from those of a firm with a very high leverage ratio whose cash flows may be barely sufficient to cover its interest payments. In other words, the benefits to leverage “at the margin” depend on the initial debt/equity ratio of the firm. Intuitively, it would seem that the *lower* the leverage the *greater* the tax advantage to debt. The upward fixed-weight KF aggregation procedure leads to the conclusion that effective tax rates decline with leverage.

2.2 Capital Market Underpinnings

In the KF framework, there is no overall capital market equilibrium. Because of “corner” financial policies followed by the firm, each investor is *the* marginal investor for the specific simulation. This clearly is not satisfactory: it is difficult to imagine a firm having at the margin a multiplicity of tax clienteles. Recent research regarding the ex-dividend behavior of stock prices has highlighted that identifying the marginal investor is very important in assessing the likely impact of the tax advantages to debt. In summary (Graham, 2003), the evidence that personal taxes on dividends affect asset prices is very tenuous. Most research for markets where the costs of arbitrage are low suggests that trading around ex-days results in share prices falling by the full amount of the dividend payout. Under these circumstances personal taxes are not impounded into share prices and the marginal investor for the purpose of company valuation is *de facto* tax-free. This would seem to suggest that the personal tax penalty on using debt might be quite high. However, even debt instruments appear increasingly to be priced on a tax free basis. In contrast to the findings of McCulloch (1975), current estimations of interest rate term structures find little evidence of any tax effects (Elton and Green, 1998). While these findings may merely be suggestive of the absence of coupon effects resulting from inefficient taxation of bonds with differing maturities, they are compatible with the increasing role of tax exempt institutions in the international capital markets. If the marginal bond investor is tax exempt, at the margin the tax advantage to debt would be significantly higher than that suggested by observing personal tax rates.

2.3 Effective tax measures and company behaviour

Another important deficiency of the King-Fullerton approach is that the effective tax rate measures cannot be related to the “actual” measurable average tax burden displayed by companies. More significantly the KF measures cannot be easily related to the characteristics of companies that can be retrieved from financial statements (leverage, historic volatility of earnings, etc.) or that are implicit in share prices or bond yields (expected growth in earnings per share, “ β ”, price/earnings ratios etc.).

Moreover, the concept of marginal refers to the additional tax paid on an additional unit of investment. There is a unique marginal tax for each investment/financing/ownership combination. This type of marginal tax rate measure does not lend itself to easy empirical application since neither aggregate nor company data can be used to obtain data that can be easily correlated with measurable variables such as those present in company accounts.

As will be shown below, we believe that there is a more meaningful measure of “marginal tax rate” that is closer in line with the notion of marginal cost. Indeed this different notion of effective marginal tax rate *can* be related to a measurable metric of average effective tax rates. It also has the characteristic of being an endogenous variable that is affected by corporate decisions.

Finally, as has been noted repeatedly in the past, most measures of marginal effective tax rates assume that the tax code remains unchanged forever.

3. The model

3.1 *Antecedents*

Our model represents a synthesis of various ideas which have until now been considered separately.

We view governments as possessing a stake - through the corporate and personal taxes - in the future earnings generated by companies: the present value of this stream of tax payments can be viewed as the government's share in the company (Meade, 1978). The composition of government revenues will depend on companies' financial policy. Within such a framework taxes on corporate cash flows can be viewed as claims on corporate wealth, in the same manner as equity and debt.

The second strand of literature relates to the longstanding debate regarding the influence of corporate and personal taxation on the financing decision of companies (Auerbach (2001) and Graham (2003)). From a theoretical and empirical standpoint this literature has not arrived at a full consensus but there appears to be a broad agreement that existing tax laws favor debt over equity unless interest is not a fully deductible expense. The latter might result from the existence of competing non-debt tax shields, the expectation of tax losses resulting from the uncertain future stream of profits or outright corporate insolvency. Our model allows for these asymmetries that reduce the tax benefits of debt and allows for the borrowing rate of companies to vary with the riskiness of projects and the financial policy¹.

The third strand of literature, based on the Black and Scholes (1973) approach as extended by Merton (1974) and Brennan and Schwartz (1978), views corporate liabilities – both bonds and taxes – as claims on company cash flows with different types of payoffs². This approach allows us to model the effects of the volatility of earnings on the risk structure of interest rates, and the impact of bankruptcy costs on the distribution of returns to shareholders and bondholders. It also allows us to model the asymmetric character of tax payments induced by the existence of imperfect tax carryforward provisions. Being able to establish a link between effective tax rates and measurable variables (such as volatility of earnings, debt/equity ratios and bond yields) has two consequences: (a) effective tax rates can be measured directly from company data; (b) cross-sectional variation in effective tax rates may be then correlated to the underlying determinants of security prices.

Finally our approach is related to the literature on project valuation in the presence of uncertainty where investment decisions are partly irreversible (Majd and Myers, 1985 and 1987; Dixit and Pindyck, 1994; Abel and Eberly, 2002). This approach allows us to extend the risk framework to examine the implications of technical uncertainty, increasing returns and market power on the incidence of taxes. It also gives operational meaning to the source of “economic rents” and to the effects of taxes for rents arising from different competitive advantages. While we do not explicitly consider these factors in this paper, in section 4 we discuss how these considerations could be addressed in future extensions of the model.

¹ This gives rise to the result that all companies with an internal optimum financing face the same cost of capital irrespective of the riskiness of their projects (Mayer, 1986).

² Schnabel and Roumi (1990) view the government tax claim as a combination of a call and a put option written on the firm's pretax value. Majd and Myers (1985), van Wijnbergen and Estache (1999) and Draaisma and Gordon (1996) discuss alternative option based approaches to interpreting tax liabilities.

3.2 Effective tax rates and financial policy

Modigliani and Miller (1958) (M-M) showed that under a number of circumstances it is possible to separate the financial and real decisions of firms. In particular, they argued that in the absence of taxation the cost of capital would be independent of financial policy. This same finding could also be interpreted as stating that the optimal financial policy was indeterminate. The introduction of corporate taxes in this framework (Modigliani-Miller 1963) leads to a diametrically opposite conclusion, namely that a company should maximize its leverage. Much of the academic effort since then has focused on identifying factors conducive to the M-M indeterminacy result or to an optimal debt-equity ratio. These approaches share one common feature: in equilibrium the *expected* tax rates on interest payments, retained earnings or dividends should be equal.

The basic differences underlying these varying approaches are most easily understood in the simple case where a mature firm can either finance its investments via retained earnings or debt. The firm's opportunity cost of funds for financing the marginal investment out of retained earnings is given by $r/(1-z)$, where r is the required rate of return for the investor and z is the accruals equivalent capital gains tax. In the case of debt finance, the net cost of borrowing is $C(1-\tau)$ where C is the marginal cost of issuing debt (the coupon rate on a consol) and τ is the statutory corporate tax rate.

A firm is indifferent between these two forms of finance if

$$r = C(1-\tau)(1-z) \quad (1.)$$

For a single, representative investor to be indifferent between debt and equity, it must also be the case that

$$r = C(1-m_B) \quad (2.)$$

where m_B is the marginal tax rate on borrowing.

In an internal equilibrium with both debt and equity financing it follows that

$$(1-m_B) = (1-z)(1-\tau) \quad (3.)$$

The literature has focused on various ways in which this "equilibrium" with both debt and equity issuance might occur. Initially, much attention was focused on extreme "corner" financial policies induced by the structure of corporate and personal taxes and the arbitrage possibilities opened up by differences in tax rates. Many considered it unlikely that such an equilibrium might be achieved unless legal constraints were imposed on extreme financial policies. The nature of these legal constraints (such as "thin capitalization rules") was explored at length in King (1977).

Another strand of literature focused on equilibrating market mechanisms that might limit "corner" financing. Miller (1977), for example, argued that in the United States owing to the progressive nature of personal tax rates and to low effective taxes on capital gains, after tax returns on debt and equity financed investments would be equal for some individuals ("the marginal investor"). Individuals with marginal tax rates below $m_B^* = 1 - (1-z)(1-\tau)$ would finance companies through debt whereas individuals with $m_B > m_B^*$ would supply equity. At the aggregate level leverage in the whole economy would be determined by the distribution of tax rates across individuals. At the individual company level, the debt-equity ratio would be indeterminate.

Another approach has been to focus on the non-linearity of the corporate tax schedule. A company may have insufficient earnings to be taxed (i.e. be taxed exhausted) as a result of low corporate cash flows or high allowable tax deductions (such as depreciation allowances and interest

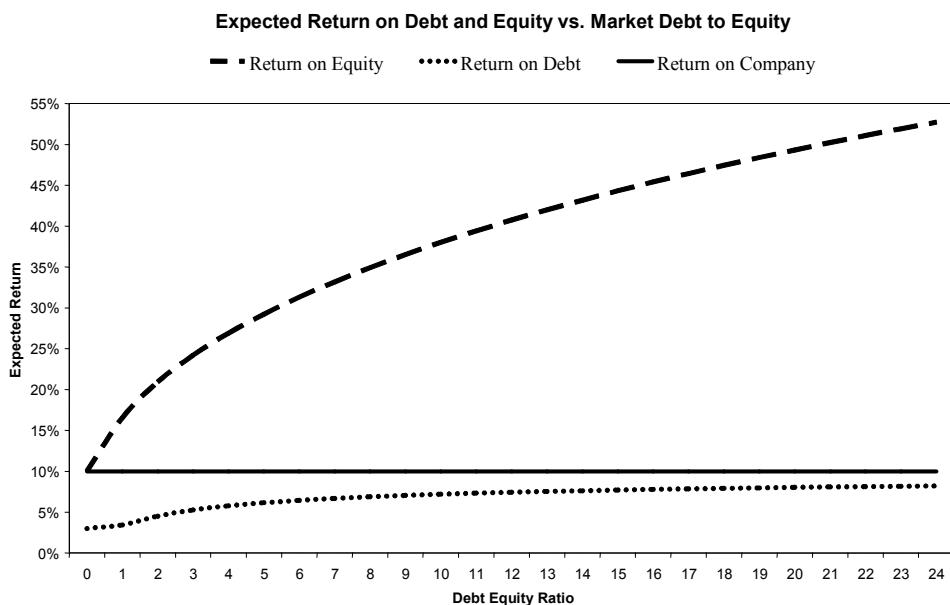
payments). Governments do not subsidize loss making companies through the tax system although limited tax loss carryforward provisions (and in some instances carryback provisions) are envisaged by most tax codes. As a result the "expected" corporate tax rate differs from the statutory tax rate. In the extreme case of the absence of carryforward or carryback provisions, the expected tax on corporate profits is given by

$$\bar{\tau} = \tau(1 - p_E) \quad (4.)$$

where p_E is the probability that the company will be tax exhausted. In tax exhausted states companies can no longer benefit fully of interest deductibility and consequently the tax advantage to leverage is considerably reduced. Since p_E will be a function of the debt-equity ratio, unless companies have extremely stable and high levels of cash flow there will be an endogenous limit to corporate indebtedness (Auerbach (1986), DeAngelo-Masulis (1980) and Mayer (1986)). Assuming that m_B is identical across individuals the equilibrium debt-equity ratio will vary among companies depending on p_E . In contrast to the Miller equilibrium each firm will face a specific optimal debt-equity ratio³.

The loss of tax shields is not equivalent to insolvency and the consequent restructuring of claims on companies. There are two implications for the cost of capital from the risk of insolvency even in the absence of external costs associated with bankruptcy. The first is that the interest rate on debt becomes a function of the debt-equity ratio. As leverage increases, the return to bondholders approximates increasingly the return on the risky cash flows of the company (Figure 1).

Figure 1



Second, the tax rate on debt issued by the firm is no longer given by m_B . The expected value of the personal tax rate on debt becomes

$$1 - \bar{m}_B = (1 - m_B) - p_B [(1 - m_B) - (1 - z)(1 - \tau(1 - p_E))] \quad (5.)$$

³ In the Miller equilibrium the aggregate supply of debt and equity is determined by the marginal investor but individual companies are indifferent between debt and equity. In the presence of a unique marginal tax rate on debt and the possibility of tax exhaustion each company will have an optimal debt-equity ratio but the aggregate debt/equity ratio will be undefined.

where p_B is the probability of default and the second half of the expression represents the reduction in after tax return to bondholders resulting from the increase in probability that the company will be bankrupt.

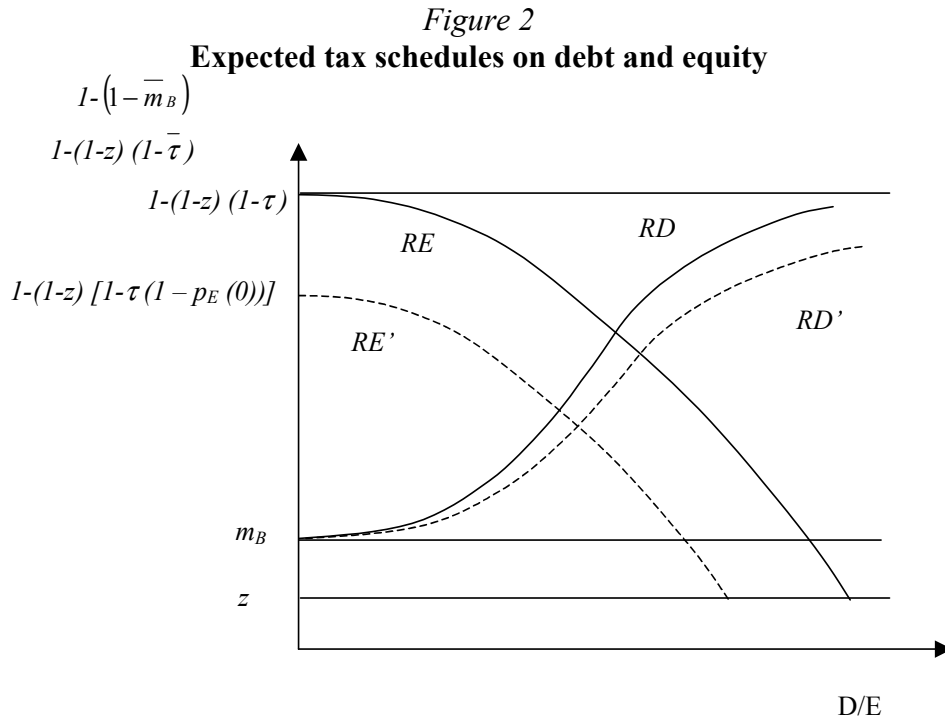


Figure 2 shows how the risk of tax exhaustion and the risk of insolvency interact to determine the company's optimal debt equity ratio. An increase in the debt/equity ratio raises both the probability of default and the probability that the company will be tax exhausted. An increase in leverage, for a given distribution of returns, decreases the expected tax on equity from a maximum of $1 - (1 - z)(1 - \tau)$ to a minimum of z along the tax schedule RE . However, as shown in Figure 2 even an unleveraged firm may start from a lower after-tax position if there are sufficient non-debt tax shields such that $p_E(0) > 0$. This case is represented by the schedule RE' which lies below RE . The slope of these curve will depend on the relationship between leverage and credit risk.

The expected tax rate on debt (RD) raises asymptotically from m_B to $1 - (1 - z)(1 - \tau)$. The slope of the curve at any point will depend on the functional relationship between the probability of default p_B and leverage. This will vary depending on the volatility of company earnings. RD' illustrates a case of lower volatility of earnings.

Figure 2 illustrates how non-debt tax shields and the volatility of earnings interact with one another; this subject has recently been the object of extensive empirical research (Graham, 2000; MacKie Mason, 1990). As we shall show below in our simulations it is possible that companies with equal debt-equity ratios may or may not have different effective tax rates depending on the nature of their earnings stream and the probability that they become tax-exhausted.

3.3 A multi-period model

We now consider a more complex multi-period model that allows for a wider array of tax interaction and dynamic of corporate financial policies. The principal assumption of this model (which is described in greater detail in Appendix 1) is that corporate earnings before interest and

taxes, EBIT, are determined exogenously⁴. In other words that taxation and leverage have no impact on the returns to investment⁵: the value of the firm is neither created nor destroyed by changes in leverage. This assumption has intuitive appeal since claims to future EBIT should be relatively insensitive to changes in capital structure. It is also reminiscent of the so-called fixed-p approach adopted by KF.

This assumption entails that in a contingent claims framework it is possible to attach a valuation to the payments to government (G) in the same fashion as the claims to cash flow payments to shareholders (E) and bondholders (D). Valuations of the claims to cash flows occur in two states of the world. In the first, the company is solvent and its total value is given by V_{solv} . Government holds claims to company profits and interest payments. In the second state, when the company is in default its value is given by V_{Def} . The government holds claims on the residual cash flows accruing to bondholders after paying for any bankruptcy costs.

The value of the various claims of government divided by the present value of cash flows produced by the firm, G/V , represents a composite (wealth tax equivalent) measure of effective tax burden. Under the assumption that the company will operate in perpetuity this is also a measure of the annualized tax burden on the pre-tax cash flows generated by the company. In other words G/V can be viewed as akin to the fixed-p measure of effective tax rates adopted in the KF framework.

As already noted, the essential element behind the approach taken in this paper is to recognize that there are three classes of claims on the value of a firm: those of shareholders, of bondholders and of the government. The division of EBIT between these three classes of claims at any moment in time will depend on the solvency or bankruptcy status of the company. Let V_B be the value of the company at which shareholders decide to default on their debt obligations and hence bankrupt the firm⁶. It is possible to show (see technical appendix) that given a total value of the firm of V , the claim to payouts above the bankruptcy level will be given by:

$$V_{solv} = V - V_B p_B(V) \quad (6.)$$

where $p_B(V)$.

We can now turn to the valuation of the three types of claims to EBIT.

A. Debt

The value of a consol when the firm is solvent is given by

$$V_{int} = \frac{C}{r} [1 - p_B(V)] \quad (7.)$$

In other words the market value of the future stream of debt so long as the firm is solvent is given by the value of future coupon payments discounted at the riskless rate times the probability that the firm will not go into default. The value of this stream of payments to bondholders D_{int} will be net of any payments of personal tax at a rate m_b on the stream of coupons, i.e.

$$D_{int} = (1 - m_b) \frac{C}{r} [1 - p_B(V)] \quad (8.)$$

⁴ We assume for simplicity that investment projects have an infinite life and that depreciation is nil.

⁵ This assumption can be taken as valid assuming that the degree of riskiness of projects is taken a priori at the company level.

⁶ We exclude debt re-negotiation as an option.

Following default, after paying any obligations to government and bankruptcy costs to third parties the residual value of the company $V_{def} = V_B p_B(V)$ reverts to bondholders. Consequently the value of the company to the bondholders in these circumstances is given by

$$D_{def} = (1 - \alpha) \frac{(1 - m_s)\theta(1 - \tau)}{(1 - z)} V_B p_B(V) \quad (9.)$$

where α is the cost of bankruptcy which is taken to be proportional to the residual value of the company, θ is the opportunity cost of gross dividends in terms of retained earnings (King, 1977), m_s is the marginal personal tax rate on gross dividends, z is the accrual equivalent rate of capital gains taxes and τ is the rate of company taxes. In other words when the company becomes bankrupt, bondholders become the shareholders of the firm. Defining $\nabla = \frac{(1 - m_s)\theta}{(1 - z)}$ as the marginal value of dividends (Mayer, 1986) the value of debt outstanding will be given as

$$D = D_{int} + D_{def} = (1 - m_B) \frac{C}{r} [1 - p_B(V)] + (1 - \alpha) \nabla (1 - \tau) V_B p_B(V) \quad (10.)$$

Rearranging terms the value of bondholders claims on EBIT can be re-written as

$$D = (1 - m_B) \frac{C}{r} + (1 - m_B) \frac{C}{r} p_B(V) \left[1 - \frac{(1 - \alpha) \nabla (1 - \tau) V_B}{(1 - m_B) \frac{C}{r}} \right] \quad (11.)$$

This particular formulation of the value of company debt and in particular of D_{def} contains two important assumptions: (a) the cost of financial distress α is significant and (b) the benefits of debt tax shields are lost following bankruptcy and taxes on negative EBIT can be offset against other tax liabilities. As is well known (Graham, 2003), the ex post costs of bankruptcy appear to be relatively small and cannot account for the tax advantage to debt. The simulations reported below rely on fairly small costs of bankruptcy costs (a value of α less than 3%). The assumption that the interest shield is lost following bankruptcy depends on the nature of corporate restructuring following bankruptcy. Indeed, in the presence of a secondary market for distressed firms or recapitalisations it is arguable that tax shields are not lost. In the absence of empirical evidence on the prevalence of such transactions and of the residual value attached to interest deductions on restructured debt, we shall assume that D_{def} takes the form an equity claim.

B. Equity

Shareholders have a claim to payouts so long as the firm is not bankrupt, i.e. $V > V_B$. The value of the claims of shareholders on the value of the company under a risk neutral valuation will be given at time 0 by:

$$E = (1 - \tau) \nabla E_0^Q \int_0^{\tilde{T}} (\delta - C) e^{-r/(1-z)s} ds \quad (12.)$$

This simple formula does not capture fully the workings of the tax system since it assumes that a firm can obtain full loss offset, i.e. it is refunded tax when coupon payments C exceed the flow to EBIT. In practice, firms cannot offset losses perfectly and benefit only from carry-forward and carry-back provisions. As shown by Shevlin (1990) and Graham (1996) this gives rise to a tax rate which depends in a complex fashion from the forecast of the future stream of earnings. The

closed form solution adopted by Goldstein, Ju and Leland (2001) does not allow for a solution to this problem. In the empirical stimations reported in section 3 below we allow for the asymmetric treatment of tax losses.

Another important assumption in the Goldstein, Ju and Leland approach is the absence of share buybacks as a manner of distributing EBIT to shareholders. Where such buybacks are explicitly possible (without limitation) or can be carried out through mergers they provide a tax minimizing strategy relative to dividend distributions (Green and Hollifield, 2003).

C. Government claims and effective tax rates

One special case to which we shall return below is where $\alpha = 0$ and $z=0$. In this case the value of the government claim is equal to:

$$G = G_{solv} + G_{def} = \Gamma(V_{solv}-V_{int}) + m_B V_{int} + \Gamma V_{def} \quad (13.)$$

where $\Gamma = 1 - (1 - m_s)\theta(1 - \tau)$. Since $V_{def} = V_B p_B(V)$, we may write:

$$G = \Gamma V - (\Gamma - m_B)V_{int}. \quad (14.)$$

However, the government cannot exercise the option of closing down the firm and retrieving and its part of the liquidation value of the firm. Unlike a debtor's claim the government's claim on the deferred tax liability can only be clawed back from future profits or if the company goes into liquidation and the liquidation value of the firm exceeds that of the bondholders' claim.

Table 1 reports the distribution of claims across various types of claims to EBIT.

Table 1: Distribution of claims on company EBIT

	Status of the company	
	Solvency	Bankruptcy
Equity	$E_{solv} = (1 - \tau^*)(V_{solv} - V_{int})$	-
Bondholders	$D_{solv} = (1 - m_B)V_{int}$	$D_{def} = (1 - \alpha)(1 - \tau^*)V_{def}$
Government	$G_{solv} = \tau^*(V_{solv} - V_{int}) + m_B V_{int}$	$G_{def} = (1 - \alpha)\tau^*V_{def}$
Bankruptcy cost		$BC_{def} = \alpha V_{def}$

D. Some simple examples

A simple example can illustrate the impact of various assumptions regarding the effects of taxation in this simple stochastic framework with only one dynamic element affecting the firm. Our initial scenario is based on the Italian tax system prior to the Tax Reform of 1998⁷. The parameters for the tax variables are taken as $z = 0$, $m_B = 0.125$, $\tau = 0.532$, $\theta(1 - m_s) = 0.875$ and consequently $\Gamma = 0.5905$. The other base case parameter values are $r = 0.045$, $\sigma = 0.25$, $\alpha = 0$ and $\delta = 0.07$. We assume that the tax system is symmetric with perfect loss offsets.

⁷ We focus on the Italian tax system for two reasons. First, it offers a good example of very significant changes in the level of statutory tax rates which one would expect to result in major changes in effective tax rates. Secondly, the so-called "dual income tax" introduced in 1998 was explicitly meant to redress the existing incentives to debt finance. This change would be expected to change the average effective level of taxes but also, significantly, the marginal tax advantage to debt for all levels of the debt-equity ratio.

Figure 3

Effective tax rates - Baseline case - $s = 0,25$

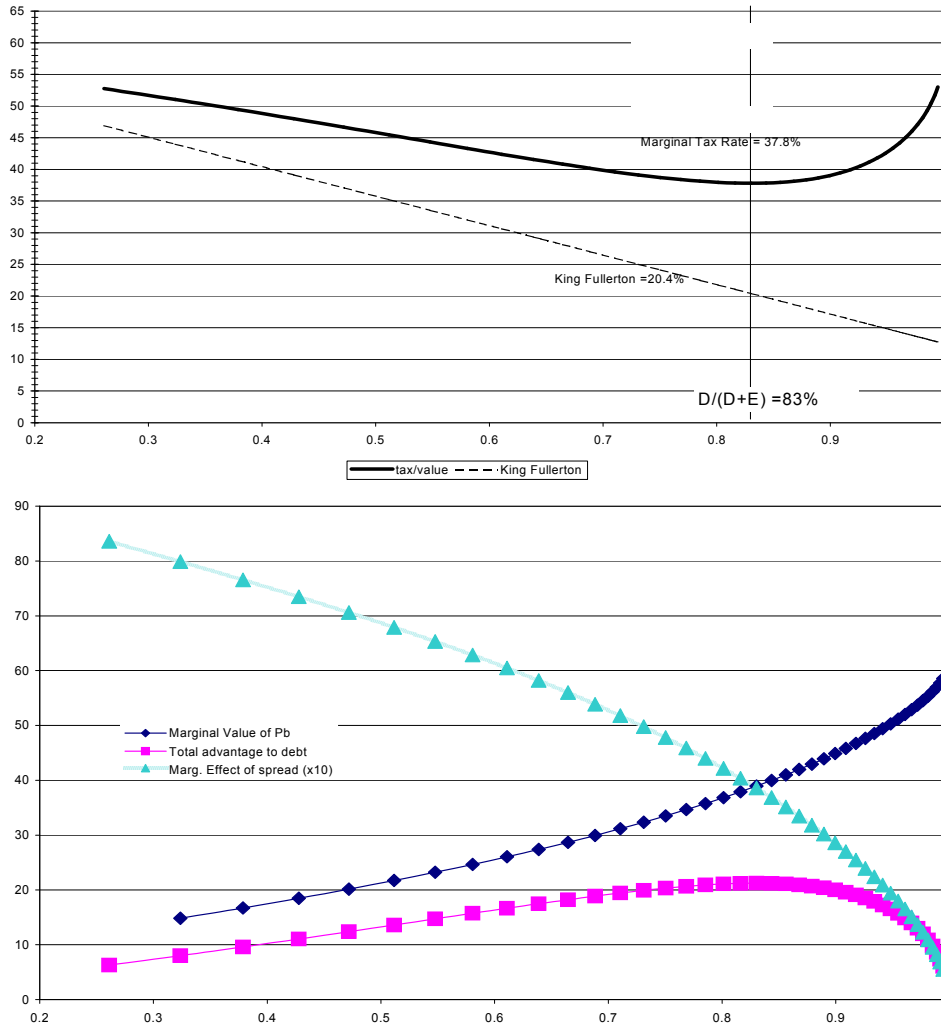


Figure 3 plots the value of the effective tax rates for different values of $D/D+E$. The KF measure is weighted average of the tax rates on debt and equity respectively. It has a maximum given by 0.5905 where the firm finances its marginal investment via equity and a minimum of 0.125 where the firm is fully financed by debt. The marginal advantage to debt issuance in this case is given by 0.4655 ($=0.5905-0.125$). In other words the value of private claims on EBIT ($D+E$) increases by 0.4655 per unit increase of debt.

The alternative measure of effective tax rates G/V is convex and achieves a minimum of 37.8% at a $D/(D+E) = 83\%$. In a general sense the optimum leverage ratio is obtained by maximizing the value of the claims to EBIT of shareholders and bondholders. However, in this specific example where $\alpha = 0$ the optimum leverage ratio can be viewed more simply as minimizing the effective tax rate or, alternatively as maximizing the tax advantage to debt $(I - m_B) V_{int}$. This maximization is the net outcome of two offsetting elements: the increase in the probability of a firm going bankrupt, which lowers the tax advantage to debt, and the increase in the credit spread, which raises the tax advantage to debt. By substituting for V_{int} the tax advantage to debt can be written as

$$(I - m_B) \cdot (1 - p_B(V)) \cdot (C/r). \quad (15.)$$

The optimal coupon⁸ satisfies the following first order condition:

$$[(1 - m_B) C / r] * \partial p_B / \partial C = (1 - p_B) (1 - m_B) / r. \quad (16.)$$

The left hand side represents the effect on the tax advantage to debt of a marginal increase in the probability of going bankrupt, the right hand side is the effect on the tax advantage to debt of a marginal increase in the credit spread.

The curves for these two offsetting factors are plotted in the lower part of the graph. Their intersection where the marginal increase in expected taxes resulting from an increase in P_b equals the marginal tax saving resulting from the widening credit spread is the optimum leverage ratio.

Table 2 presents some comparative statics. The table highlights three major features. First the tax advantage to debt is highly sensitive to the rates of tax but somewhat more to personal tax rates than τ . However, the exact sensitivity depends on the value taken by α . Second, the optimal leverage ratio is not highly sensitive to the volatility of EBIT. This is not so surprising since the tax system is assumed to be perfectly symmetric with full loss offsets. Finally, it should be noted that credit spreads are sensitive to both volatility and tax rates.

Table 2: Sensitivity of effective tax rates to select parameters (in percentages)

	Average tax rate	$G_{\text{solv}}/V_{\text{solv}}$	$G_{\text{def}}/V_{\text{def}}$	Tax advantage to debt	Optimal Leverage Ratio
Base	37.81	59.05	31.08	21.24	83.03
$\sigma = 0.005$	29.53	59.05	27.79	29.52	87.74
$\sigma = 0.45$	39.94	59.05	31.63	19.11	82.09
$\alpha = 0.03$	36.97	56.10	31.61	21.27	82.07
$\alpha = 0.10$	36.24	53.15	32.13	21.27	81.12
$\tau = 0.35$	29.15	43.13	24.72	13.97	79.32
$\tau = 0.35; m_b = 0.35$	39.42	43.13	38.24	3.71	75.81
$\tau = 0.35; \alpha = 0.10$	28.15	38.81	26.14	13.95	74.81
$\tau = 0.35; m_b = 0.35; \alpha = 0.10$	39.00	38.81	39.45	3.48	59.68

4. Montecarlo simulations

4.1 The least squares Montecarlo approach

The model of the previous sections made a number of simplifying assumptions in order to obtain a simple analytical solution. Tax policy, however, places a far greater number of discontinuities than those highlighted by expression. In particular carry-forward provisions with expiration dates are available in the presence of tax losses. Such provisions preclude full analytical solutions such as those of the previous section. In order to assess tax burdens, it is necessary to make use of simulation techniques under different assumptions regarding the level and variability of earnings. In this section we report the results of carrying out Montecarlo simulations for the Italian tax system extending the results reported in Alworth and Arachi (2001) and Alworth and Lovisolo (1998).

⁸ Expression (16) is similar to (5) but with two significant differences. First, in expression (16) the tax system is assumed to be symmetric and hence $p_E = 0$. Second, in (16) debt finance is compared to a new share issue, whereas previously in (5) the comparison was with retained earnings.

The existence of imperfect loss offsets and carryforward provisions pose many of the same problems faced in approximating the value of American style path-dependent options. Longstaff and Schwartz (2001) have recently proposed a very flexible approach to solving these types of problem that allows for a number of extensions discussed in the next sections⁹. Montecarlo simulations allow state variables (in our instance EBIT) to follow very general stochastic processes. In the framework of valuing the claims to EBIT the shareholder must compare at each instant in time the value of keeping the firm alive versus closing it down. This is similar to the American put option strategy of comparing the payoff from immediate exercise with the expected payoff from continuation. The conditional expectation of the value of the firm can be estimated by OLS using a cross section of the simulated values of EBIT. By estimating the conditional expectation of the value of the firm in time $t+1$ it is possible to determine the value of E at time t . The value of E at time 0 is then determined recursively.

4.2 An evaluation of the Italian Tax System

A. The Italian Tax System prior to the 1998 tax reform: an overview

Corporate taxation and the personal taxation of capital income have undergone numerous changes in recent years. Prior to these changes, corporate profits were taxed at a rate of 37% under the company income tax (IRPEG) and a 16.2% non-deductible surcharge (ILOR). In the case of IRPEG losses could be carried forward against future profits for five years; whereas no carryforward provisions were envisioned for ILOR.

At the personal tax level, dividends could benefit from an imputation system; however, individuals could opt to be taxed at flat rate tax of 12.5% (but not benefit from the imputation credit). Capital gains on the appreciation of shares of Italian companies were exempt from tax. Interest income was subject to a wide range of withholding tax depending on the type of financial instrument. Over time there was a gradual reduction in the number of tax rates with the most important rates being 27% for bank-related financial instruments and 12.5% for government bonds. In our simulations we shall assume that interest income was subject to a tax of 12.5%, dividends were taxed at a flat rate of 12.5% (without imputation credits) and capital gains were tax exempt.

B. Financial policy and effective tax rates prior to the 1998 tax reform

The first part of table 3 reports the results of Montecarlo simulations for the Italian tax system prior to the 1998 tax reform. The first row contains the values of the average tax rate, the tax advantage to debt and the optimal leverage ratio calculated from the analytical model using the Italian tax rates and assuming a symmetric tax system. The second row reports estimates based on the Longstaff and Schwartz (2001) approach under similar assumptions. The simulated results display a higher optimal leverage ratio and tax advantage to debt. As a consequence there appears to be a downward bias in the estimation of the average tax rate.

The remaining rows contain the estimates obtained using the actual Italian loss carryforward provisions. As expected, the tax asymmetries reduce the tax advantage to debt and increase the effective tax rate. The tax increase is not evenly distributed across firms. In particular the system entails a heavy burden on firms with a highly volatile EBIT as they have a higher probability of incurring losses.

⁹ The details of the technique utilised by Longstaff and Schwartz are described at length in Appendix 2.

The relationship between EBIT volatility and optimal leverage ratio appears highly non-linear. The optimal leverage ratio declines sharply when the standard deviation rises from 0.05 to 0.25 but rises only marginally when volatility is further increased to 0.45.

Surprisingly, the results do not appear to be very sensitive to the level of bankruptcy costs.

Table 3: An evaluation of the Italian Tax System

Prior to the 1998 tax reform			
	Average tax rate	Tax advantage to debt	Optimal Leverage Ratio
Base – Analytical solution	37.81%	21.24%	83.03%
Base – Simulated	34.05%	24.68%	88.74%
Symmetric tax system			
Base – Simulated	37.61%	21.25%	81.81%
Asymmetric tax system			
$\sigma = 0.05$	35.40%	23.31%	92.95%
$\sigma = 0.45$	55.35%	5.00%	82.47%
$\alpha = 0.1$	37.56%	21.29%	81.46%
Dual taxation of corporate income and Irap			
	Average tax rate	Tax advantage to debt	Optimal Leverage Ratio
Base	29.72%	13.31%	79.57%
$\sigma = 0.05$	32.90%	9.38%	96.20%
$\sigma = 0.45$	41.39%	3.24%	79.22%
Base no Irap	24.82%	14.05%	78.31%

4.3 *The impact of the Italian dual income tax (DIT)*

The Tax Reform of 1998 resulted in two changes¹⁰. First, ILOR was abolished and IRAP, a new regional tax on value added (measured as wages plus EBIT less depreciation) at a rate of 4.25% was introduced in its place. Secondly, in order to reduce the tax cost of equity the corporate tax was amended. Profits were split into two components. One component was categorised as “ordinary income”, the opportunity cost of new equity financing, and taxed at a rate of 19%. “Ordinary income” was computed by multiplying the interest rate on long-term government bonds plus a measure of the equity risk premium times the value of new share issues and retained earnings¹¹. Another element of the tax base was “extra normal profits” measured as the difference total profits and “ordinary income”. This second component was taxed at the IRPEG rate of 37%.

The new tax system had two main sources of asymmetries. Corporate losses could be carried forward for five years. The same provision applies to the computed “Ordinary income” exceeding actual profits.

The results of Montecarlo simulations, reported in the second part of table 3, show that the new system succeeds in reducing the tax advantage to debt and effective tax rates. Yet the impact on financing decisions seems rather limited: optimal leverage ratios are in general marginally lower

¹⁰ For a description of the Italian tax reform, see Bordinon, Giannini and Panteghini (2001)

¹¹ The “new equity financing” was defined from the date of introduction of this new “dual income tax regime”. Over time the opportunity cost would cover the majority of equity. In 1999, given the slow take up of the new regime the definition of equity to which the new regime applied was considerably widened in order to speed up the transition to the new regime. Partnerships and individual firms could apply the deduction to the entire stock of equity. Corporations were allowed to deduct multiples of new equity (1.2 in 2000 and 1.4 from 2001 onwards).

than in the pre-1998 situation. The data also show that more volatile firms continue to bear a higher effective tax burden.

Surprisingly, when EBIT volatility is very low, the total tax advantage to debt sharply falls and the effective tax rate remains quite high. At the same time the optimal leverage ratio increases with respect to the value estimated in the pre-1998 scenario.

The last row in the table allow to appreciate the difference between the corporate tax and IRAP. As expected, IRAP seems to be neutral with respect to financing decisions: the optimal leverage when only the corporate income tax is considered is very close to the optimal leverage in presence of IRAP. It is interesting to see that the tax advantage to debt decreases (albeit slightly) when IRAP is taken into account. This may be due to the fact that the presence of this tax increases the probability of default.

5. Extensions of the model

The model described in the previous two sections assumes that only one state variable (EBIT) determines the evolution of valuations and of tax burdens over time. There are basically two possible types of extension that can be easily incorporated in this framework. The first consist in adding a number of state variables as potential determinants of valuations. The second extension entails looking at more closely at the underlying determinants of EBIT. The model can also be used as a basis for constructing measures of effective tax rates based on company balance sheet and other financial data.

5.1 Multiple state variables

The general Least Squares Montecarlo algorithm developed by Longstaff and Schwartz (2001) allows for higher dimensional problems with a multiplicity of state variables. In practice this entails that estimates of the conditional expectations of the state variables in the simulations should include terms in all of the state variables considered as well as their cross products. In other words, the simulations need to take account of the co-variation between the state variables. This suggests that higher dimensional problems need to be treated carefully in order to avoid that the determination of conditional expectations may become unduly complicated¹².

Two types of state variables spring to mind as potential candidates for further elaboration. The first consist of exogenous macroeconomic variables which might impact on the determinants of valuations such as those considered by standard multifactor APT models: nominal interest rates, inflation, industrial production and commodity prices. The second type of state variables which can be considered in this framework relate to policy uncertainty.

A. Stochastic nominal interest rates and inflation

With stochastic interest rates even the simplest paths of cash flow generate an option value. In the particular context described in the previous sections, interest rate uncertainty creates a timing option in respect of the decision to default. As a result shifts in the term structure and the existence of term premia may influence the levels of effective tax rates and the debt-equity decision¹³.

¹² However, Longstaff and Schwartz (2001) suggest that these complications are quite manageable.

¹³ Stochastic interest rates may also capture cyclical effects on effective tax rates.

B. Uncertain tax policy

It is well known that the effects of changes in tax policy can differ quite considerably depending on the extent to which they are anticipated. A number of articles have addressed these issues within perfect foresight or rational expectations models (Nickell, 1978; Lucas, 1976). The simulation approach taken in this paper allows a number of different features of the tax system to be examined that have remained until now outside the purview of research. For example, it is possible to examine the impact of changes in loss offset provisions. However, the most important additional features that can be modelled by the options approach concern uncertainty regarding the magnitude and timing of changes in tax policy.

5.2 Effective tax rates and the investment decision

The model described in the previous section relies on considering the path of EBIT as determined by a dynamic stochastic process. The evolution of EBIT itself is determined by a number of underlying forces that can in turn be modeled as stochastic processes.

Another aspect of tax policy connected to investment decisions that can be modeled through this approach is the impact of depreciation allowances and other forms of non-debt tax shields. This has two important for effective tax rates. First it allows the computation of effective tax rates that are directly comparable to those of KF. Secondly, the approach taken in this paper allows a direct assessment of the impact of the existence of non-debt tax shields on the debt-equity decision in an optimising framework¹⁴. It is possible to disentangle the effects of high depreciation allowances resulting from investments from those used to lower taxes.

6. Commentary

6.1 Relationship to other measures of effective tax rates

The EBIT model presented above has several interesting additional features. First the interpretation of “effective tax rates” is very simple in spite of its forward-looking character. Average effective tax rates represent the present value of the claims of government on future cash flows. Marginal effective tax rates represent the additional tax (or tax saving) that is obtained from an increase in leverage. These are concepts that have an immediate intuitive appeal to practitioners. Second, the measures of effective tax rates are based largely on observable variables. The only unobservable unknowns concern the volatility of EBIT and the maturity of debt. However, the Merton model which views the value of equity as a call option on firm’s assets from implies that in the absence of taxes if the volatility of share prices (σ_E) is known the volatility of the assets of the firm s will be given by

$$\sigma_E = \sigma N(d)_1 / [1 - (D / D + E)] \quad (17.)$$

where $N(\cdot)$ is the cumulative normal distribution and d_1 is given by

$$d_1 = \left[-\ln(D / D + E) - (y - \sigma^2)T \right] / \sigma \sqrt{T} . \quad (18.)$$

¹⁴ Graham (1996) and Alworth and Arachi (2001) find that NDTs do not affect the leverage decision of companies contrary to the conjecture of DeAngelo and Masulis (1980). See Graham (2003) for a further discussion.

Since D , E , the yield spread (y) and σ_E are observable it is possible to obtain an estimate of the volatility of σ .

6.2 Limitations

There are many implicit assumptions underlying the model utilised in this paper that would be difficult to accept in a fully optimising framework. We assume, for example, that once the company becomes insolvent it will continue to operate indefinitely with ownership being transferred to bondholders. If there were a secondary market for capital goods closure, sale of the underlying assets could be a more plausible alternative. However, under these circumstances the advantages of interest rate deductibility would be lost. The sale of an insolvent company to take possible advantage of “tax synergies” (i.e. the availability of unused interest rate deductions) would be under many circumstances an even better outcome. The value of V_{def} in both instances would be very different from that assumed in this paper.

Another assumption driving the results of this paper is that firms do not exploit the option to readjust leverage in response to stochastic changes in firm value. For example if EBIT were to increase unexpectedly the firm may find it optimal to issue more debt. This option has implications for credit spreads, equity valuations and effective tax rates (Dangl and Zechner, 2001; Goldstein, Ju and Leland, 2001).

7. Conclusions

This paper has proposed an integrated framework for examining the impact of risk and financial decisions on effective tax rates. The new framework is based on the modern theory of corporate finance and allows the endogenous determination of the optimal debt/equity ratio and of the resulting effective tax rates. The paper shows how this approach can cope with the complexity of real world tax systems through Montecarlo simulations. By applying the new methodology to the Italian tax system prior and post the 1998 tax reform, the paper provides an evaluation of the impact of various tax asymmetries (such as loss carryforward provisions and allowances for corporate equity) on effective tax rates and optimal leverage ratios.

One of the principal findings of this paper is that the tax burden faced by companies over some ranges of the debt equity ratio are relatively invariant to leverage.. This occurs because the marginal benefit to issuing debt varies with the debt-equity ratio unlike the traditional King-Fullerton assumption that the marginal benefit to issuing debt is a constant. The King-Fullerton framework does not take into account the loss of the tax shield for deducting interest as a factor in reducing the marginal benefit to debt and the change in the nature of the tax burden faced by debt holders when companies become insolvent.

Technical appendix

A1 A simple model of firm dynamics in a stochastic framework

Following Goldstein, Ju and Leland (2001) the company EBIT, that represents the total payout δ flow to shareholders, bondholders and the tax authorities, can be specified by the process:

$$\frac{d\delta}{\delta} = \mu_p dt + \sigma dz \quad (A1)$$

where μ_p and σ are constants. There is no inflation and only one representative investor. If the risk and return characteristics of EBIT can be replicated using traded assets of known value, firm's assets can be priced by discounting expected cash flows under the risk-neutral measure. Hence the value of the entire payout flow is

$$V(t) = E_t^Q \left(\int_t^{\infty} \delta_s e^{-r(s-t)} ds \right) = \frac{\delta_t}{r - \mu} \quad (A2)$$

where $\mu = \mu_p - \phi\sigma$ is the risk-neutral drift of the payout flow, ϕ is the risk premium and r is the nominal interest rate. Since r and μ_p are constants, the dynamics of V and δ are given by

$$\frac{dV + \delta dt}{V} = r dt + \sigma dz^Q \quad (A3)$$

In other words, the risk neutral expected return on the claim to a company's EBIT is the risk-free rate¹⁵.

As we have already mentioned there are three types of claims on firms: shareholders, bondholders and government. In general any of these claims must satisfy the partial differential equation

$$\mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + F_t + P = rF \quad (A4)$$

where P is the payout flow to the particular claim. If we assume for expository purposes that all debt issuance is in the form of perpetual bonds, all claims are time independent. This implies that $F_t = 0$, and the expression reduces to an ordinary differential equation:

$$\mu V F_V + \frac{\sigma^2}{2} V^2 F_{VV} + P - rF = 0 \quad (A5)$$

The general solution to this equation is given by

$$F_{GS}(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2} \quad (A6)$$

where A_1 and A_2 are constants to be determined by the boundary conditions which apply to each of the claims on various types of payout flows P and β_i are the roots of the quadratic equation

$$\frac{\sigma^2}{2} \beta(\beta-1) + \mu\beta - r = 0 \quad (A7)$$

The roots of this equation have opposite signs. As a consequence the term in (A6) with the positive root (say β_2) explodes as V becomes large. This would violate the boundary conditions for all claims of interest. Hence in the following we set $A_2 = 0$ and use as a general solution:

$$F_{GS}^R(V) = A_1 V^{\beta_1} \quad (A6')$$

The particular solution when the relevant cash flow is the entire EBIT (i.e. $P = \delta$) is:

¹⁵ See Dixit and Pindyck (1994), chap. 4 and Cochrane (2001) chap.18

$$F_{PS}^{\delta}(V) = V \quad (\text{A8})$$

while if the relevant cash flow is the coupon payment (i.e. $P = C$) the particular solution is:

$$F_{PS}^C(V) = \frac{C}{r} \quad (\text{A9})$$

As long as the firm is solvent the total value of shareholder, bondholders and government (V_{Solv}) is given by sum of the general solution (A6') and the particular solution (A8):

$$V_{Solv} = V + A_1 V^{\beta_1} \quad (\text{A10})$$

The value of this claim vanishes when the firm becomes insolvent. Let V_B be the value of the company at which shareholders decide to default on their debt obligations and hence bankrupt the firm. The boundary condition is:

$$0 = V_B + A_1 V_B^{\beta_1} \quad (\text{A11})$$

which implies:

$$A_1 = -V_B^{1-\beta_1}. \quad (\text{A12})$$

Substituting into (A10) and rearranging yields:

$$V_{Solv} = V - V_B p_B(V)$$

where $p_B(V) = \left(\frac{V}{V_B}\right)^{\beta_1}$ is the probability of reaching V_B .

When shareholders decide to default, the company is sold to competitors. The residual value, after paying any obligations to government and bankruptcy costs to third parties reverts to bondholders. The value of the claim on the default value (V_{def}) is therefore equal to the general solution (A6'). When $V=V_B$ the claim on default value is equal to V_B itself. This boundary condition is equivalent to (A11). By substituting (A12) into (A6') we get:

$$V_{def} = V_B p_B(V).$$

The value of the claim on coupon payments while the company is solvent is given by the sum of the general solution (A6') and the particular solution (A9):

$$V_{int} = \frac{C}{r} + A_1 V^{\beta_1}. \quad (\text{A13})$$

When the company goes bankrupt the value of this claim vanishes. This leads to the following boundary condition:

$$0 = \frac{C}{r} + A_1 V_B^{\beta_1} \quad (\text{A14})$$

which implies

$$A_1 = -\frac{C}{r} V_B^{-\beta_1}. \quad (\text{A15})$$

By substituting in (A13) and rearranging yields:

$$V_{int} = \frac{C}{r} [1 - p_B(V)]. \quad (A16)$$

A2 Montecarlo simulations

We approximated the EBIT dynamics

$$\frac{\delta dt}{\delta} = \mu dt + \sigma dz^Q \quad (A17)$$

by

$$\delta(t) - \delta(t-1) = \delta(t-1) (\mu + \sigma \varepsilon_i(t)) \quad (A18)$$

where $\mu = r - (\delta/V_0)$ and $\varepsilon_i(t)$ is drawn from a standardized normal distribution.

We simulated 1000 different paths for the company's EBIT up to $t = 20$. For each path, we first calculated taxes due and shareholders' cash flows going forward from period 1 to period 20, under the assumption that the company was solvent and that bondholders received the coupon payment C . In order to approximate the infinite horizon framework we assumed that shareholders will receive in perpetuity, starting from $t = 21$, the same cash flow they obtained at the end of the simulated path ($t = 20$).

We then proceeded backward to evaluate the shareholders' claim by discounting their future cash flows using the risk free interest rate r . This recursive evaluation proceeds until the beginning of the path is reached or a negative cash flow is encountered. In the latter case the shareholders must decide whether to keep the company alive or to close it down. The decision is taken by comparing the negative cash flow with the expected value of cash flows from continuation.

In order to estimate the conditional expectation of the value of future cash flows we followed the Longstaff and Schwartz (2001) approach. At each time the discounted future shareholders' cash flows are regressed on the first three powers of the present cash flows using OLS. The fitted value is an efficient unbiased estimate of the conditional expectation of the firm value. If this value exceeds the present negative cash flows, the firm continues to operate and the evaluation proceeds backwards. If not, the firm goes into default. In this case, all future cash flows to shareholders are set equal to zero. The backward evaluation of equity resumes from a value of zero.

This procedure yields 1000 different values of equity at time 0. The estimated value of equity is the mean over these simulations.

Once the bankruptcy time has been determined for each path, the estimates of G , V_{int} and V_{def} can be obtained by discounting future taxes, coupon payments and EBIT after default. These values, as well as E , depend on the level of the coupon payment C . By repeating the Montecarlo simulation for different values of C the minimum tax rate and the maximum tax advantage to debt is found through the Newton-Raphson method.

Figure 4

Tax/value and leverage

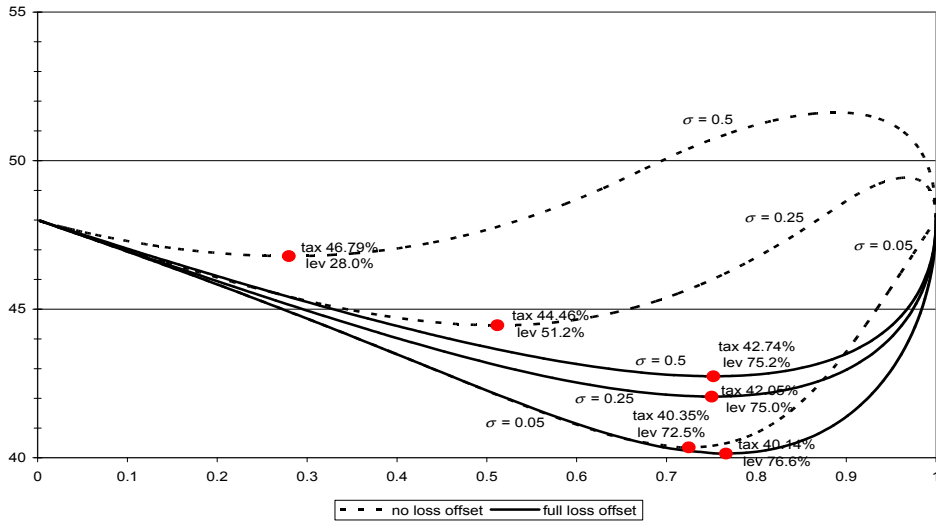


Figure 5

Tax/value and leverage open economy no personal taxes

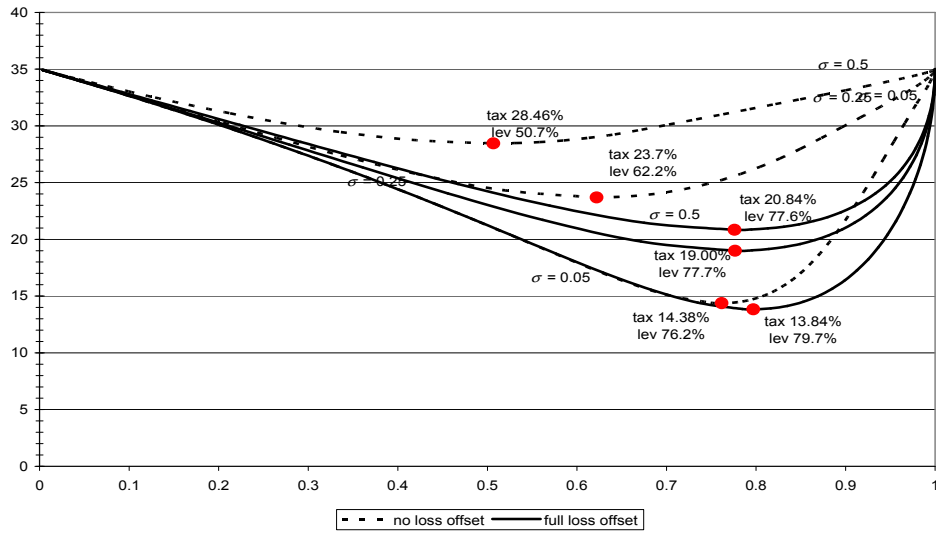
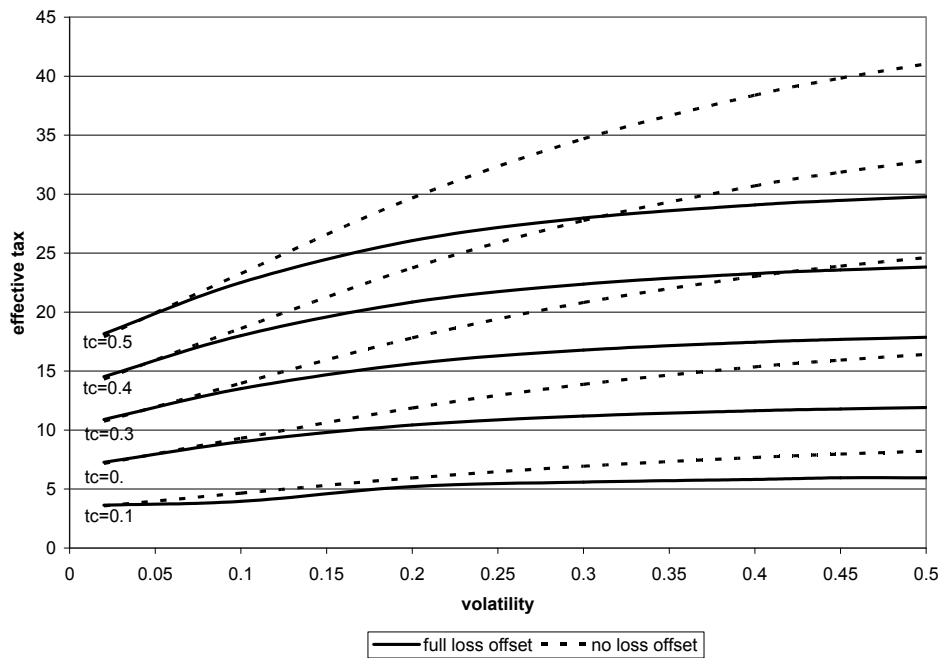


Figure 6

Effective tax and volatility



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